

**Distributed Coordination and Control –A bottom up design
framework for Highly Renewable Grids and Microgrids**

A THESIS

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Dedication

To my parents and my grandparents for their love and support throughout this journey.

Abstract

The modern power grid is undergoing an inflection point where the role of distributed energy resources (DERs) including renewable energy sources (RESs) has surpassed parity with conventional bulk power generation system as the prevalent source of energy generation. This transition from the conventional to modern power grids is also favored by economies of integrating distributed generation technologies, proliferation of intelligent communication devices such as sensors and data acquisition systems providing a global and persistent view into the state of the power system and, low footprint computational devices that have accelerated the distributed nature of the coordination of these DER units and if desired, to appear as a single aggregated unit to leverage the benefits of scale that a large centralized bulk power system can provide. Most of the design philosophy for the power-electronics interfaced DERs and highly renewable grid has been developed keeping in mind the operation of a conventional power system which is a large inertial system and highly centralized in nature. The existing grid needs to undergo transformations including control design methodologies keeping DERs and RESs at the focus and utilization of distributed communication algorithms at DERs allowing renewable energy resources and battery energy storage systems to participate in grid interactive services that can meet aggressive response timescales to maintain the operation and stability of the power system. This thesis makes several contributions, firstly towards enabling better active and harmonic power sharing in multi-inverter microgrid systems (MMGs) with DERs interfaced with low-inertia voltage source inverters (VSIs) while reducing the complexity of reactive power sharing and control. Secondly, investigations and contributions towards developing a novel aggregation strategy and implementation methodology for smaller DER units for enabling challenging grid interactive services in

presence of real-world non-idealities such as time delay in communication channel are also presented and validated by developing a novel end-to-end power-hardware-in-the-loop (PHIL) configuration. The finite-time termination distributed schemes suffer from vulnerability to cyber-attacks that are aimed at manipulating data and control flow. In the final part we develop a novel distributed method for detecting the presence of such intruders for a multi-agent system implementing ratio consensus protocol.

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Chapter 1

Introduction

The modern power system is undergoing a transformation where the power utilities and infrastructure across the world are trying to address challenges such as incorporating renewable generation, diversification of generation assets while reducing carbon footprint from conventional generation units. The conventional power system is lending its way from unidirectional flow of power from large centralized generation plants to multidirectional power flow with distributed generation (DG) units scattered throughout and interacting with the rest of the power grid as shown in Fig. 1.1.

This transition from the conventional power grids to modern power grids is also favored by new and cheaper distributed generation technologies, intelligent communication devices such as sensors and data acquisition systems providing a global and persistent view into the state of the power system and, low footprint computational devices that have accelerated the distributed nature of the coordination of these DG units and if desired, to appear as a single aggregated unit (referred to as a ‘*virtual power plant*’).

One of the natural extensions that has arisen as a result of the proliferation of the DG units or Distributed Energy Resources (DERs) is the capability to form microgrids. Multi-inverter microgrids (MMGs) are systems that allow interaction of various DERs

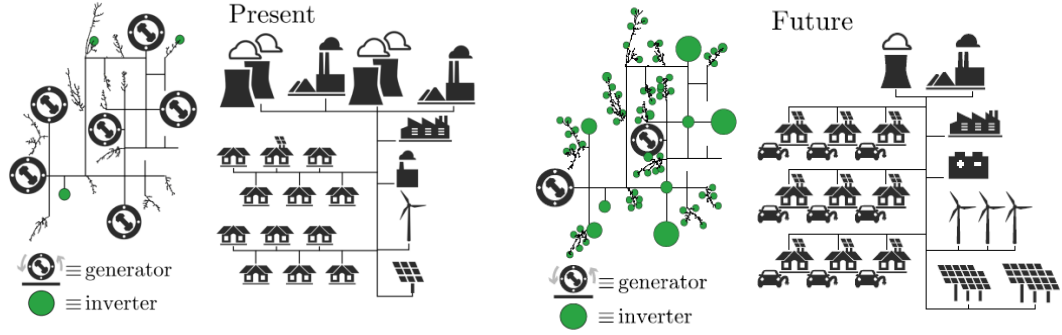


Figure 1.1: (a) Conventional power grid with large centralized generation and unidirectional flow of power; (b) A Modern power grid with Distributed Generation units interacting with the grid for a multi-directional flow of power

such as photovoltaics (PV) and battery energy storage systems connected to loads with DC/DC converters and DC/AC inverters (referred to as voltage source inverters (VSIs)) at the point of common coupling (PCC) as shown in Fig. 1.3. MMGs can be operated in both grid-connected as well as islanded modes.

DER interfaced grids and microgrids follow a hierarchical control structure in order to comply to design objectives and performance criteria, namely, fast timescale (millisecond) regulation of system voltage and frequency at the point of common coupling (PCC), tracking of active and reactive power set-points, restoration of system frequency following a disturbance in the system at sub-second to second timescales, power flow control at seconds-minutes timescales while maintaining economic dispatch of assets (minutes-days). This hierarchical control structure consists of four levels, namely the *Level 0* or *inner* control, *Level 1* or *primary* control, *Level 2* or *secondary* control, and *Level 3* or *tertiary* control. The order is provided in a bottom-up manner and each higher level commands the next immediate lower level and provides supervisory control. In order to maintain stability and robustness, it is necessary to ensure that the bandwidth is

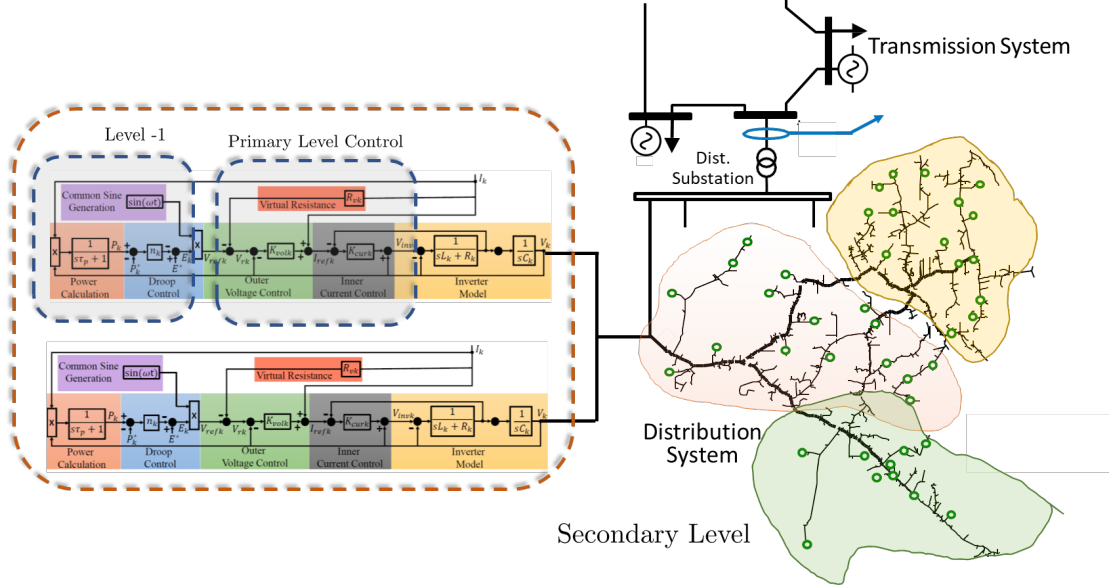


Figure 1.2: Layers of control in a Power system

decreased with an increase in the control level

The *Level 0* or *inner* control hierarchy consists of the voltage and current regulation control of each DERs. This control strategy employed to the DERs are developed based on the nature and dynamic behavior of the energy resources. The *Level 1* or *primary* control provides references for *grid-forming* VSIs and *grid-feeding* VSIs. Either *P-Q* reference for *grid-feeding* VSIs or voltage and frequency reference for *grid-forming* VSIs are provided in order to stabilize the network, enabling plug-and-play capability for DERs and maintaining proper active and reactive power share among DERs with reduced circulating currents. Level 2 or *secondary* control is realized with a centralized controller that restores the voltage and frequency of microgrid compensating the inevitable deviations caused by the *primary* control, especially droop-based control. Lastly, *Level 3* or *tertiary* control considers the economical concerns in the optimal operation of the microgrid, and manages power flow between microgrid and main grid.

In the islanded mode of operation, DERs (as grid-forming inverters) support local

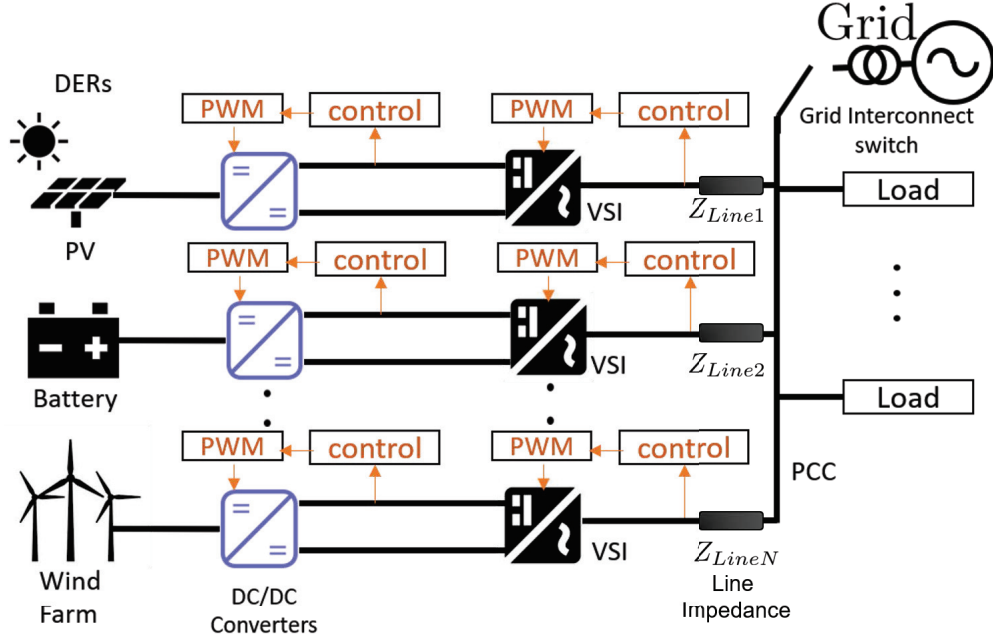


Figure 1.3: Microgrid with DERs interfaced with DC/DC converters and VSIs with control, to regulate voltage and frequency at PCC and power sharing between DERs, terminated at AC loads.

loads in the microgrid through either centralized, distributed or de-centralized control architectures. Such primary controllers are employed either locally at individual DERs or at the microgrid level and are responsible for, stabilizing microgrid voltage [3, 4], maintaining power quality [5] with plug-and-play capabilities [6], apportioning load between various distributed generations (DGs) [7], and primary frequency response support [8–10]. However, control of VSIs in MMGs pose the additional challenge of regulating system frequency and output voltage in the absence of a stiff grid. Furthermore, these VSIs need to be controlled such that the DERs share the load demand, satisfying active and reactive power requirements according to a prescribed sharing ratio. High performance in meeting these objectives are difficult to achieve due to the uncertainty in load demand size and load schedules, and, due to the complexity that arises as a result

of the coupling of paralleled VSIs in an MMG.

Moreover, most of the existing secondary control (*level 2*) system and associating power infrastructure has been designed based on a conventional centralized approach in mind and hence the services developed around it are more naturally suited to a centralized methodology. However, a distributed method for grid ancillary services has a number of advantages over centralized architectures —i) Distributed co-ordination is achieved using only local computations (which reduces communication overheads and congestion) pertaining only to the neighborhood of the DER., ii) Distributed coordination facilitates ‘plug and play’ capability, where a new DER that needs to connect to the network requires communication only to its nearest available DER whereas in a centralized case all new and existing DERs need to communicate with the centralized center through their own communication channel directly thereby increasing the network setup cost and, iii) Distributed architecture is resilient to failures as any particular node failure does not lead to failure of the entire network.

Thus, in order to utilize the full potential of a DER-interfaced highly renewable system, it is imperative to investigate the control design methodologies keeping DERs and RESs at the focus and utilization of distributed communication algorithms at local DERs allowing renewable energy resources and battery energy storage systems to participate in grid interactive services that can meet aggressive response timescales to maintain the operation and stability of the power system.

This thesis targets contributions towards inner control (*Level 0*), primary control (*Level 1*) and, secondary control (*Level 2*) designs in the light of DER-focused grids and microgrids with high penetration of renewable energy sources that makes these achieving these objectives challenging in modern power systems.

1.1 Thesis Overview

Chapter 2 challenges and investigates alternatives to the conventional full droop-based architectures that primarily rely on inherent high rotating inertia directly coupled with the network by proposing a voltage-active power only droop architecture which is more suited towards low voltage (LV) microgrids that exhibit low natural inertia and thereby are more susceptible to large inherent deviations of the microgrid system frequency that are caused during uncertain load changes while implementing frequency droop control. The proposed voltage-active power half-droop (VP-D) control is augmented with access to a global positioning systems (GPS)-based common clock to disseminate timing pulses to generate the frequency reference for inverters interfaced with LV microgrid networks. Furthermore, the lack of explicit control over reactive power flows in the network is mitigated by proposing a novel control design based on loop shaping and the internal model principle as opposed to standard proportional-resonant (PR) controllers that are frequency dependent designs. The resulting control law guarantees that the parallel inverters share reactive power such that deviations from desired reactive power sharing remain bounded while serving a common load and therefore improves the steady state response of multi-inverter systems. Finally, the effect of delays in receiving GPS clock signals by an inverter is experimentally characterized and an inverter isolation mechanism is proposed that allows inverters facing large GPS communication delays to disconnect from the network. Hardware experiments on an 1.2 kVA-prototype and controller-hardware-in-the-loop validation on the CIGRE distribution network are conducted to demonstrate the effectiveness of the proposed architecture towards active and reactive power sharing between inverters with load scenarios.

Chapter 3 discusses the issue of harmonic current sharing between various DERs in a LV microgrid in presence of highly non-linear loads. The design of an outer virtual impedance loop to shape the output voltage in such microgrids is proposed. In

order to achieve this, a virtual impedance extracting and shaping filter is first proposed to extract and weigh selected frequency components of the line current. Secondly, design of impedances to weigh the extracted frequency components is obtained in order to achieve harmonic current sharing and reducing total harmonic distortion (THD) even in scenarios of asymmetric sharing of highly non-linear loads. We show that the isochronous architecture developed above aids in the design and implementation of the output impedances. The control design is tested on numerical simulations and in 4 kW/1 kVAR two inverter configuration in a controller-hardware-in-the-loop (CHIL) setup.

Chapter 4 discusses the challenges of formulating and adopting existing distributed frameworks utilizing consensus protocols, specifically ratio-consensus protocols, for coordinating a multi-agent system towards real-time implementation. The ratio consensus protocol is thoroughly investigated and is augmented with additional Min-Max protocols and the monotonicity properties of these agents' states following min-max updates are established. This methodology facilitates a finite time termination criteria rendering the ratio consensus protocol to be suitable for real-time applications where finite time convergence of algorithms, within a pre-specified user-defined tolerance, is imperative.

Chapter 5 extends the results of finite-time termination of ratio consensus algorithm developed in Chapter 4 towards distributed apportioning of load in the presence of communication delays for providing demand response services requested by the grid. This work contributes towards developing a resource apportioning algorithm which is completely distributed in nature, from selection of communication weights to the finite time termination criteria. This framework enables unique and simultaneous, yet local and independent realization of power reference commands by the distributed control units and a subsequent coordinated action. The algorithm proposed in this paper also enables prioritization of RES available in the power network and autonomous adjustments of energy contributions from other non-RES units to allow for increased renewable

penetration. Novel early dispatch and brown start mechanisms are synthesized to enable the framework to efficiently respond to time-varying ancillary service requests, namely initial DER response to changing SFR commands that is faster than state-of-the-art distributed approaches. A novel end-to-end power-hardware-in-the-loop setup is developed with 40+ physical hardware DERs (ranging from 3 kW devices to 100 kW utility scale hardware), a 7 MW real-world distribution system model from Aurora, Colorado, in real-time simulator with 250 DER nodes network including 48 Raspberry Pi agents as distributed control units communicating over a real-world communication network with channel delays forming a first-of-its-kind real-time setup implementing the distributed framework with dispatch of the power reference commands generated by the resource apportioning algorithm to the local power electronics interface. The experimental setup is used to demonstrate aggregated response of aggregator commands within secondary frequency response timescales —full activation times of 1 minute with initial response time of 5 seconds which are faster response time objectives that exist today in North American and European electricity markets.

Chapter 6 investigates the cybersecurity issue of distributed networks implementing ratio consensus protocols where an intruder in the network can inject falsified data to manipulate convergence to a false ratio value in the network. We first propose a novel intruder model that has not been considered in the literature and show that the intruder can successfully steer the ratio consensus states to any desired ratio. In order to detect such intrusions, we then propose a distributed detection algorithm by modifying the Max-Min protocol of the ratio consensus algorithm to develop a low cost, easy to implement detection strategy where each participating node identifies the intruder independently, eliminating the need for a trusted certifying agent or computationally intensive intruder detection algorithms in the network. The effectiveness of the detection method is demonstrated by numerical simulations on large scale networks.

1.2 Scope and contributions

This dissertation is focused towards control and coordination of DER units in a highly renewable grid.

The contributions of this thesis are delineated as under:

1. A Control design and Isochronous architecture to maintain system frequency and active power sharing in multi-inverter microgrid systems

- *Design of control loop independent of frequency variations:* Conventionally, in the multi-loop controller, outer-voltage controller is designed to be a proportional + Resonant (P+R) controller based on a nominal system frequency. However, in a conventional full-droop control scheme, the system frequency inherently varies based on the $P - f$ (or $Q - f$) droop curve, and the corresponding control design (P+R) does not achieve good tracking results. However, in the proposed controller, both the inner-current and outer-voltage controllers are designed based on internal model principle and loop-shaping to achieve better regulation in output voltage of the inverter. This becomes possible since the frequency of the multi-inverter microgrid system (MMG) is maintained at a single common frequency (ω_0 rad/s) even in the presence of system disturbances. VP-D control decouples voltage regulation objectives from frequency variations thereby alleviating issues such as frequency dependent control resulting in poor voltage regulation and poor disturbance rejection.
- *Reduced controller complexity - No explicit reactive control:* The architecture does away with issues stemming from active frequency-regulation in full-droop methods. VP-D control allows direct control of active power with regulation of voltage magnitude. The resulting control law guarantees that the parallel inverters share

reactive power such that deviations from desired reactive power sharing remain bounded while serving a common load even without explicitly controlling the reactive power flow. The resulting controller design is simpler as no explicit $Q - f$ control is required. The architecture also enables improved start up transients, reduced circulating currents between participating VSIs and easy plug-and-play of VSIs in the MMG.

- *Facilitate design analysis: Close form relationships between phase difference and system parameters* The proposed framework facilitates a comprehensive analysis that provides: i) delineation analytically of how circulating current between VSIs depends on output voltage regulation error; ii) small-signal based stability analysis of the ac microgrid by having access to the common clock signal to generate $\alpha - \beta$ components without added complexities of PLL dynamics. These provide useful design guidelines for selecting droop gains and virtual output resistances to maintain system stability.
- A *VSI isolation mechanism* is proposed to allow VSIs to detect locally, delays in GPS signals (with respect to a tolerance value) and the ability to disconnect from the MMG. The resulting flexibility in provides delay tolerance based on performance requirements of the MMG. Furthermore, the effect of GPS communication delays on MMG is studied experimentally and characterization of time delay is obtained for the operational stability of the MMG.

Results show that the proposed architecture exhibits improved transients over full-droop implementations during start-up and different plug-and-play scenarios. Accuracy of active power and reactive power sharing is observed to improve whereas circulating currents among inverters greatly reduced. Moreover, the transient response time to

reach steady state improved in proposed VP-D architecture when compared to full-droop control. Moreover, the proposed VP-D control exhibits tighter regulation of the PCC voltage regulation during plug-and-play of the MMG system.

2. Control design to improve harmonic sharing and total harmonic distortion (THD) of PCC voltage in LV microgrids

- *Impedance shaping and extracting filter in synchronous reference frame:* We propose a synchronous reference frame based extraction of the fundamental and selective harmonic components of the line current for each VSI by extracting the direct and quadrature component by an extraction filter. The SRF framework show best performance under non-sinusoidal conditions which are suitable for LV microgrids with large harmonic loads. The accuracy of extraction filter results in obtaining the harmonic components of the current reduces additional circulating currents between VSIs and facilitates better design of the impedance shaping loop.
- *Reduced complexity and implementation viability:* The isochronous architecture is adopted here which allows for the local inverter controllers to have access to a common clock aiding in generation of modulation signals of appropriate frequency without additional complexities of implementing SRF-based phase-locked loops. Validations are obtained through SIMULINK/MATLAB simulations as well as CHIL-based real time simulation on a two inverter set-up to demonstrate the viability of implementation on low cost micro-controller devices and performance of the proposed methodology.

3. Finite time termination of ratio consensus protocols towards real-time applications

- *Strict monotonicity property of Ratio Consensus:* The key idea presented in [2] is the strictly decreasing (increasing) nature of max (min) consensus after every D

number of iterations, where D is the diameter of the network. However, it is not clear if such a nature of max-min consensus is preserved in case of ratio consensus, when using the algorithm proposed in [11]. In this work we provide a rigorous analytical justification of the algorithm proposed in [11]. We establish the monotonicity of maximum and minimum ratios when executing ratio consensus which is absent in earlier works. The results developed are generalized to both directed and undirected graphs with the communication weight matrix being column stochastic. We provide a non-trivial extension and a generalization of the results proposed in [2] without using some of its restrictive assumptions.

- *Validation with real-world low cost Raspberry Pi (system on a chip)-based networks:* The validity of the theoretical results are demonstrated with numerical simulations as well as hardware experiments on an actual communication network realized with Raspberry Pis.

4. Distributed aggregation framework for grid interactive services

- *Distributed framework amenable towards real-time implementation and RES prioritization:* We develop a distributed framework for coordinated power apportioning in the presence of communication delays that preserves the privacy of private values (capacity, output power) of the participating DERs. This work extends the preliminary results of distributed stopping criteria for ratio consensus protocols where each DER in the network can detect convergence within a tolerance independently in a finite time for a specified tolerance, ϵ . Moreover, the states communicated with neighbor DERs do not compromise private values such as total demand, capacity, or current generation, thus, allowing for a privacy-preserving and non-discriminatory participation of DERs while always prioritizing RES (such as PV) output power in the network .

- *Achieving fast time scale secondary frequency response distributedly —Early Dispatch and Brown Start Mechanisms:*

The proposed framework challenges the existing centralized secondary control approaches in providing secondary frequency response (SFR) services while achieving faster response timescales and accuracy of the dispatched solution as achieved in a centralized approach at the same time providing the benefits of a distributed framework, namely, DER coordination using only local computations, plug-and-play capabilities and, resiliency to communication channel delays. This framework targets initial response time of 5 seconds and full activation (ramp response) times of 1 minute after receiving an aggregator command. Our proposed distributed framework achieves these performance metrics for a large network system of DERs.

Towards this end, we devise an Early Dispatch mechanism. This mechanism leverages the monotonicity property of the Min/Max protocols and utilizes the ratio state value of the node from the most recent epoch (much before convergence) to obtain reference power commands and dispatch for each DER. This mechanism allows the network to achieve SFR faster than state-of-the-art distributed approaches. In order to enable the network to dispatch to subsequent aggregator demands, a brown-start approach is also proposed whereby DERs, already participating in SFR, smoothly transition to new states in order to meet new system operator commands in the presence of changes in generation capacities. Furthermore, We provide a bound on error in the converged ratio state due to brown start mechanism for any DER node in the network, to the finite time converged state that increases only linearly in the change in flexibility in each node and the tolerance value. These results allow for design consideration allowing the network to meet the SFR demand with desired accuracy.

- *Large scale PHIL system with first-of-its-kind distributed network integration and validation:* The proposed distributed finite time termination framework for aggregating DERs, demonstrates full activation times of 1 minute with initial response time of 5 seconds are achieved for experimental setup with power hardware and real-world communication networks. A large scale validation employing 40+ physical hardware DERs and 250 nodes on a real-world distribution system model. Experimental results show that, with the apportioning protocol, dispatch objectives are met within the targeted metrics. To the best of our knowledge, distributed DER coordination and control at such a scale is not demonstrated earlier with a large-scale validation.

The proposed framework is uniquely placed where we can achieve these fast response times while adding the advantages of robustness and distributed computation which are the limitations of a centralized architecture.

5. A distributed framework for Detecting Intruders in consensus networks

- *Efficient topology independent Distributed approach and simultaneous detection of malicious node:* A distributed scheme for detection of malicious node in a network implementing ratio consensus protocols is proposed eliminates the need of a unique identifier for each node in the network which allows each node to individually detect the intrusion. The results developed here enable a truly distributed way of detection as an agent utilizes information from its neighbors. The only global information needed is an upper bound on the graph diameter D . Furthermore, it also obviates the need for any centralized planning beforehand; and thus even the network synthesis is distributed. In addition, the scheme proposed has a small memory requirement; specifically, each node needs to store D values for each of its neighbors. The computations involved in the proposed algorithm are simple mathematical operations like multiplication of scalar values, in contrast to techniques

using matrix inversions and rank calculations which can easily become intractable as the size of the network increases. This makes the proposed algorithm suitable for *ad-hoc* networks where the agents have less computational power and storage capacity. The proposed scheme is topology independent unlike schemes where the graph topology is the determining factor for tolerance towards malicious agents

- *Successful Intruder attack models:* We consider a class of new intruder models and analytically show that the intruder has a successful attack strategy thereby steering the ratio consensus protocol to achieve a desired ratio value in the network in the absence of any attack strategy. Our analytical result does not require doubly-stochastic weight matrix and strong assumptions on the induced sub-graph after removing the intruder in the network be strongly connected as required in previous works.
- *High Success rate of detection:* We validate the proposed algorithm by obtaining numerical simulation results by running simulations for a set of 10,000 graphs with 20 nodes with a maximum possible diameter of 19 with arbitrary initial conditions for all nodes. The attack strength of the intruder is varied from 10% to 90% of the consensus convergence value. We created 30 million possible scenarios by varying the graph topology, attack strength, intruder node, time instant of attack. Our algorithm detects smaller attacks with an average accuracy of 96.65%. In such cases the intruder cannot steer the final consensus to achieve larger deviations. Stronger attacks ($\theta > 30\%$) are detected with an accuracy $> 99.5\%$.

Chapter 2

Isochronous Architecture for Voltage-Active Power Droop for Multi-inverter Microgrid Systems

Modernization of rapidly dispatchable DERs to provide demand response and ancillary services are transforming emergent microgrids into advanced microgrids enabling additional flexibility, resilience and reliability for local resources as well as support of the large scale grid when connected [12]. These multi-inverter microgrids (MMGs) [13] are systems that allow interaction of various DERs such as photovoltaics (PV) and battery energy storage systems connected to loads with DC/DC converters and DC/AC inverters (referred to as voltage source inverters (VSIs)) at the point of common coupling (PCC) as shown in Fig. 1.3. MMGs can be operated in both grid-connected as well as islanded modes. In the islanded mode, DERs support local loads in the microgrid through either centralized, distributed or de-centralized control architectures. Such controllers are employed either locally at individual DERs or at the microgrid level and are responsible for, stabilizing microgrid voltage [3,4], maintaining power quality [5] with plug-and-play

capabilities [6], apportioning load between various distributed generations (DGs) [7], and primary frequency response support [8–10]. An H_∞ based controller for grid feeding VSIs is developed in our other work while providing sufficient LCL resonance damping under a large range of variations of grid impedance [14]. However, control of VSIs in MMGs pose the additional challenge of regulating system frequency and output voltage in the absence of a stiff grid. Furthermore, these VSIs need to be controlled such that the DERs share the load demand, satisfying active and reactive power requirements according to a prescribed sharing ratio. High performance in meeting these objectives are difficult to achieve due to the uncertainty in load demand size and load schedules, and, due to the complexity that arises as a result of the coupling of paralleled VSIs in an MMG.

A distributed and communication-less approach of controlling VSIs is typically achieved through droop control methods. These methods use measurements obtained locally at the DERs and leverage the relationships between system voltage, line frequency, and power supplied by the DERs to ensure regulation of voltage and frequency in the network [15]. In microgrids with high line inductance (high X/R ratio), DERs are generally controlled by conventional $P - f$ and $Q - V$ droop control [16, 17], where P , Q , f , and V are active power, reactive power, frequency, and voltage respectively. The droop strategies in microgrids are motivated by the autonomous control of synchronous generators, with large rotating inertia directly coupled with the network that primarily rely on conventional droop strategies for regulation of system voltage and frequency. However, low voltage (LV) microgrids use more converter-interfaced DERs, and thus, exhibit low natural inertia with significant R/X ratio because of the small spatial expanse of the networks [18]. Consequently, non-conventional $P - V$ and $Q - f$ droop control strategies [18, 19] have gained attention for LV microgrids with the advantages of direct voltage control [20], improved harmonic power sharing [21] and seamless transition to islanded

mode of operation from grid connected mode in the case of critical infrastructures for providing resiliency against disasters [22] and contingencies as compared to conventional droop control approaches. Moreover, in order to improve transient performance and leverage decoupling compensations for droop controlled MMG, there are several works in the literature concentrating on modifying the droop characteristic equations [23–27]. However, these more complicated droop characteristic equations are, difficult to analyze, give stability guarantees, and difficult to incorporate in applications.

In traditional full droop based strategies that employ a frequency related droop strategy ($P - f$ or $Q - f$), the droop laws are designed to allow participating DERs to agree upon a system frequency based on the operating conditions and the specified droop curves. As a result, deviation of microgrid system frequency from nominal is inevitable. In the case of LV microgrids that have low inertia due to converter-interfaced DERs, the system frequency may deviate considerably from nominal [16, 28, 29]. In order to mitigate the detrimental effects of large excursions in frequency, typically, a secondary controller is employed that utilizes communication between DERs to restore the steady state system frequency to nominal [30]; however, complete mitigation of transient frequency deviations caused by sudden load changes in islanded microgrids is not achieved. Frequency regulation becomes even more difficult in the presence of uncertain renewable energy resources. Furthermore, traditional full droop methods suffer from the fundamental limitation of the trade-offs between accurate power sharing on one hand and the desire for stringent regulation of system voltage and frequency on the other.

An alternative to traditional full droop strategies is to adopt a half-droop strategy where only the system voltage is regulated based on the changes of active power while keeping the system frequency constant by utilizing intelligent communication technologies such as GPS-based timing signals [31]. Reference [32] presents a decentralized

control method for meshed microgrids to allow plug-and-play of DERs. However, additional measurements are required that render such strategies difficult and expensive to implement. A power-angle droop strategy is proposed in [29] to allow DERs to share the power demand based on the voltage angles. However, one drawback of this approach is the slow dynamic response because of delays in measuring power and obtaining voltage angles. A local rotating reference frame based synchronization using GPS timing signals is proposed in [33] to synchronize DERs utilizing the direct and quadrature-axis components of the reference voltage and output current which involve additional computations that can be avoided in single phase MMGs. Similarly, [34] proposes an augmented strategy where the GPS timing signal is used in conjunction with an adaptive $Q - f$ droop characteristics as a backup to ensure stable operation of the microgrid during GPS interruptions. Reference [34] uses a voltage-current droop law for maintaining active power sharing whereas in this article the active-power and voltage coupling of LV microgrids is used for controlling active power sharing.

In this work, we propose using only VP-D control for voltage regulation and address frequency regulation by providing a common timing signal from a GPS receiver to all the inverters. The framework developed enables generation of a common frequency reference for all the inverters and avoids the need for explicit frequency control and obviates the need for communication between inverters. Conventionally, $P - V/Q - f$ droop characteristics results in an inevitable deviation in system frequency as a result of which the designed controllers are effected if the system frequency is off-nominal due to the Q-f droop characteristics. A GPS based isochronous VP-D half droop controller allows the system frequency to be always maintained at nominal thereby resulting in a better decoupling of the active and reactive power control. Moreover, the half-droop is easier to implement on low cost controller devices due to reduced complexity of the controller design. Additionally, the dependence of VSI output voltage and current on

network parameters such as droop gains, line parameters can be obtained analytically, in closed-form, that allows for better design of the controllers for MMGs and provides insights on the effect of various parameters on the stability and performance.

In contrast to existing GPS-based methods [32, 34], our approach simultaneously achieves multiple objectives including reduced circulating currents between VSIs with better voltage regulation, reduced complexity due to the absence of phase locked loops (PLL) required for frequency synchronization and plug-and-play of VSIs.

Results show that the proposed architecture exhibits improved transients over full-droop implementations during start-up and different plug-and-play scenarios. Accuracy of active power and reactive power sharing is observed to improve whereas circulating currents among inverters greatly reduced. Moreover, the transient response time to reach steady state improved in proposed VP-D architecture when compared to full-droop control. Moreover, the proposed VP-D control exhibits tighter regulation of the PCC voltage regulation during plug-and-play of the MMG system. Experimental results are provided for a system of two 0.6 kVA inverters under different load variations and sharing ratios to validate the feasibility and effectiveness of the proposed isochronous architecture.

Note: More details on this work can be found in the published articles [35].

2.1 System Description and Controller Design

This section provides the details of an LV resistive MMG network containing N parallel VSIs, serving a common load, Z_L , connected at a single point of common coupling (PCC) as shown in Fig. 2.1. Each inverter in the MMG includes a communication module that receives synchronized clock pulses from the GPS segment [1] through a GPS receiver. These clock pulses are then used by the proposed outer VP-D control to

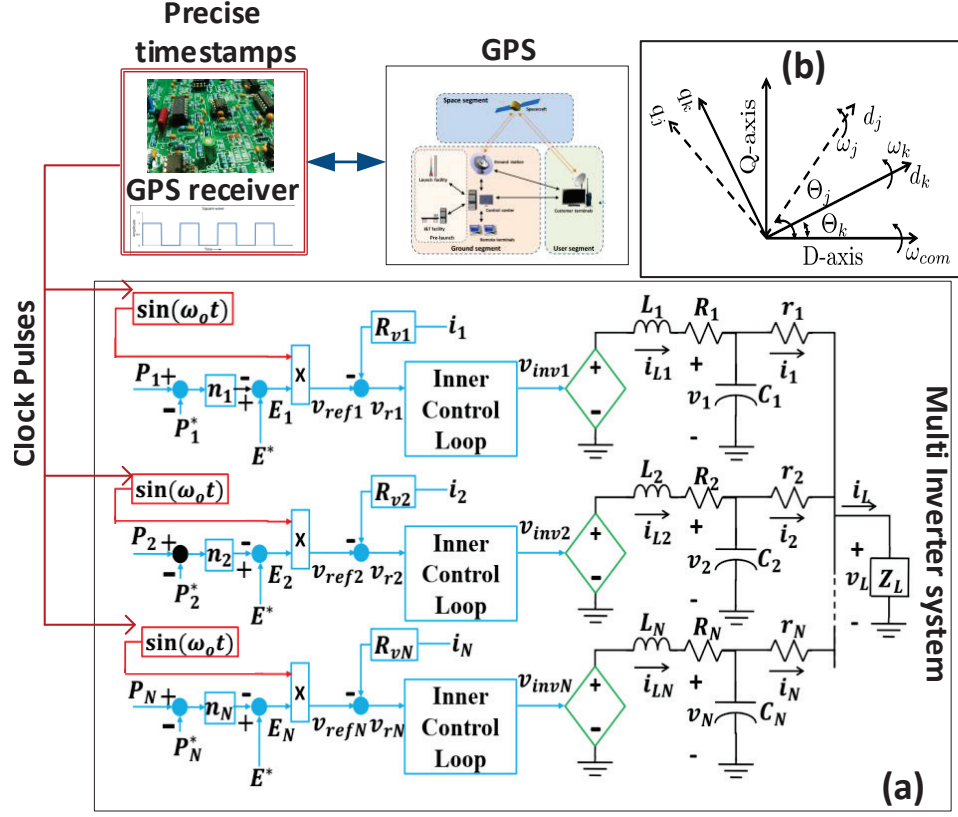


Figure 2.1: (a) A parallel N -inverter MMG with outer-droop and inner multi-loop controllers receiving GPS signals [1] to disseminate clock signals for frequency regulation, (b) representation of VSI j and VSI k with respect to common reference frame.

provide a synchronized sinusoidal reference voltage signal to inner-loop controllers located at each VSI forming the isochronous system architecture. We begin our discussion by describing first the dynamics of the k^{th} single inverter system and the controllers employing the proposed control law for achieving output voltage regulation and load disturbance rejection through a cascaded multi-loop control architecture.

2.1.1 Single Inverter System Dynamics

A single phase H-bridge inverter with an output LC filter is connected to load Z_L through a line resistance r_k . The VSI is modeled as a controllable voltage source, V_{invk} , by employing an average model of the inverter [36] as shown in Figure 2.2,

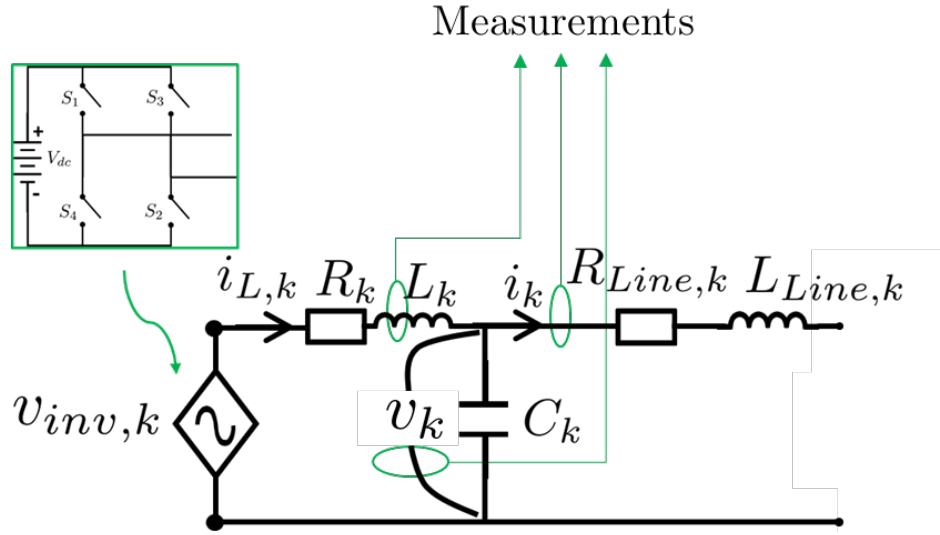


Figure 2.2: Averaged model of a single inverter with a linear load Z .

Thus for the k^{th} inverter (see Figure 2.2) we have

$$L_k \frac{di_{Lk}}{dt} + i_{Lk} R_k = v_{invk} - v_k \quad (2.1a)$$

$$C_k \frac{dv_k}{dt} = i_{Lk} - i_k \quad (2.1b)$$

where v_{invk} , i_{Lk} , v_k , and i_k are average values over one switching cycle (T_s) of control signal, inductor current, output voltage, and output current, respectively (See Fig. 2.3).

Taking Laplace transforms and eliminating $I_{Lk}(s)$, we get,

$$V_k(s) = \underbrace{\frac{1}{L_k C_k s^2 + R_k C_k s + 1}}_{G_{v_{invk}}} V_{invk}(s) - \underbrace{\frac{(L_k s + R_k)}{L_k C_k s^2 + R_k C_k s + 1}}_{G_{ik}} I_k(s). \quad (2.2)$$

2.1.2 Single Inverter Controller Design

In this section, we propose a control methodology to perfectly track sinusoidal reference voltage commands generated from an outermost droop law during matched conditions and droop within acceptable bounds during mismatched conditions even in the presence of complex loads.

The control design objectives are to: i) regulate the output voltage of the inverter (V_k) to track a reference signal (V_{rk}); ii) regulate the output inductor current of the inverter (I_{Lk}) to track a reference signal (I_{refk}); iii) reject disturbances due to load variations. We propose the following control law:

$$V_{invk}(s) = K_{refk} V_{refk}(s) - K_{vk} V_k(s) + K_{ik} I_k(s), \quad (2.3)$$

where $K_{refk} = K_{curk} K_{volk}$, $K_{vk} = K_{refk} + K_{curk} s C_k - 1$, and $K_{ik} = -K_{refk} R_{vk}$. As shown in Fig. 2.3, the control law takes the form of cascaded voltage and current controllers. The outer voltage controller, K_{volk} , acts on a sinusoidal reference signal provided by the outer droop law with the incorporation of a voltage drop, $i_k R_{vk}$, on a virtual resistance, R_{vk} . A feed-forward of the line current, i_k is used to minimize inrush currents and improve the transient response of the outer voltage controller. The output of the voltage controller, I_{refk} , provides the sinusoidal reference current signal to be tracked by the inductor current which generates the controllable voltage signal, V_{invk} , for the k^{th} VSI. The main advantages of the inner-outer loop structure are the decoupled design of the fast inner current controller and the slow outer voltage controller as well as the

increased robustness of the VSI to load variations. By combining (2.2) and (2.3), the closed-loop averaged output voltage dynamics of k^{th} inverter are given by:

$$V_k(s) = G_k(V_{refk}(s) - R_{vk}I_k(s))(s) - Z_kI_k(s), \quad (2.4)$$

where $G_k = S_k G_{v_{invk}} K_{refk}$, $S_k = (1 + G_{v_{invk}} K_{vk})^{-1}$ is the sensitivity transfer function, $T_k = 1 - S_k$ is the complementary transfer function, and $Z_k(s) = S_k G_{ik}$. Note that, where $G_k := (1 + G_{v_{invk}} K_{vk})^{-1} G_{v_{invk}} K_{refk}$ is the voltage gain (with complementary sensitivity transfer function, $\mathbf{T}_{V_{rk} \rightarrow V_k}(s)$, sensitivity transfer function, $\mathbf{S}_{V_{rk} \rightarrow V_k}(s) := 1 - \mathbf{T}_{V_{rk} \rightarrow V_k}(s)$), and $Z_k(j\omega_0) := (1 + G_{v_{invk}} K_{vk})^{-1} G_{ik}|_{s=j\omega_0}$ is the equivalent Thevenin output impedance of the k^{th} inverter at the fundamental frequency, $\omega_0 = 2\pi 60$ rad/s. The primary control objectives thus translate to $G_k(j\omega_0) \approx 1$ for voltage regulation and the closed-loop inner- current loop transfer function from I_{refk} to I_{Lk} be such that $T_{I_{refk} \rightarrow I_{Lk}}(j\omega_0) \approx 1$ and $Z_k(j\omega_0) \ll 1$. Thus, the controller design should achieve, $|\mathbf{T}_{V_{rk} \rightarrow V_k}(s)|_{s=j\omega_0} \approx 1$, $\angle \mathbf{T}_{V_{rk} \rightarrow V_k}(s)|_{s=j\omega_0} \approx 0^\circ$ and $|\mathbf{T}_{I_{refk} \rightarrow I_{Lk}}(s)|_{s=j\omega_0} \approx 1$, $\angle \mathbf{T}_{I_{refk} \rightarrow I_{Lk}}(s)|_{s=j\omega_0} \approx 0^\circ$ for the voltage and current controllers respectively to meet the regulation objectives. Moreover, $|\mathbf{S}_{V_{rk} \rightarrow V_k}(s)|_{s=j\omega_0} \ll 1$, $|\mathbf{S}_{I_{refk} \rightarrow I_{Lk}}(s)|_{s=j\omega_0} \ll 1$ and $|Z_k(s)|_{s=j\omega_0} \ll 1$ for ensuring good disturbance rejection. The current-control and voltage-control designs are based on the internal model principle [36,37], where the controllers K_{curk} and K_{volk} are designed to have poles $s = \pm j\omega_0$ and loop-shaping-based design to cancel the pole located at $s = R_k/L_k$ by the current controller. Moreover, the design ensures proper separation in bandwidth of both the controllers (current control loop has the larger bandwidth), which enables the inner current loop to make the inductor current I_{Lk} to track the reference current I_{refk} .

The closed-loop bandwidth is sufficiently larger than the system frequency allowing for compensation from load variations while ensuring good tracking of I_{refk} . The

bandwidth of the outer voltage control loop is designed to be sufficiently slower than the current control loop. This separation in bandwidth allows for treating the output of the voltage controller as a reference signal to the inner-current loop. Moreover, current control provides over-current protection for active current-limiting output under fault conditions.

An output virtual resistance loop [21] is also adopted to alleviate the sensitivity of active power sharing to the line and output impedances. Here, the virtual resistance, R_{vk} , is designed to dominate the line impedance of the k^{th} VSI, rendering the design insensitive to line and filter parameters. However, R_{vk} is bounded above to ensure VSI output voltage regulation to be within prescribed limits as larger R_{vk} results in larger regulation error by reducing V_{rk} . When we need to emphasize the fundamental frequency, ω_0 , we make the following assumption.

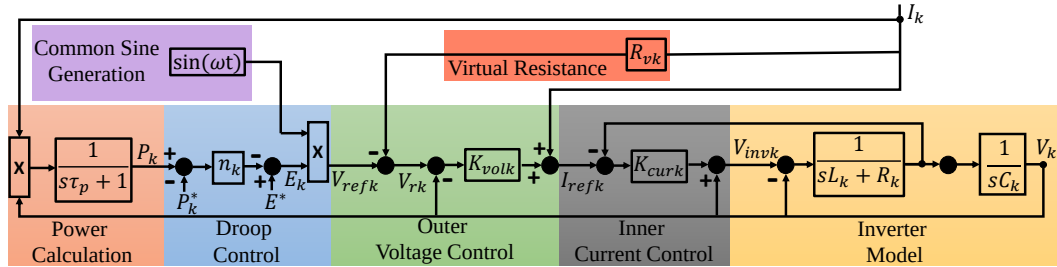


Figure 2.3: The inner-control loop for voltage regulation where a virtual resistance is incorporated.

Assumption 1 In Fig. 2.3 the following conditions are met:

1. $v_{refk}(t)$ and $i_k(t)$ are sinusoidal with frequency ω_0 .
2. $K_{volk}(s)$ has poles at $\pm j\omega_0$. ($\frac{1}{s^2 + \omega_0^2}$ is a factor.)

3. The closed-loop system from reference voltage, $v_{rk}(s)$, to output voltage, $v_k(s)$, is stable.

The following theorem shows that if all external inputs (i.e., $v_{refk}(t)$ and $i_k(t)$) to the system are sinusoidal with frequency ω_0 , then the inverter voltage follows the sinusoidal reference $v_{rk}(t)$ with zero steady-state error.

Theorem 1 Under Assumption 1, the closed loop system in steady state satisfies,

$$v_k(t) = v_{rk}(t) = v_{refk}(t) - R_{vk}i_k(t).$$

Furthermore, $G_k(j\omega_0) = 1$ and $Z_k(j\omega_0) = R_{vk}$ in (2.4).

Proof: See Appendix (Section 2.6).

The theorem above establishes the performance of the proposed VP-D controller towards regulation of output voltage and disturbance rejection of output line current for an inverter at the nominal frequency.

2.1.3 Relationship between active, reactive power and system parameters for a VSI connected to a reference bus

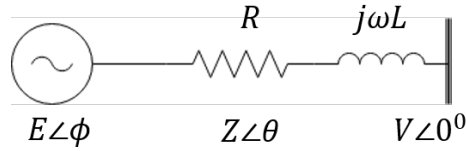


Figure 2.4: Voltage source inverter connected to a reference Bus.

In this section, we review the effect of impedance parameters on active and reactive power control for a single voltage source connected to a reference bus. Consider an

inverter-bus system as shown in Fig. 2.4 where $v_{inv}(t) = E \sin(\omega_o t + \phi)$ and $v_{PCC}(t) = V \sin(\omega_o t)$ are the inverter output and PCC bus voltages, respectively, ω_o (rad/s) is the line frequency and $Z\angle\theta$ is the output impedance. The current vector from E to V is given by $\vec{I}_o = \frac{E\angle\phi - V\angle 0}{Z\angle\theta}$ and the complex power delivered to the bus is given by $S = V\vec{I}_o^*$. Since, $S = P + jQ$, where P and Q are the active and reactive power respectively, we get,

$$P = \frac{V}{Z} \left((E \cos \phi - V) \cos \theta + E \sin \theta \sin \phi \right) \quad (2.5a)$$

$$Q = \frac{V}{Z} \left((E \cos \phi - V) \sin \theta - E \cos \theta \sin \phi \right) \quad (2.5b)$$

It can be seen from (2.5a) and (2.5b) that for a complex impedance $Z\angle\theta$, the power control strategy is relatively difficult since it requires simultaneous control of both E and ϕ to effect control of P and Q .

Usually, Inverter output voltages are controlled to have small ϕ values. As a result of which, the P/Q decoupling approximation for small ϕ , $\sin \phi = \phi$ and $\cos \phi = 1$, thus

$$P \approx \frac{V}{Z} \left((E - V) \cos \theta + E \phi \sin \theta \right) \quad (2.6a)$$

$$Q \approx \frac{V}{Z} \left((E - V) \sin \theta - E \phi \cos \theta \right) \quad (2.6b)$$

LV microgrids with purely Resistive Output Impedances

LV microgrids exhibit the special case where output impedance $Z = R + jX$ can be approximated to be purely resistive ($\frac{X}{R} \ll 1$) i.e. $Z = R$ and $\theta = 0$. This provides a fair degree of decoupling between P and Q control as given by (2.7) facilitating the control of P by regulating only E .

$$S = \left(-\frac{V^2}{R} + \frac{EV}{R} e^{-j\phi} \right)$$

Using small ϕ value approximation for inverter output voltages, we get,

$$P \approx \frac{V}{R}(E - V) \quad \text{and} \quad Q \approx -\frac{EV}{R} \phi \quad (2.7)$$

HV grid connected system with purely Inductive Output Impedances

High voltage (HV) grids are mostly grid connected system which comprise of transmission lines that span several hundreds of miles exhibiting high inductance values. Moreover, the connecting DERs interface with the grid using LCL filters that allows output impedance $Z = R + jX$ approximation to be purely inductive ($\frac{X}{R} \gg 1$) i.e. $Z = \omega_0 L$ and $\theta = 90^\circ$. This provides the conventional P and Q control as given by (2.8) facilitating the control of P by regulating only E .

$$S = \left(-\frac{V^2}{R} + \frac{EV}{R} e^{j\phi} \right)$$

$$P \approx \frac{EV}{R} \phi \quad \text{and} \quad Q \approx -\frac{V}{R}(E - V) \quad (2.8)$$

2.1.4 Full Droop Architecture

In AC microgrid during *islanded* mode, grid-forming VSIs are operated in voltage control mode to perform the tasks of regulating the PCC voltage and frequency. Level 1 or primary control of the microgrid provides the reference voltage and frequency set point to the controllers of grid-forming VSIs. These control methods are usually implemented either by using the communication-based active load sharing techniques or by mimicking the droop characteristics of a conventional synchronous generator system followed for multiple interconnected synchronous machines. The common droop characteristics used for multiple interconnected synchronous machine-based system use linear dependence

slopes for the i^{th} unit as n_i and m_i selected as:

$$n_i = \frac{f_{NL} - f_{FL}}{P_{FL,i}}, \quad (2.9a)$$

$$m_i = \frac{V_{NL} - V_{FL}}{Q_{FL,i}}, \quad (2.9b)$$

where, f_{NL} , V_{NL} and f_{FL} , V_{FL} are the steady-state frequency and RMS voltage of the system at no-load and full-load respectively. $P_{FL,i}$ and $Q_{FL,i}$ are the full-load active and reactive power capacity of i^{th} unit. For both frequency-active power and voltage-reactive power droop it can be shown that the active and reactive power sharing ratio among generating units can be controlled by properly choosing the slope of the characteristics.

Chandorkar et. al. in [38] first proposed the droop characteristic-based independent, autonomous, and wireless control of multiple VSIs by eliminating the inter-communication links between them. The droop characteristic equations for VSIs used in [38] are as follows:

$$\omega = \omega^* - n_i(P_i - P_i^*), \quad (2.10a)$$

$$V = V^* - m_i(Q_i - Q_i^*), \quad (2.10b)$$

where, ω^* , V^* are nominal system frequency (in rad/sec) and nominal system voltage (RMS) respectively. P_i^* , Q_i^* are active and reactive power reference set-points respectively of the i^{th} VSI. n_i and m_i are the corresponding droop coefficients and P_i and Q_i are output active and reactive power outputs of i^{th} VSI.

In [38], the droop characteristic-based control is directly adapted for multiple VSIs in an islanded microgrid by mimicking the interconnected synchronous machine case. However, the behavior of droop-control in an islanded microgrid is complex where its precise and accurate operation rely significantly on the VSI control techniques, its parameters

and microgrid network parameters.

2.1.5 GPS as timing signals

Apart from providing navigational data, GPS systems can provide accurate clock signals for civilian usage via the Standard Positioning Service (SPS). SPS is available for civilian use and has an accuracy of 340 ns. GPS-SPS satellite signals contain digitally modulated information in the carrier wave and the GPS signal contains clock pulse data. GPS receivers can compare the clock pulses received from the GPS satellite m and compare it to a local clock to obtain a time difference, which we denote as $\Delta\tau_m$ [31]. Inherent clock biases, C_b , are always present due to deviations in local clock and GPS clock. At any given instant the coordinates of GPS satellite m are known, we can determine the spatial position of the receiver (user) and the clock bias uniquely, with signals from four different GPS satellites, given as the solution to:

$$(x_m - x_r)^2 + (y_m - y_r)^2 + (z_m - z_r)^2 = (c\Delta\tau_m - C_b)^2,$$

where $[x_m, y_m, z_m]$ for $m = \{1, 2, 3, 4\}$ are the x, y and z coordinates of the four satellites at any given time, c is the speed of light and $[x_r, y_r, z_r]$ are the spatial coordinates of the receiver. If C_b is known then receiver (in this case a VSI) can accurately obtain universal coordinated time (UTC) required to synchronize with the accurate universal clock. GPS is used to obtain absolute time, time interval and frequency with precision up to hundred of nanosecond (ns) [1]. Low cost GPS modules can obtain time-stamping even triggers to about 50 nanosecond accuracy in Coordinated Universal Time (UTC) between widely separated sites [39]. GPS receiver modules have the ability to measure high precision timing signals which are readily available and inexpensive making them an

ideal candidate for time synchronization applications. We emulate low-cost Raspberry-Pis as GPS receiver modules to receive timing signals from an emulated GPS. It is assumed here that there is accurate and precise synchronization of the real-time clocks of the networked distributed devices to the common clock and is compliant with IEEE 1588-2008 standards [40].

2.1.6 Isochronous Architecture

An isochronous architecture can be achieved by providing a common timing signal from the GPS-SPS to all VSIs in the network. Based on the position of the VSI receiver, each VSI can thus determine its clock bias, C_b , and thereby obtain timing pulses with high accuracy. The advantages of such an architecture over frequency droop are delineated below: i) the ability to operate the microgrid at a single frequency, thereby eliminating deviation of frequency that arises when using frequency-droop control; ii) simplified control of the inverters by implementing only VP-D control without explicitly controlling reactive power and, iii) enabling a ‘plug and play’ nature of the microgrid.

It is imperative that such an architecture be robust to any momentary loss of access to the common clock by one or more inverters in the system. Mitigation of such scenarios can be achieved by employing a corrective mechanism, for example by employing a local clock at each VSI for satisfactory operation. This is possible since a local clock can provide a reasonable estimate of GPS time signal, eventually shifting the operating point in response to load changes causing degradation in power system performance. For conditions of continued signal loss, the VSI unit cannot participate in system wide load sharing and should be isolated from the rest of the system. A radio reference signal is used in [41] in order to mitigate system wide loss of GPS reference signal. Similarly, [42] utilizes three different PLL structures to guarantee robustness versus delay or loss in communication to synchronize the phase and the frequency of all of the

devices connected to the microgrid.

In order to mitigate the uncertainty in GPS communication, a VSI isolation mechanism is also proposed to allow VSIs to detect locally, delays in GPS signals (with respect to a tolerance value) along with the ability to disconnect from the MMG to maintain operational stability of the system. This article, however, mainly focuses on analyzing and obtaining system performance guarantees assuming clock synchronization is reasonably maintained at all times.

2.2 Multi-Inverter System Isochronous Operation

A network of N inverters in a parallel configuration is assumed as servicing a common load (see Figure 2.1).

In this section, a detailed analysis of isochronous operation of multi-inverter system is conducted by analyzing the performance of individual inverters, the nature of circulating currents and bounds on voltage regulation at PCC. The network of Fig. 2.1 is considered to be purely resistive (also aided by incorporating a virtual resistance term) and that the active power is appreciably affected by the magnitude of the voltage. Also, situations like mismatch (denoted as Δ) in total commanded power and actual total load demand as well as non-zero values of $P_k - P_k^*$ for k^{th} inverter are taken into consideration as these are unavoidable in practical systems.

2.2.1 Performance of Individual Inverters

In this section, the performance of the multi-inverter system with the proposed VP-D approach is analyzed. A benefit of the VP-D approach is that reactive power sharing among individual inverters is maintained within a bound without having control of reactive power explicitly defined as a control objective. Theorem 2 provides analytical expressions for output voltage and line currents of networked VSI units.

Theorem 2 *If the conditions of Assumption 1 hold, then for any complex load Z_L for the network shown in Fig. 2.1, the steady-state voltage and current at the output of the k^{th} inverter are, respectively, $v_k(t) = V_k \sin(\omega_0 t + \psi_k)$ and $i_k(t) = I_k \sin(\omega_0 t + \phi_k)$, where,*

$$I_k = \frac{\gamma_k}{r_k + R_{vk}} \sum_{m=1}^N \frac{E_m}{r_m + R_{vm}}, \quad \tan \phi_k = \frac{\sin \angle \alpha}{\cos \angle \alpha - \frac{\beta_k}{|\alpha|}}$$

$$V_k = \sqrt{(E_k - R_{vk} I_k)^2 + (R_{vk} I_k)^2}, \quad \tan \psi_k = \frac{\sin \phi_k}{\cos \phi_k - \frac{E_k}{R_{vk} I_k}}$$

where, $\alpha^{-1} = Z_L^{-1} + \sum_m (r_m + R_{vm})^{-1}$, $\gamma_k = \sqrt{(\beta_k - |\alpha| \cos \angle \alpha)^2 + |\alpha|^2 \sin^2 \angle \alpha}$, $\beta_k = E_k (\sum_{m=1}^N E_m (r_m + R_{vm})^{-1})^{-1}$.

Proof: See Appendix (Section 2.6).

Corollary 2.1 *In the case of purely resistive load, i.e., $Z_L = R_L$, the steady state voltage and current at the output of the k^{th} inverter are $v_k(t) = V_k \sin \omega_0 t$ and $i_k(t) = I_k \sin \omega_0 t$ for $V_k, I_k \in \mathbb{R}_+$. In other words, phase lags $\phi_k = \psi_k = 0$.*

Proof: From the expression of α in Theorem 2, it can be clearly stated that if $Z_L = R_L$, then α is a pure real number and thus $\angle \alpha = 0$. Therefore,

$$\tan \phi_k = 0 \implies \phi_k = 0 \implies \tan \psi_k = 0 \implies \psi_k = 0 \quad \square$$

Remark 2.1 One of the outcomes of Theorem 2 and Corollary 2.1 is that with the assumed control architecture and without any explicit control of reactive power, there will be no extraneous reactive power flows if the load is purely resistive. This also holds true when there is an active power mismatch.

Remark 2.2 It is important to note that Theorem 2 provides analytical expressions of output current and output voltage of a single VSI when connected to an MMG. The

direct dependence on line parameters, virtual impedance parameters and regulation of voltage at PCC is evident; such analytical insights are difficult to obtain in the conventional architecture. This forms the basis for designing parameters to largely improve active power sharing between VSIs. This is validated in Section 2.5.2.

The droop model

The voltage-power droop law which sets the target reference voltage at the load is given by

$$v_{ref,k}(t) = [E^* - n_k(P_k - P_k^*)] \sin \omega_0 t \quad (2.11)$$

which in the matrix form is

$$\mathbf{v}_{ref,k}(t) = [E^* \mathbf{1} - \mathcal{N}(\mathbf{P} - \mathbf{P}^*)] \sin \omega_0 t \quad (2.12)$$

where $\mathcal{N} = \text{diag}(n_1, \dots, n_N)$. We will further assume that the active power is derived from the instantaneous power $p(t) := v(t)i(t)$ via a first order filter with a time constant τ_P . Thus we assume that

$$\hat{P}_k = \frac{1}{\tau_P s + 1} \hat{p}_k. \quad (2.13)$$

Note that there is no explicit control of reactive power.

2.2.2 Design Guideline for Droop Coefficients

In the following theorem the phase difference between inverter currents i_k and i_j is characterized in the general setting for complex linear loads.

Theorem 3 Let $\delta_k = \frac{E_k - E^*}{E^*}$, $\boldsymbol{\delta} = \begin{pmatrix} \delta_1 & \dots & \delta_N \end{pmatrix}^T$, $\boldsymbol{\lambda}_v := \begin{pmatrix} \frac{1}{r_1 + Z_v} & \dots & \frac{1}{r_N + Z_v} \end{pmatrix}^T$, $\boldsymbol{\xi} = \frac{\boldsymbol{\lambda}_v}{\boldsymbol{\lambda}_v^T \mathbf{1}}$, $\nu = \frac{Z_L \boldsymbol{\lambda}_v^T \mathbf{1}}{1 + Z_L \boldsymbol{\lambda}_v^T \mathbf{1}}$. Suppose the conditions of Assumption 1 hold, then the phase

ϕ_k of i_k differs from the phase ϕ_j of i_j according to:

$$\tan(\phi_k - \phi_j) = \frac{\text{Im}g(\nu) \frac{\delta_k - \delta_j}{1 + \xi^T \delta}}{\left(\text{Re}(\nu) - \frac{1 + \delta_k}{(1 + \xi^T \delta)} \right) \left(\text{Re}(\nu) - \frac{1 + \delta_j}{(1 + \xi^T \delta)} \right) + \text{Im}g^2(\nu)}.$$

Proof: See Appendix (Section 2.6). □

Remark 3.1 For purely resistive load, $\tan(\phi_k - \phi_j) = 0$ as ν becomes real which also follows from Corollary 2.1.

Remark 3.2 Theorem 3 can be extended to show that to ensure the phase difference in the inverter currents to be small, the values E_k have to be close to E^* . Indeed, for the phase difference of the currents to be smaller than $\epsilon \ll 1$, it follows that $|\delta_k - \delta_j| \leq \epsilon \frac{\sum_m \frac{1}{r_m + Z_v}}{|Z_L| \sin \theta_L}$ needs to be satisfied, where $\cos \theta_L$ is the p.f. of the load, Z_L .

Remark 3.3 The expression for δ_k depends only on E_k , whereas all other parameters are load and network parameters. Moreover, the deviation of E_k from E^* is dictated by the outer droop law. Thus, this analysis aids the appropriate choice of n_k and the allowable mismatch from the reference active power P_k^* that is to be delivered, for example., selecting droop gains n_i, n_j for any two inverters i, j , such that $n_i(P_i^* - P_i) = n_j(P_j^* - P_j)$ will keep reactive power flows in check.

Remark 3.4 Theorem 3 shows that for any two inverters j, k , the phase difference between inverter output currents i_j and i_k is related to the tightness of the output voltage regulation with regard to the reference voltage which also provides practical guidelines for designing the droop control.

The following corollary provides a more intuitive expression for the phase difference between currents which can provide guidelines for practical design.

Corollary 3.1 *Assume the notation and the assumptions of Theorem 3. Suppose $\delta_k = O(\epsilon)$ and $|Z_L \boldsymbol{\lambda}^T \mathbf{1}| \gg 1$ then*

$$\tan(\phi_k - \phi_j) \approx \left(\sum_m \frac{|Z_L| \sin \theta_L}{r_m + Z_v} \right) (\delta_k - \delta_j). \quad (2.14)$$

Proof: See Appendix (Section 2.6). $\square$

Remark 3.5 *Assuming that E_k is close to E^* , resulting in small $\delta_k := \frac{E_k - E^*}{E^*} = \sum_m \frac{1}{r_m + Z_v}$ it is expected that $\delta_k - \delta_j$ is small. Typically, the interconnect resistance r_k and the virtual resistance Z_v are both small and thus for typical cases $|Z_v \boldsymbol{\lambda}_v^T \mathbf{1}| \gg 1$ should also hold. Thus assumptions in Corollary 3.1 are not restrictive.*

Remark 3.6 *The relationship in (2.14) shows that to ensure the phase difference in the inverter currents to be small, the values E_k have to be close to E^* . Indeed, for the phase difference of the currents to be smaller than $\epsilon \ll 1$, it follows that $|\delta_k - \delta_j| \leq \epsilon \frac{\sum_m \frac{1}{r_m + Z_v}}{|Z_L| \sin \theta_L}$ needs to be satisfied.*

Remark 3.7 *The amplitude of the voltage $v_k = E_k \sin \omega_0 t - Z_v i_k$ at the inverter output depends on the inverter current i_k where the dependence can be quite complex. However, the expression in (2.14) depends only on E'_k s with $E_k = E^* - n_k(P_k - P_k^*)$ and thus how much E_k deviates from E^* is dictated by the outer droop law. Thus the analysis above translates the concerns of limiting reactive flows to the design of the outer droop law where the appropriate choice of n_k and the allowable mismatch of the active power from the reference active power P_k^* can be estimated to keep reactive power flows in check.*

2.3 Eigen-analysis for Stability assessment

One of the important aspects to be investigated for the multi-inverter system operation is its stability. The non-linear multi-inverter closed-loop system can be represented as,

$\dot{X}(t) = f(X(t), u(t))$, where $X(t)$ denotes states of the system including both physical variables and internal control variables as shown in Table 2.1 and the input to the system, $u(t)$ is $v_{PCC}(t)$. The objective is to develop a small signal model of the multi-inverter system by linearizing the above system around a stable operating point. We consider the $d_k - q_k$ axis as the local reference frame for the k^{th} VSI operating at a rotating frequency ω_k . The individual VSI state equations are derived in terms of their individual local reference frame. The local reference frames are transformed to the global reference (D-Q) frame which is chosen to be the PCC reference frame operating at ω_{com} . This allows for generalizing the VP-D architecture beyond single-phase microgrids. We first consider the state equations of individual VSIs in their local reference frames represented by $\Theta_k := \theta_{com} - \theta_k$, where θ_{com} represents the synchronous reference frame angle at PCC and θ_k represents the deviation of the k^{th} VSI from the synchronous reference frame (Fig. 2.1 b). The translation between local $d_k - q_k$ frame to global D-Q frame is given by,

$$T_{\Theta_k} = \begin{bmatrix} \cos \Theta_k & -\sin \Theta_k \\ \sin \Theta_k & \cos \Theta_k \end{bmatrix},$$

$$v_{kdq} = [v_{kd} \ v_{kq}]^T, \ i_{kdq} = [i_{kd} \ i_{kq}]^T, \ i_{Lkdq} = [i_{Lkd} \ i_{Lkq}]^T,$$

where, $\begin{bmatrix} x_{kd} \\ x_{kq} \end{bmatrix} = T_{\Theta_k} \begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix}$. x_α and x_β are the $\alpha - \beta$ components of a given state x . For the single phase case, $x_\beta = 0$. Note that in the case of perfect synchronization with the GPS clock signal, $\theta_k = 0$ for all VSI as a result of which all inverters can be represented in the reference frame eliminating the need for translating each VSI's states into the common reference frame. Each of these VSI units comprise an outermost droop control, inner multi-loop voltage and current controllers, output LC-filter connected to the PCC through line impedances and are terminated at the common load. Combining these dynamics, the small signal state space model of the k^{th} single inverter system is

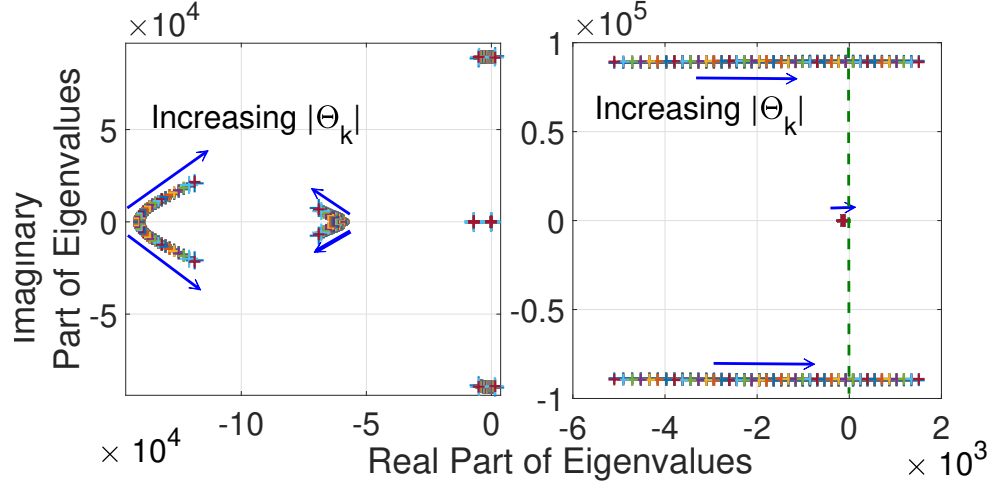


Figure 2.5: Eigenvalues of the two inverter network for stability analysis with δ_k changing from 0° to $\pm 10^\circ$ for a resistive load.

given as,

$$\Delta \dot{X}_k = A_k \Delta X_k + B_k T_{\Theta_k}^{-1} \Delta V_{PCCDQ} \quad (2.15)$$

where $\Delta X_k = [\Delta P_k \ \Delta \phi_{kdq} \ \Delta \gamma_{kdq} \ \Delta i_{Lkdq} \ \Delta v_{kdq} \ \Delta i_{kdq}]^T$,

Combined model of the N inverter system can now be written as [43, 44],

$$\Delta \dot{X} = \mathbf{A} \Delta X + \mathbf{B} \Delta V_{PCC,DQ}, \quad (2.16)$$

Table 2.1: States of the multi-inverter system

Parameters	States
Droop & LPF	P_k
Voltage Controller	$\phi_{k,dq}$
Current Controller	$\gamma_{k,dq}$
LC filter	$i_{Lk,dq}, v_{k,dq}$
Line parameters	$i_{k,dq}$
Load, Z_L	$i_{Load,DQ}$

where $\Delta X = [\Delta X_1 \ \Delta X_2 \ \dots \ \Delta X_N]^T$, $\mathbf{A} = \text{diag}(A_1, A_2, \dots, A_N)$ and $\mathbf{B} = \begin{bmatrix} B_1 T_{\Theta_1}^{-1} & B_2 T_{\Theta_2}^{-1} & \dots & B_N T_{\Theta_N}^{-1} \end{bmatrix}^T$.

$$\Delta \dot{i}_{Load,DQ} = -\frac{R_L}{L_L} \Delta i_{Load,DQ} + \frac{1}{L_L} \Delta V_{PCC,DQ} \quad (2.17)$$

where, $\Delta i_{LoadDQ} = T_\theta \sum_{k=1}^N \Delta i_{kdq}$. Thus,

$$\begin{bmatrix} \Delta \dot{X} \\ \Delta \dot{i}_{Load} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & 0 \\ \mathbf{0} & \frac{R_L}{L_L} \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta i_{Load} \end{bmatrix} + \begin{bmatrix} \mathbf{B} T_\theta^{-1} \\ \frac{1}{L_L} \end{bmatrix} \Delta V_{PCC} \quad (2.18)$$

Considering the dynamics of the lumped complex (Z_L) load and considering only a resistive load for simplicity, we obtain:

$$\Delta V_{PCC,DQ} = \sum_{k=1}^N T_{\Theta_k} \Delta i_{kdq} R_L \quad (2.19)$$

$$\text{Thus, using (2.19) in (2.16) we get, } \Delta \dot{X} = \underbrace{(\mathbf{A} + R_L \mathbf{B} \mathbf{T}_\Theta)}_{\mathbf{A}_{sys}} \Delta X,$$

where, $\mathbf{T}_\Theta$ is a matrix that captures the coupling of VSI interconnection at PCC. Stability analysis in the presence of a complex $R - L$ load can be obtained in a similar manner.

where the block matrix of $\mathbf{A}_{sys}$ for the k^{th} VSI is given as

$$\begin{bmatrix} \ddots & & & \vdots & & & \ddots \\ & & & \tilde{A}_k & & & \\ \dots & 0_{1 \times 7} & \frac{1}{L_k} & 0 & -\frac{(r_k + R_L)}{L_k} & \omega_0 & \dots \\ & 0_{1 \times 8} & \frac{1}{L_k} & -\omega_0 & -\frac{(r_k + R_L)}{L_k} & & \\ \ddots & & & \vdots & & & \ddots \end{bmatrix}$$

$\tilde{A}_k$ is the state matrix for k^{th} VSI associated with states given as,

$$\tilde{X}_k = [\Delta P_k \ \Delta \phi_{kdq} \ \Delta \gamma_{kdq} \ \Delta i_{Lkdq} \ \Delta v_{kdq}]^T.$$

The stability analysis of the system is given by the eigenvalues of $\mathbf{A}_{sys}$. For a two inverter setup the eigenvalues of the system are shown in Fig. 2.5 setup configured with the parameters of Table 2.3. The results are obtained for the case of an asymmetric power sharing ratio of 2.2. The line parameters are chosen to be $\text{Line}_1 = 0.2 \ \Omega + j\omega_o \ 0.1 \ mH$ and $\text{Line}_2 = 0.2 \ \Omega + j\omega_o \ 0.15 \ mH$.

Remark 3.8 *The small signal stability analysis reveals that the stability of the system is maintained in the VP-D architecture when deviation, θ_k , for all individual VSI units with respect to the common reference frame are $\leq \pm 5^\circ$. The presence of larger inductive loads (low p.f.) results in driving the eigenvalues of the linearized system towards the origin.*

$$\begin{bmatrix} \dot{P}_k \\ \dot{\phi}_k \\ \dot{\gamma}_k \\ \dot{\psi}_k \\ \dot{\Gamma}_k \\ \dot{i}_{Lk} \\ \dot{v}_k \\ \dot{i}_k \end{bmatrix} = \begin{bmatrix} -\omega_p P_k(t) + \omega_p v_k(t) i_k(t) \\ \gamma_k \\ (E_k^* - n_k(P_k - P_k^*)) \sin \omega_0 t - R_{vk} i_k \dots \\ \dots - v_k - \omega_0^2 \phi_k \\ \Gamma_k \\ -\omega_0^2 \psi_k + i_k + k_{vp}((E_k^* - n_k(P_k - P_k^*)) \dots \\ \dots \sin \omega_0 t - R_{vk} i_k - v_k) + k_{vi} \phi_k - i_{Lk} \\ -\frac{R_f}{L_f} i_{Lk} + \frac{1}{L_f} (v_k + k_{cp}(i_{refk} - i_{Lk}) - k_{ci} \psi_k) \dots \\ \dots - \frac{1}{L_f} v_k(t) \\ -\frac{1}{C_f} i_k(t) + \frac{1}{C_f} i_{Lk}(t) \\ -\frac{r_k}{L_k} i_k(t) + \frac{1}{L_k} v_k(t) - \frac{1}{L_k} v_{PCC}(t) \end{bmatrix}$$

2.4 Numerical Simulation Results

To validate the viability of VP-D-based control of a multi-inverter system, a MATLAB/SIMULINK based switching model of two inverters sharing a common load at PCC in a LV resistive network is simulated. The network and inverter parameters used in the simulation are listed in Table 2.3. Fig. 2.6 shows the voltage and current output results from a simulation where two parallel inverters are serving a common load of 0.58 kVA with $p.f. = 0.9$ (lagging). Both inverters are operated with $n_1 P_{inv1}^* = n_2 P_{inv2}^* = 2.5$ and $P_{inv1}^* = P_{inv2}^* = 250W$. Fig. 2.6(a) shows that the P share between two inverters, i.e., P_{inv1}/P_{inv2} , is maintained at 1.07. Note here that without having a separate Q control strategy, the reactive power sharing between the inverters is nearly even. Here, Q_{inv1}/Q_{inv2} is 0.88. The mismatch between actual measured and reference output active

power for both the inverters, primarily because of finite line losses, results in a small voltage deviation from E^* (around -0.5%) at the PCC. The voltage waveform at the PCC and the output current are shown in Fig. 2.6(b). Fig. 2.6(c) shows the circulating current between the two inverters, which is significantly low compared to the rated output current.

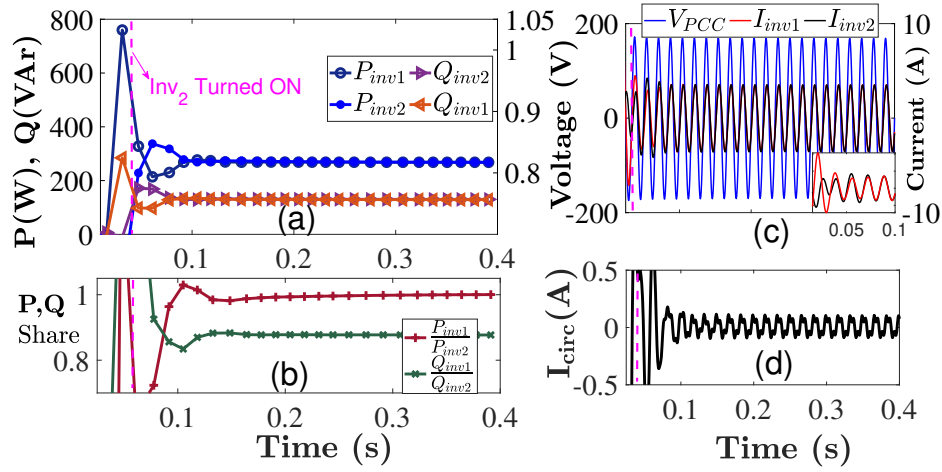


Figure 2.6: Simulation results of (a) P - Q share, (b) P , Q sharing ratio, (c) voltage at the PCC and inverter output currents, and (c) circulating current between two parallel inverters.

2.4.1 Mismatched line parameters

Table. 2.2. depicts simulation results for active and reactive power sharing between two inverters employing VP-D control when active power reference values for outer droop laws are mismatched from actual active power injected by the two inverters. A number of scenarios are considered each for symmetric and asymmetric active power sharing where total reference power commanded from each inverter is $P_{inv1}^* = P_{inv2}^* = 0.6$ kW whereas the actual load, Z_L , is a series R - L load with $R = 11.52 \Omega$ and $L = 0.02293$ H. $R_v = 0.2 \Omega$ for both inverters. Results clearly showcase that the reactive power share is

Table 2.2: Active and Reactive Power Share

Droop Coefficients (V/kW)	Line Resistance (Ω)	P_{Inv1} (kW)	P_{Inv2} (kW)	Q_{Inv1} (kVAr)	Q_{Inv2} (kVAr)	$\frac{Q_{\text{Inv1}}}{Q_{\text{Inv2}}}$
$n_1 = 0.025$ $n_2 = 0.025$	$r_1 = 0.10$ $r_2 = 0.10$	0.403	0.397	0.304	0.294	1.030
$n_1 = 0.025$ $n_2 = 0.016$	$r_1 = 0.10$ $r_2 = 0.10$	0.475	0.317	0.300	0.291	1.032
$n_1 = 0.025$ $n_2 = 0.025$	$r_1 = 0.10$ $r_2 = 0.09$	0.398	0.397	0.297	0.297	1.000
$n_1 = 0.025$ $n_2 = 0.016$	$r_1 = 0.10$ $r_2 = 0.09$	0.416	0.378	0.296	0.297	0.999
$n_1 = 0.025$ $n_2 = 0.025$	$r_1 = 0.10$ $r_2 = 0.08$	0.394	0.401	0.292	0.302	0.966
$n_1 = 0.025$ $n_2 = 0.016$	$r_1 = 0.10$ $r_2 = 0.08$	0.411	0.383	0.292	0.302	0.966
$n_1 = 0.025$ $n_2 = 0.025$	$r_1 = 0.10$ $r_2 = 0.07$	0.390	0.405	0.287	0.307	0.935
$n_1 = 0.025$ $n_2 = 0.016$	$r_1 = 0.10$ $r_2 = 0.07$	0.407	0.387	0.287	0.307	0.935
$n_1 = 0.025$ $n_2 = 0.025$	$r_1 = 0.10$ $r_2 = 0.06$	0.386	0.409	0.282	0.313	0.902
$n_1 = 0.025$ $n_2 = 0.016$	$r_1 = 0.10$ $r_2 = 0.06$	0.403	0.391	0.281	0.312	0.900
$n_1 = 0.025$ $n_2 = 0.025$	$r_1 = 0.10$ $r_2 = 0.05$	0.382	0.413	0.276	0.318	0.868
$n_1 = 0.025$ $n_2 = 0.016$	$r_1 = 0.10$ $r_2 = 0.05$	0.398	0.396	0.276	0.318	0.868
$n_1 = 0.025$ $n_2 = 0.025$	$r_1 = 0.10$ $r_2 = 0.01$	0.364	0.431	0.252	0.343	0.735
$n_1 = 0.025$ $n_2 = 0.016$	$r_1 = 0.10$ $r_2 = 0.01$	0.433	0.359	0.258	0.332	0.775

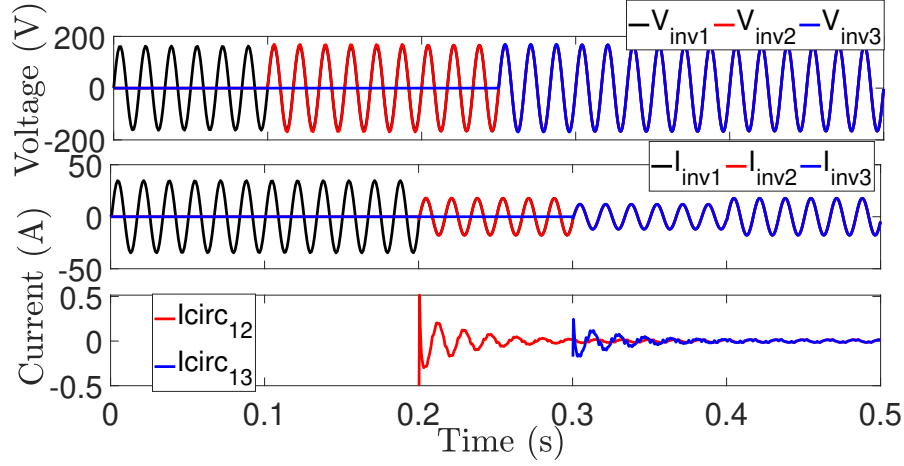


Figure 2.7: MMG with VP-D for demonstrating plug and play.

$0.9 \leq \frac{Q_{inv1}}{Q_{inv2}} \leq 1.03$, when the line resistance of inverter 2, satisfies $60\%r_{k1} \leq r_{k2} \leq r_{k1}$. For the cases when the line resistances are highly mismatched (i.e., $r_{k2} = 50\%r_{k1}$), $\frac{Q_{inv1}}{Q_{inv2}}$, is 0.868 with a worst-case sharing ratio of 0.735 obtained when $r_{k2} = 10\%r_{k1}$. This result demonstrates the performance of the proposed architecture in worse-case conditions toward mismatches in active power references and line parameters as equal reactive power sharing is desirable between the inverters.

2.4.2 Plug-and-play capability

A three VSI network is considered. At $t = 0$ s VSI-1 is on and synchronized with the local clock signal serving a load, $Z_L = 3\text{kW}$ (0.97 pf lagging). At $t = 0.1$ s and $t = 0.25$ s, VSI-2 and VSI-3 set individual clock receive flags high respectively to receive synchronized clock pulses for reference generation. The output voltages at the capacitor of VSI-2 and VSI-3 are thus generated at these instants. In order to demonstrate the plug and play capability, VSI-2 is connected to the common load and VSI-1 at $t = 0.2$ s and VSI-3 is then connected to the rest of the network at $t = 0.3$ s. Symmetric power sharing is desired at all VSIs. Output voltages, line currents and circulating currents

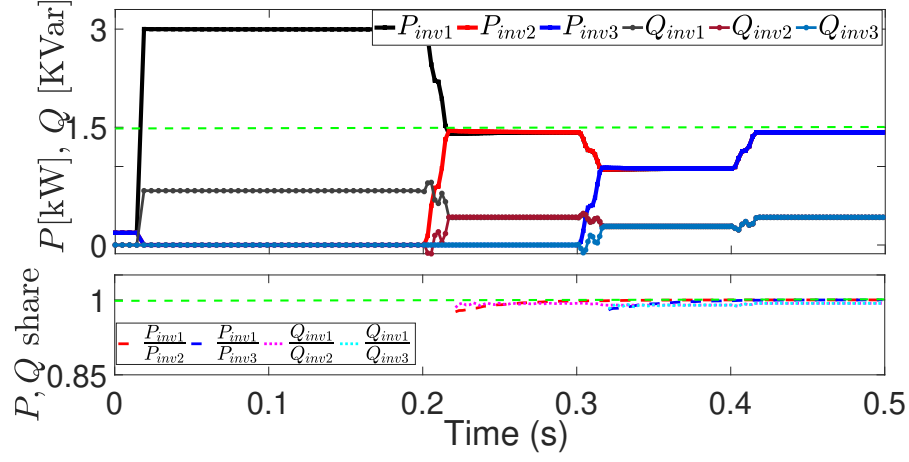


Figure 2.8: P - Q sharing performance for the MMG with VP-D.

are shown in Fig. 2.7. Fig. 2.8 shows the symmetric sharing of the load, Z_L , with active power sharing ratio of 1 as well as reactive power sharing ratio of > 0.95 at steady state even when no Q control is implemented. Finally, at $t = 0.4$ s, a load transition to 4.5 kW (0.97 p.f.lagging) occurs.

Table 2.3: Parameters for Experimental Prototype

Parameters	Values
Cut off frequency (ω_p)	1 (Hz)
Nominal Voltage/Frequency at PCC	120 $V/60$ Hz
Branch Resistance	0.2 Ω
Inverter DC Link Voltage	250 V
Inverter Filter Inductance	0.063 mH
Inverter Filter Capacitance	1 μF
Rated Power of Inverters	0.6 kVA

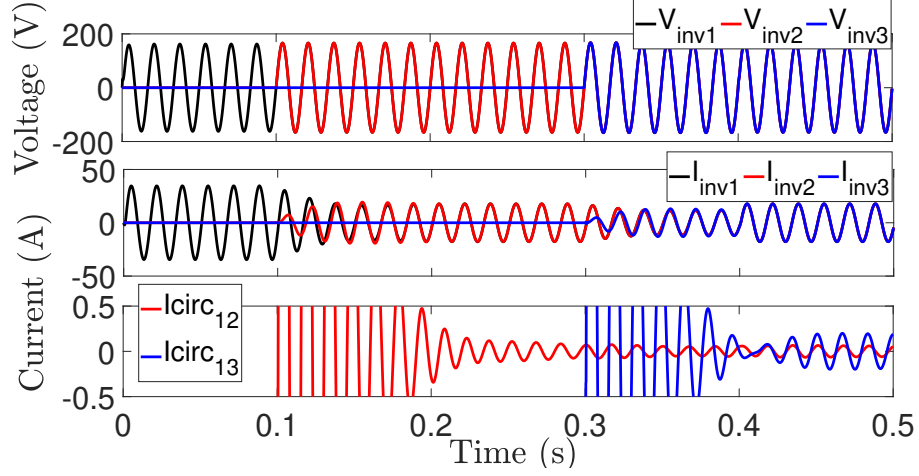


Figure 2.9: MMG with full droop for demonstrating plug and play.

2.4.3 Comparison with full droop

The three VSI network as above is considered while implementing a full droop architecture with $P - V$ and $Q - f$ droop laws. The multi-loop controller structure is designed to effectively achieve control objectives as described in Section 2.1 A, with the same P sharing objective, network parameters and common load to derive a fair comparison with the proposed architecture. At $t = 0$ s VSI-1 is on and serving a load, $Z_L = 3\text{kW}$ (0.97 pf lagging). At $t = 0.1$ s and $t = 0.3$ s, VSI-2 and VSI-3 are synchronized and connected to the network. The presence of an integrator in the $Q - f$ droop implementation results in initial transients in output voltage and current of the VSIs that cause in higher transient and steady state circulating currents. This is apparent at $t = 0.1$ s and $t = 0.3$ s when the VSIs are turned on (see Fig. 2.9). At $t = 0.4$ s, a load transition to 4.5 kW (0.97 p.f.lagging) occurs. The full droop implementation also suffers from longer transient times to reach steady state in both active and reactive power sharing as well as poor reactive power sharing ratio (sharing ratio of 1 is desired)

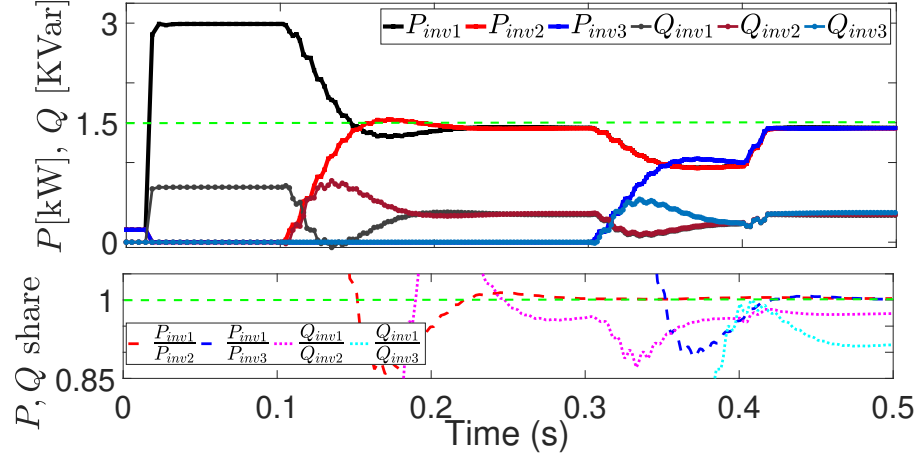


Figure 2.10: P - Q sharing performance for the MMG with full droop.

during load changes and steady state values due to mismatch in commanded and actual reactive power (see Fig. 2.10). Meanwhile, the isochronous architecture provides improved transient and steady state performance during black starts, plug-and-play of VSIs and load changes even in mismatched conditions. High power sharing accuracy during plug-and-play and fast transient response demonstrates the superiority of the proposed architecture over full droop methods in MMGs.

2.5 Experimental Results

2.5.1 Experimental Validation (Two VSI system)

Experiments are carried out in a laboratory setup as shown in Fig. 2.11. The experimental configuration consists of two single-phase insulated-gate bipolar transistor-based H-bridge inverter units along with LC output filter and controller unit for each, a purely resistive network, a variable common load served by both inverter units, and a Raspberry Pi-based (RPi) clock pulse generator. The RPi unit provides clock pulses with precise time-stamping. Each inverter consists of a digital input module to emulate a

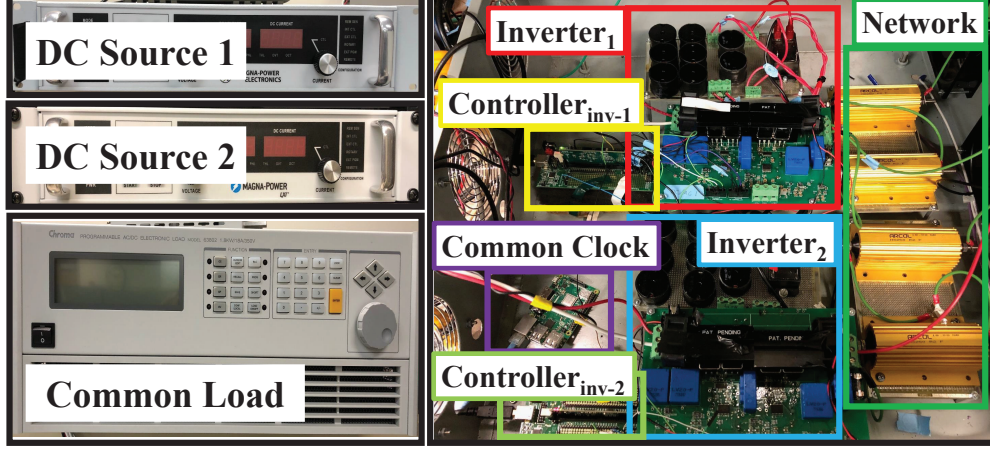


Figure 2.11: Experimental prototype for 1.2 kVA two inverter setup with common clock and network resistances.

GPS receiver. All inverters synchronize their local clocks, using zero-crossing detection and pulse counting, to the RPi clock signal transmitted as a square wave. This clock signal is then used to generate sinusoidal references to individual inner-loop controllers. Each inverter unit is rated as 0.6 kVA operating and utilizing a switching frequency of 20 kHz. The active power control including active power calculation and droop control, reference voltage generation, inner voltage and current controller, and PWM generation, digital input module are implemented on a TMS320F28335 digital signal processor. The inverter and network parameters are provided in Table 2.3. The value of the droop coefficients for both inverters – i.e., n_1, n_2 are selected as 2×10^{-4} V/W. The loop-shaping controllers K_{curk} and K_{volk} are designed as described in Section 2.1.2 with bandwidths 980 Hz and 600 Hz, respectively, for closely tracking sinusoidal references.

To verify the proposed VP-D control of the multi-inverter system, two case studies are conducted for experimental validation of the active and reactive power sharing performance (as shown in Fig. 2.12) using the hardware prototype. Case 1 considers two inverters serving a common load of $S_{tot} = 0.87$ kVA with transition at $t = 2.22$ s to $S_{tot} = 0.61$ kVA maintaining unity power factor throughout. Meanwhile, Case 2 studies two

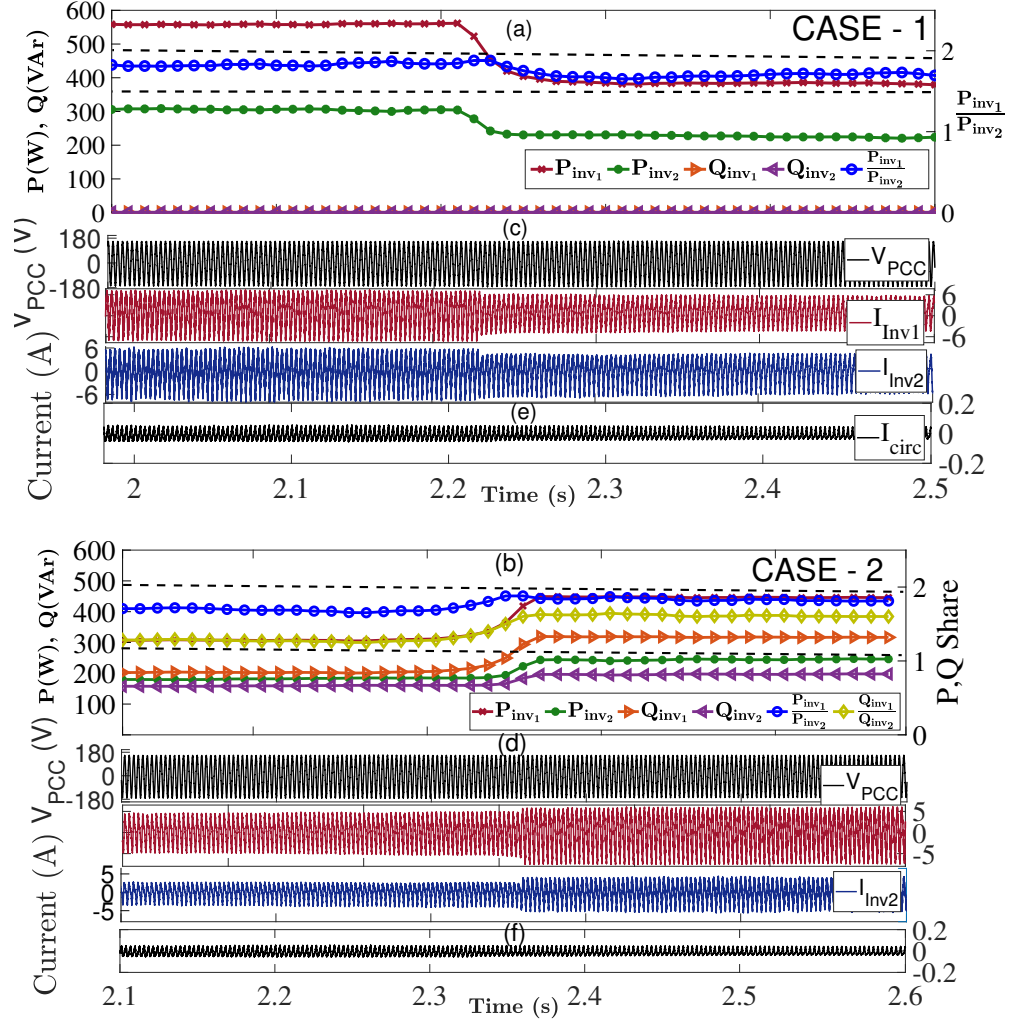


Figure 2.12: Hardware results of (a) P - Q share between two parallel inverters in Case 1, (b) P - Q share in Case 2 (c) common grid voltage and output currents in Case 1, (d) common grid voltage and output currents in Case 2 and (e) circulating current in Case 1, (f) circulating current in Case 2. Case 1 has a load of 0.87 kVA with transition at $t = 2.22$ s to 0.61 kVA with $p.f. = 1.0$ and Case 2 has a load of 0.6 kVA with transition at $t = 2.34$ s to 0.84 kVA with $p.f. = 0.8$.

inverters serving a common load of $S_{tot} = 0.6$ kVA with transition at $t = 2.34$ s to $S_{tot} = 0.84$ kVA maintaining 0.8 power factor throughout.

In Case 1, both inverters are assigned with asymmetric active power references of

$P_{\text{inv}_1}^* = 0.56 \text{ kW}$ and $P_{\text{inv}_2}^* = 0.31 \text{ kW}$ throughout the operation. The active power sharing ratio of $1.8 \pm 10\%$ is maintained throughout the operation, as shown in Fig. 2.12(a). Moreover, the output reactive powers of both the inverters are zero. The transient time of the inverter, which here is defined as the 90% settling time of the inverter's output current peak after a load step, is about 0.037 s (i.e., 2.2 cycles of 60 Hz). This demonstrates the ability of the proposed controller to inject real power into the network to provide ancillary support to the microgrid in fast- (primary response) timescale regimes. Fig. 2.12(c) shows the voltage and output current waveforms of both inverters before and after the load transition. There is a finite active power mismatch ($\Delta \approx +4.44\%$) that causes a deviation ($\approx +2.5\%$) of voltage magnitude at the PCC from its nominal value. These deviations are attributed to measurement error in active power at relatively low power operating conditions. Fig. 2.12(e) illustrates the circulating current between two inverters throughout the operation [45]. Clearly, the magnitude of the circulating current is significantly less than the nominal output current ratings of the inverters. The total harmonic distortion (THD) values of the voltage waveform at the PCC before and after the transition are 1.86% and 1.47%. These values are also maintained within acceptable standards [46]. Similarly, in Case 2, the inverters are assigned asymmetric active power references of $P_{\text{inv}_1}^* = 0.3 \text{ kW}$ and $P_{\text{inv}_2}^* = 0.18 \text{ kW}$ throughout the operation. The sharing ratio of active power is maintained at 1.67 throughout the operation. Moreover, reactive power is shared with a ratio of $1.2 \pm 10\%$ without implementing any dedicated Q control strategies throughout the operation. Fig. 2.12(d) illustrates the voltage and output current waveforms of both inverters before and after the load transition. The transient time of inverter current is 0.05 s (i.e., 2.99 cycles of 60 Hz). Here, $\Delta \approx -5.56\%$ that results in a deviation ($\approx -3\%$) in PCC voltage magnitude from its nominal. Fig. 2.12(f) illustrates the circulating current between the two inverters.

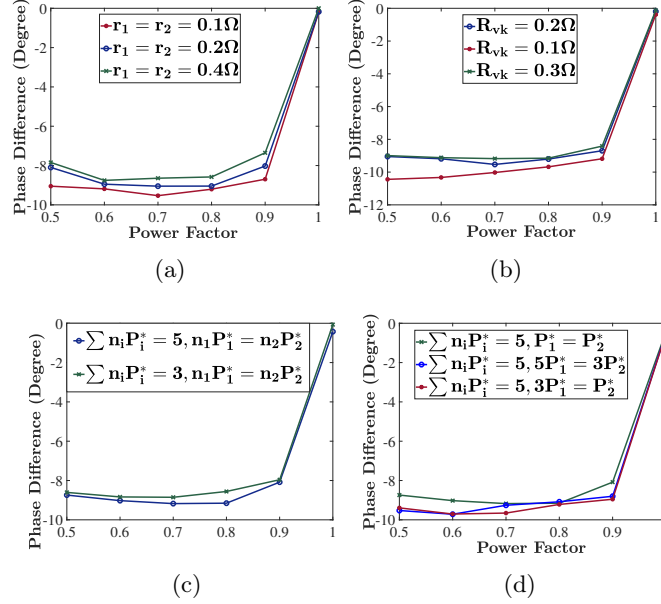


Figure 2.13: Experimental results of phase difference between output currents of two inverters serving common load with varied power factors in the scenario of having various values of (a) $\Sigma_i n_i P_i^*$ with $n_1 P_1^* = n_2 P_2^*$, (b) $\Sigma_i n_i P_i^*$ with $n_1 P_1^* \neq n_2 P_2^*$.

2.5.2 Design Considerations for the Multi-Inverter System

This section provides design guidelines to restrict limits on reactive power flows for implementing VP-D control in a multi-inverter system. Fig. 2.13 shows experimental results for the variation in phase difference (inverter currents) when an electronic load (common load in Fig. 2.11), Z_L with apparent Power $S = 0.6$ kVA is varied in power factor from $p.f. = 0.5$ to unity. It is important to emphasize here that even with such a low power factor of the load to be served and no explicit control over reactive power flows of the system, the phase difference between currents is kept small ($\approx 10^\circ$ or less) by the proposed controller architecture.

Fig. 2.13(a) and Fig. 2.13(b) show that higher values of branch resistance and virtual resistance lead to smaller phase differences. This, however, may lead to a decrease in the accuracy of active power sharing in the network. From Fig. 2.13(c) and Fig.

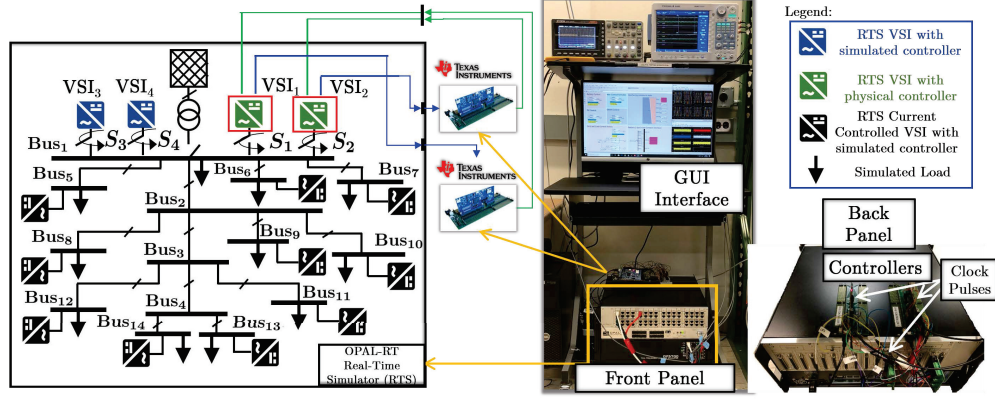


Figure 2.14: Controller-Hardware-in-the-loop setup in OPAL-RT RTS with CIGRE test distribution feeder model and Texas Instruments based Delfino TMS320F28379D controllers. VSI-1, VSI-2, VSI-3 and VSI-4 receive the clock pulses and operate in VP-D control whereas other inverters operate in current controller mode interfaced with PV as DERs.

2.13(d) it can be observed that a choice of smaller $\sum_i n_i P_i^*$ results in a smaller phase difference. These observations serve as design considerations to limit reactive power flows and implement an isochronous VP-D architecture in an LV multi-inverter system.

2.5.3 Controller-Hardware-in-the-loop: Experimental Validation on modified CIGRE Distribution Network

In order to further validate the performance of the proposed isochronous-based VP-D architecture and the viability of realizing these controllers on real-world low-cost micro-controller boards, we discuss here the experimental results of CHIL validation studies on an OPAL-RT real-time simulator (RTS). The distribution network is chosen from North American low voltage distribution feeder model ($240 V_{rms}$, $1 - \phi$, 60 Hz) from CIGRE Task Force C6.04.02 [47] (CIGRE Study committee C6) is modelled inside the OPAL-RT RTS using eMegaSim at $50 \mu s$ time-steps. The CIGRE model is modified to include four VSIs on Bus 1 and scaling of the loads. This modification is done to corroborate the

system study of VSIs connected at a PCC (in this case Bus 1) while being connected to other DERs over a distribution network as shown in Fig. 2.14. VP-D controllers for VSI-1 and 2 are implemented on TI microprocessor boards receiving clock pulses from a pulse generator (1 clock pulse per 60 Hz). PWM duty as control signals are provided by the TI DSPs and provided to OPAL-RT RTS through digital input modules to drive the average inverter models inside OPAL-RT RTS. Analog output signals (output current, capacitor voltage and inductor current) are measured and provided to TI -DSPs of corresponding VSIs via ADC inputs to close the feedback loop (Fig. 2.14). Delay blocks are simulated inside TI DSP controllers to emulate delays in GPS clock pulses. The CHIL parameter details are shown in Table 2.4.

Scenario Case 3 for CHIL validation:

The microgrid scenario case 3 tested in CHIL is described in Table 2.5. Initially, VSI-1 is on and PV interfaced current controlled VSIs are operating to meet a load of

Table 2.4: Parameters for CHIL Setup

Bus No.	Source Control Mode	Load $P + j Q$ kW/kVAR	Device
1	Voltage Source Inverters VSI-1 - VSI-4	$2.5 + j 1.5$	VSI-1, VSI-2 : TI DSP (Cap. 12 kVA) VSI-3, VSI-4: Emulated in RTS (Cap. 6 kVA)
2-13	Current Controlled VSI	$20.0 + j 9.5$	Emulated in RTS (Cap. < 2 kVA each)
14	Current Controlled VSI	$3.0 + j 0.5$	Emulated in RTS (Cap. 6 kVA)

13.6 kW/6.5 kVAR. VSI-1 is maintaining the voltage and implements a multi-loop VP-D controller as described in Section 2.1. At $t = 2.15$ s, VSI-2 is turned on and connected to the distribution feeder (demonstrating plug and play). VSI-1 and VSI-2 are prescribed $P_1^* = P_2^* = 7.1$ kW each and Bus-1 load is turned on. At $t = 9.4$ s, VSI-3 and VSI-4 are turned on and the load is increased to 22.5 kW / 11 kVAR. The active power reference commands are prescribed as $P_1^* = P_2^* = 5.5$ kW and $P_3^* = P_4^* = 2.5$ kW ($2.2 \times P_1^*$). At $t = 29.15$ s, VSI-2 is disconnected from the network due to loss of GPS signal (described in Section 2.5.4 later). Following the disconnection of VSI-2, Bus-1 load is disconnected in order to avoid large drooping in voltage, VSI-1, VSI-3 and VSI-4 along with feeder current controlled VSIs share the remaining load in the prescribed ratio. Note that P_1^*, P_3^* and P_4^* remain unchanged in this period as VSI-2 disconnection is an unscheduled event. Finally at $t = 57$ s, the load on Bus 14 is turned on and each VSI increases its active power in order to maintain its proportional sharing of load demand. The active power output of the network along with transients at transition events are shown in Fig. 2.15. Fig. 2.16 shows the CHIL results for reactive power outputs of inverters for case 3. Note that even though there is no explicit control of output reactive power, the proposed architecture allows equal sharing of reactive power from participating VSIs. Reactive power transients are shown to further demonstrate the performance of the VP-D architecture during load transitions. Fig. 2.17 summarizes the sharing ratio of active and reactive power among various VSIs. It can be clearly observed that prescribed sharing ratios are maintained well within $\pm 10\%$ even during disconnection of VSI-2 and during various load transitions. Finally, Fig. 2.18 shows the circulating currents among participating VSIs, as they become available, measured based on the VSI line currents and in accordance with the sharing ratios provided. Circulating currents are observed to be < 2 A (5% of rated output current) throughout for all the VSI pairs. Output voltages of each VSI are verified to be within $\pm 10\%$ throughout.

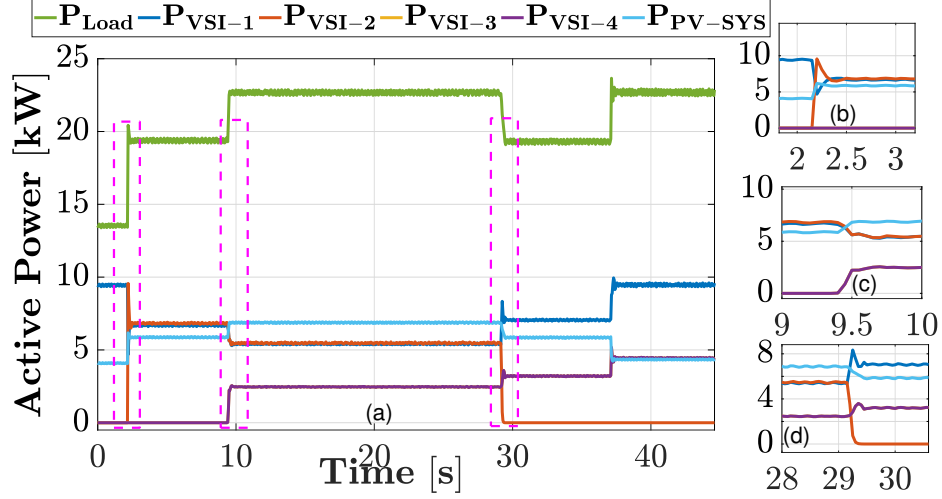


Figure 2.15: (a) Active Power for CHIL experiment Case 3, (b) transition 1 at $t = 2.15$ s (c) transition 2 at $t = 9.4$ s (d) transition 3 at $t = 29.15$ s.

Table 2.5: Scenario Case 3 for CHIL Experiments

time (in seconds)	event details	Total Load kW/kVAR	Active Power sharing ratio
$t = 0$	VSI-1 ON	13.6 / 6.5	-
$t = 2.15$	VSI-2 ON	20 / 9.5	$P1/P2 = 1:1$
$t = 9.4$	VSI-3 ON VSI-4 ON Load at Bus 1 ON	22.5 / 11.0	$P3/P4 = 1:1$ $P1/P3 = 2.2:1$
$t = 29.15$	VSI-2 OFF Load at Bus 1 OFF	20.0 / 9.5	$P3/P4 = 1:1$ $P1/P3 = 2.2:1$
$t = 57$	Load at Bus 14 ON	23.0 / 10	Same as above

2.5.4 Effect of Delay on Active Power, Reactive Power and Circulating Currents

This section studies the effect of delays in receiving GPS clock timing pulses by a VSI in the network. Furthermore, a mitigation mechanism is provided whereby each VSI locally

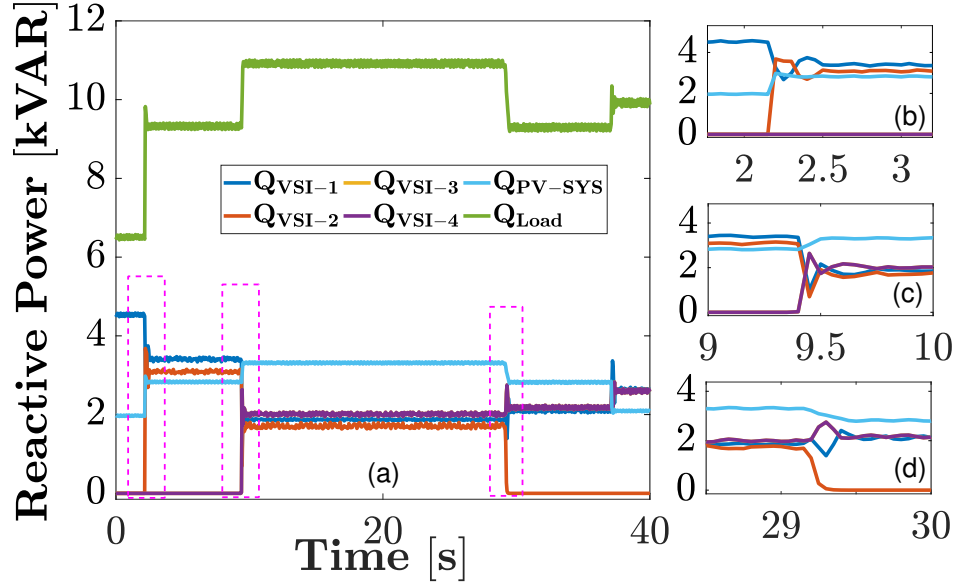


Figure 2.16: (a) Reactive Power for CHIL experiment Case 3, (b) transition 1 at $t = 2.15$ s (c) transition 2 at $t = 9.4$ s (d) transition 3 at $t = 29.15$ s

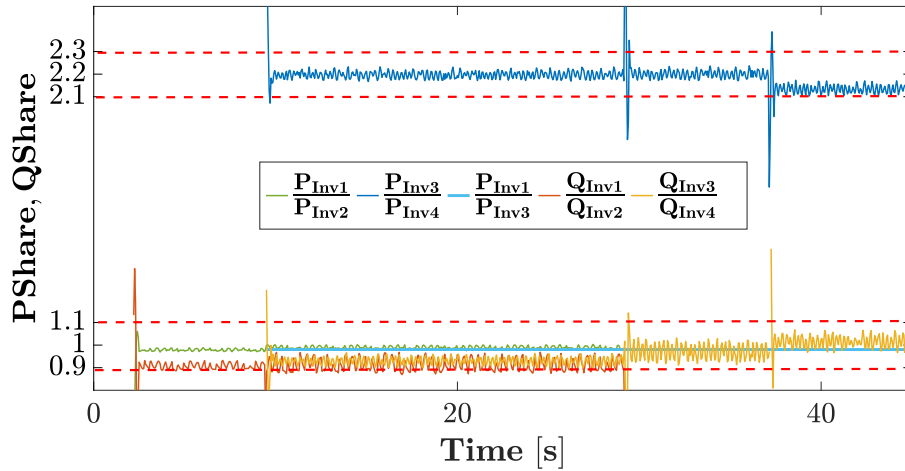


Figure 2.17: Active (P), reactive (Q) power sharing ratio among VSIs for CHIL experiments of Case 3.

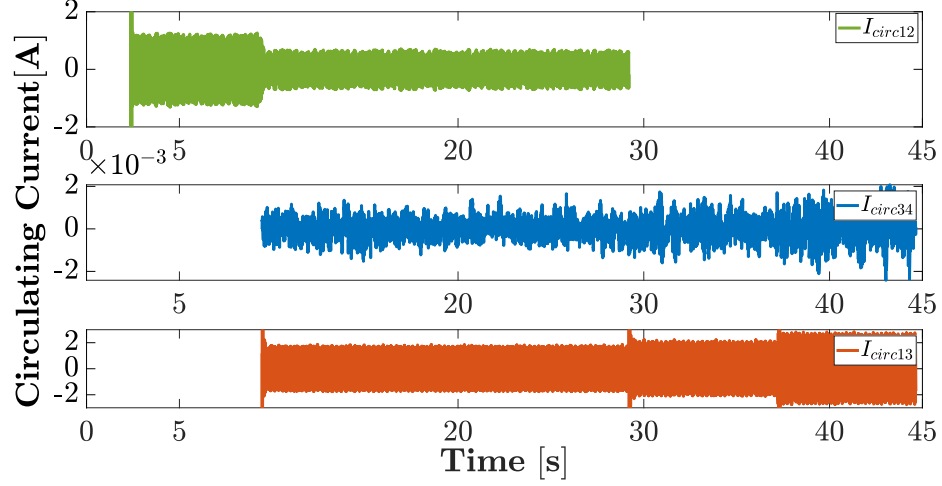


Figure 2.18: Circulating Current VSI-1 and VSI-2 (top); VSI-3 and VSI-4 (middle) and VSI-1 and VSI-3 (bottom) for CHIL experiments of Case 3.

monitors the delay in receiving the timing pulses from the GPS based on a local clock and can disconnect from the rest of the MMG based on a user-defined delay tolerance. GPS clock pulses are provided to VSI-1 and VSI-2 via digital inputs of the TI DSP boards. Delay blocks are implemented inside TI DSP controllers, running at a time step, $T_s = 100 \mu s$, to emulate delays in GPS clock pulses. In order to characterize the effect of delay, the CHIL setup of Fig. 2.14 is chosen with only VSI-1 and VSI-2 connected to a load (5.7 kW/ 2.8 kVA) on Bus 1. In this setup, VSI-1 receives GPS clock pulses at no delay whereas the delay in receiving GPS signals by VSI-2 is increased subsequently and its effect on active, reactive power and circulating currents are observed. The results are shown in Fig. 2.19. At $t = 2$ s, VSI-1 is turned on to serve load at Bus-1. At $t = 4.4$ s, VSI-2 is turned on and $P_1^* = P_2^* = 2.85$ kW for symmetric sharing. At 4.4 s $\leq t < 11$ s, there is no delay in receiving GPS clock pulses, as a result active power sharing ratio is maintained as prescribed, reactive power sharing is close to unity and circulating current is less than 1 A. However, at $t = 11$ s, VSI-2 suffers from a

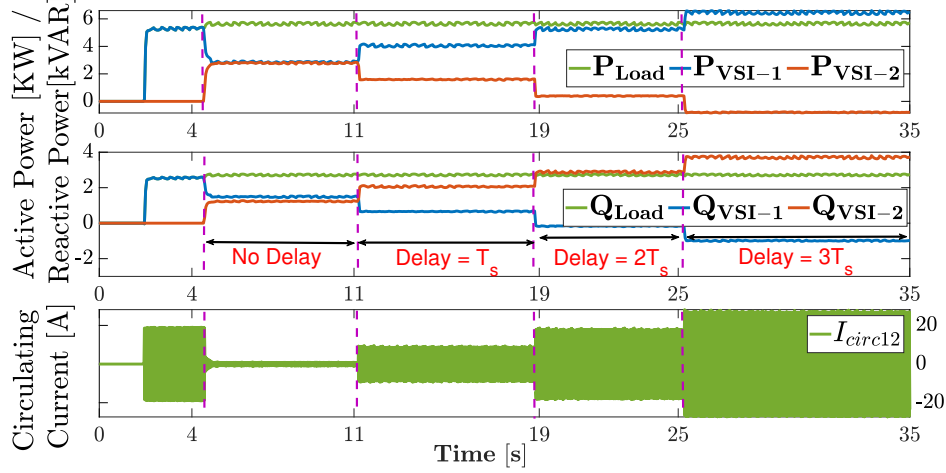


Figure 2.19: CHIL simulations for characterizing the effect of delay in clock pulses on active power, reactive power and circulating current with $T_s = 1 \times 10^{-4}$ s.

delay of $T_s = 100\mu s$ in receiving GPS clock pulses. This results in accuracy in P and Q sharing. Furthermore, the circulating current increases to ~ 6 A and the MMG operates at a degraded condition. Subsequently at $t = 18.8$ s and $t = 25.3$ s, delay is further increased to $2T_s$ and $3T_s$ respectively. Inaccuracy in P and Q sharing further increases to a point where VSI-1 starts absorbing reactive power and provides all the active power demand. Thus, for delays greater than $2T_s$, the MMG exhibits severe degradation in performance and can not be operated. It is worth noting that for delays $< 2T_s$, although the MMG has degraded performance, it remains stable demonstrating robustness of the proposed architecture towards certain delays in GPS signals experienced by the VSIs.

Remark 3.9 Note that a delay of $T_s = 100 \mu s$, $2T_s$ and $3T_s$ corresponds to phase differences of 2.16° , 4.32° and 6.48° respectively. The CHIL validation agrees with the eigenvalue analysis presented earlier where phase difference w.r.t. reference frame, $\theta_k \geq \pm 5^\circ$ results in the poles of the MMG being closer to the imaginary axis resulting in critical stability of the system (see Remark 3.8).

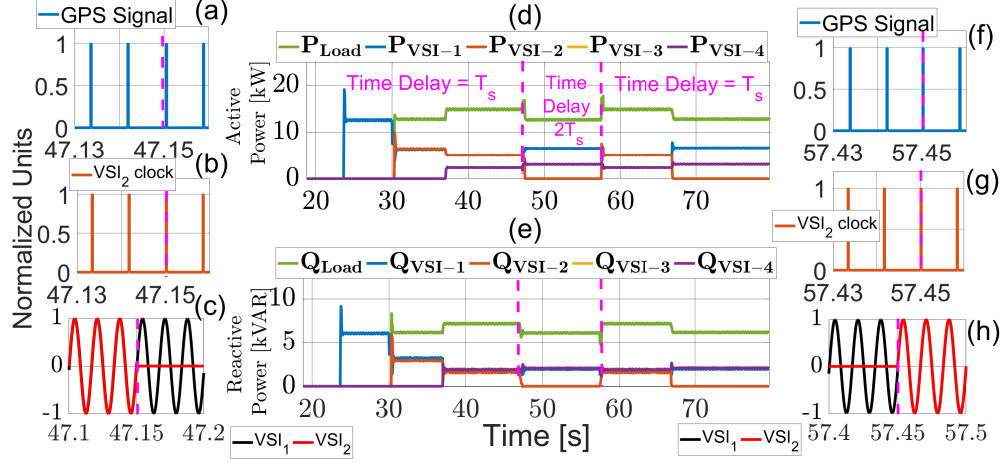


Figure 2.20: Controller-Hardware-in-the-loop experiment for demonstrating VSI isolation mechanism following a communication delay in receiving the GPS signal by VSI-2. (a),(f) GPS signal received by VSI-2, (b),(g) VSI-2 local clock pulses, (c),(h) VSI-1 and VSI-2 normalized output voltage, (d) Active power output of VSIs with VSI load only, (e) Reactive power output of VSIs with VSI load only.

In order to mitigate the issues caused due to delays in receiving GPS signals, we next propose a VSI isolation mechanism.

2.5.5 VSI Isolation Mechanism

In order to propose a VSI isolation mechanism, the following assumptions are imposed on the system.

Assumption 2 *VSI local clocks are synchronized to GPS timing signals before being connected to the MMG.*

Assumption 3 *GPS timing signals do not arrive before they are due. That is, in case of communication delays, GPS clock pulses always lag local clock pulses.*

Note that these assumptions are realistic in nature. For example, a VSI is not allowed to connect to the MMG if its local clock signals are not synchronized with GPS timing

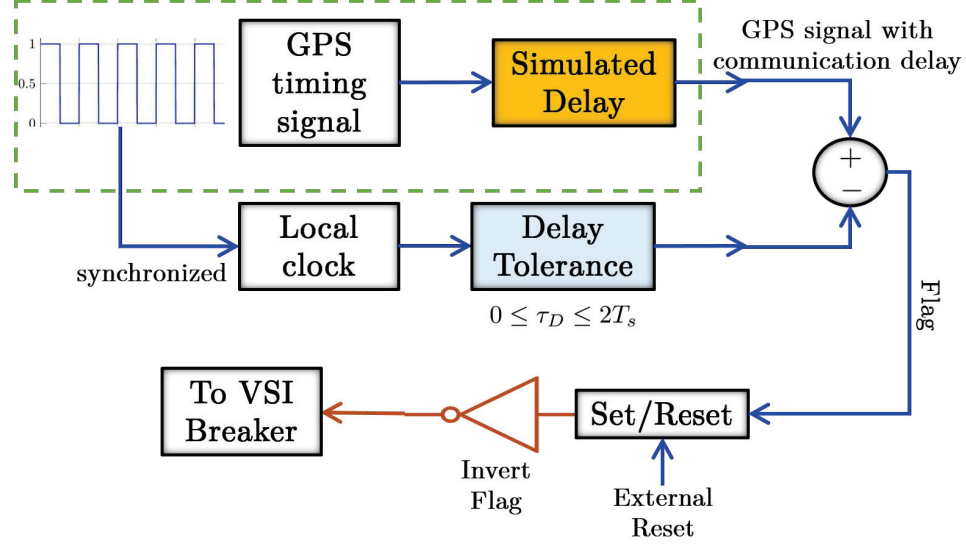


Figure 2.21: VSI isolation mechanism to locally detect delays in GPS clock signals w.r.t. to a user-specified tolerance in local clock pulse to isolate or reconnect VSI to MMG.

signals. In such a case, the VSI's local clock can wait for next GPS clock pulse and align with it using the GPS signal's rising edge while only incurring a delay of 1 cycle (for a GPS with 1 pulse every 60 Hz). For Assumption 3, the proposed mechanism can be easily modified to detect GPS signals in a window of time leads and lags and, if the GPS signal arrives before or after the window, the VSI can disconnect from the MMG to avoid large circulating currents.

The proposed VSI isolation mechanism is shown in Fig. 2.21. A delay block is used to implement delays in receiving GPS clock signals by the VSI. Following Assumptions 2 and 3, a local clock is passed through a delay tolerance block with a user-specified delay, τ_D . This block delays the local clock pulses with respect to the delayed GPS signal which are then compared using a comparator block. The comparator block generates a flag that is used as an input to a set/reset block. The output of the set/reset block is held at 'logic 0' if the delay in the GPS timing signal is less than the specified delay tolerance ($Flag = 0$) otherwise held at 'logic 1' ($Flag = 1$). The invert flag gate

inverts the output bit of set/reset to generate or set the reference generation signal to zero and to close or open the VSI breaker to MMG. Finally, the external reset signal is used to reset the set/reset block while re-synchronization to the MMG is done. Note that because of Assumption 3, the flag will only be set to 1 if GPS delay $\geq \tau_D$ s.

The VSI isolation mechanism is validated on the CHIL setup of Fig. 2.14. Note that measurements pertaining to current controlled VSIs are not shown here for simplicity. A scenario similar to Case 3 is followed. However, VSI-2 is chosen to have communication delays in receiving the GPS clock signals. For $30 \text{ s} \leq t \leq 47.15 \text{ s}$, the delay is set to be $T_s = 100 \text{ } \mu\text{s}$, for $47.15 \text{ s} \leq t \leq 57.45 \text{ s}$ delay of $2T_s$ and for $t > 57.45 \text{ s}$ delay of $3T_s$ is set. A delay tolerance of $\tau_D = T_s$ is chosen for VSI-2 local clock. The complete results of the CHIL experiment are shown in Fig. 2.20.

At $t = 47.15 \text{ s}$, the delay in receiving the GPS signal by VSI-2 increases from T_s to $2T_s$ (Fig. 2.20 (a) , (b)) as a result of which the VSI isolation mechanism sets VSI-2 breaker to off. VSI-2 output reference and hence, its output voltage, active, reactive power reduces to zero (Fig. 2.20 (c), (d), (e)). VSI-1, VSI-3 and VSI-4 compensate for the disconnection of VSI-2 and the output active and reactive power is maintained along with the desired sharing ratios. Note that VSI-2 keeps receiving GPS clock pulses while being disconnected from the MMG. At $t = 57.45 \text{ s}$, the delay between VSI-2's local clock and the GPS clock pulses reduces to T_s (Fig. 2.20 (f) , (g)). As a result, VSI-2 reference voltage generation begins (Fig. 2.20 (h)) and the VSI-2 breaker to MMG is switched On. This is further evident from the active and reactive power output of VSI-2 following re-connection at $t = 57.45 \text{ s}$. This validates the performance of the VSI isolation mechanism in the proposed isochronous VP-D architecture.

Remark 3.10 *Fig. 2.20 also demonstrates the ‘plug-and-play’ of the VSIs in the isochronous VP-D MMG as the controller of VSI-2 was never turned off, only the reference generation and VSI breaker were turned off following the detection of a delay (greater than*

tolerance) in receiving GPS signal. Once the delay was detected to be within tolerance limit, the reference generation signal started which by the virtue of the isochronous architecture is already synchronized to the MMG allowing the VSI breaker to be reconnected immediately.

2.6 Appendix

Proof of Theorem 1: The steady-state tracking error is:

$$\begin{aligned} & \lim_{s \rightarrow 0} s \left[[V_{refk}(s) - R_{vk}I_k(s)] - [G_k V_{refk}(s) - Z_k I_k(s)] \right] \\ &= \lim_{s \rightarrow 0} s(1 - G_k)V_{refk}(s) - \lim_{s \rightarrow 0} s(R_{vk} - Z_k)I_k(s) \end{aligned}$$

Now, G_k formulated in (2.4) can be written as:

$$G_k = \frac{G_{pk}K_{volk}}{1 + G_{pk}K_{volk}}$$

$$\text{where } G_{pk} = \frac{1}{C_k s} \frac{G_{ck}K_{curk}}{1 + G_{ck}K_{curk}}, \quad G_{ck} = \frac{1}{L_k s + R_k}$$

Suppose K_{volk} has poles at $\pm j\omega_o$. Hence, K_{volk} can be factored as $K_{volk} = \frac{n_{volk}}{d_{volk}(s^2 + \omega_o^2)}$. Thus, because G_{pk} has a pole at $s = 0$, it is evident that $(1 - G_k)$ will have a numerator with a factor $(s^2 + \omega_o^2)s$. This implies that $G_k(j\omega_o) = 1$. Also, from the final value theorem, it follows that:

$$\lim_{s \rightarrow 0} s(1 - G_k) \frac{\omega_o}{s^2 + \omega_o^2} = 0.$$

In a similar manner, we can rewrite the expression of Z_k as:

$$Z_k = G_k R_{vk} + (1 - G_k) \left(\frac{1}{sC_k} \right) \frac{1}{1 + G_{ck}K_{curk}}$$

Since $G_k(j\omega_o) = 1$, it follows that $Z_k(j\omega_o) = R_{vk}$. Also, from the final value theorem, it follows that:

$$\begin{aligned} & \lim_{s \rightarrow 0} s(R_{vk} - Z_k) \frac{\omega_o}{s^2 + \omega_o^2} \\ = & \lim_{s \rightarrow 0} s(1 - G_k) R_{vk} \frac{\omega_o}{s^2 + \omega_o^2} \\ & + \lim_{s \rightarrow 0} s(1 - G_k) \left(\frac{1}{sC_k} \right) \frac{1}{1 + G_{ck}K_{curk}} \frac{\omega_o}{s^2 + \omega_o^2} = 0 \end{aligned}$$

As the term $(1 - G_k)$ has a factor $s(s^2 + \omega_o^2)$ it follows that

$$\begin{aligned} & \lim_{s \rightarrow 0} s(1 - G_k) R_{vk} \frac{\omega_o}{s^2 + \omega_o^2} = 0 \\ & \lim_{s \rightarrow 0} s(1 - G_k) \left(\frac{1}{sC_k} \right) \frac{1}{1 + G_{ck}K_{curk}} \frac{\omega_o}{s^2 + \omega_o^2} = 0 \end{aligned}$$

This implies that in steady state the output of inner-loop controller tracks the input sinusoidal reference $v_{rk}(t) = v_{refk}(t) - R_{vk}i_k(t)$ with zero error.

2.6.1 Extension for Multiple Inverter System

The closed-loop averaged output voltage dynamics of (2.4) can be extended for N inverters and described by:

$$\underline{\mathbf{V}}(s) = \mathbf{G}\underline{\mathbf{V}}_{\text{ref}}(s) - \mathbf{Z}\underline{\mathbf{I}}(s) \quad (2.20)$$

where, $\underline{\mathbf{V}}(s) \triangleq [V_1(s) \ V_2(s) \ \dots \ V_N(s)]^T$, $\underline{\mathbf{V}}_{\text{ref}} \triangleq [V_{ref1}(s) \ V_{ref2}(s) \ \dots \ V_{refN}(s)]^T$, $\underline{\mathbf{I}}(s) \triangleq [I_1(s) \ I_2(s) \ \dots \ I_N(s)]^T$, $\mathbf{G} \triangleq \text{diag}(G_1, G_2, \dots, G_N)$ and $\mathbf{Z} \triangleq \text{diag}(Z_1, Z_2, \dots, Z_N)$. Assuming a linear load, the closed-loop expression of $\underline{\mathbf{V}}(s)$ and $\underline{\mathbf{I}}(s)$ can be formulated by using

the admittance matrix $\mathbf{Y}(s)$, as follows:

$$\underline{\mathbf{V}}(s) = [I - \mathbf{Z}\mathbf{Y}(s)]^{-1} \mathbf{G}\underline{\mathbf{V}}_{\text{ref}}(s) \quad (2.21a)$$

$$\underline{\mathbf{I}}(s) = \mathbf{Y}(s)[I - \mathbf{Z}\mathbf{Y}(s)]^{-1} \mathbf{G}\underline{\mathbf{V}}_{\text{ref}}(s) \quad (2.21b)$$

where $\mathbf{Y}(s) := \mathbf{\Lambda} - h(s)\mathbf{\lambda}\mathbf{\lambda}^T$, $h(s)^{-1} := Z_L(s)^{-1} + \sum_{k=1}^N r_k^{-1}$, $\mathbf{\lambda} = [r_1^{-1} \ r_2^{-1} \ \dots \ r_N^{-1}]^T$, and $\mathbf{\Lambda} = \text{diag}(r_1^{-1}, r_2^{-1}, \dots, r_N^{-1})$. In a similar way, $\underline{\mathbf{V}}_{\text{ref}}$ can be formulated by rewriting in matrix form:

$$\underline{\mathbf{v}}_{\text{ref}} = [E^* \mathbf{1} - \mathcal{N}(\underline{\mathbf{P}} - \underline{\mathbf{P}}^*)] \sin \omega_o t = \mathbf{E} \sin \omega_o t \quad (2.22)$$

where $\mathcal{N} \triangleq \text{diag}(n_1, n_2, \dots, n_N)$, $\underline{\mathbf{P}} \triangleq [P_1 \ P_2 \ \dots \ P_N]^T$, $\underline{\mathbf{P}}^* \triangleq [P_1^* \ P_2^* \ \dots \ P_N^*]^T$.

Proof of Theorem 2: For clarification, $\mathbf{R}_v := \text{diag}(R_{v1} \ R_{v2} \ \dots \ R_{vN})$. Also, $\underline{\mathbf{V}}_{\text{ref}}$ can be written as $\mathbf{E}\mu(s)$, where $\mu(s) = \frac{\omega_o}{s^2 + \omega_o^2}$ is the Laplace transform of a sinusoid with frequency ω_o . Using the Woodbury matrix identity and matrix manipulation:

$$\mathbf{Y}(s)^{-1} = \mathbf{\Lambda}^{-1} + \eta \mathbf{1}\mathbf{1}^T, \quad \eta := h(s)1 - h(s)\mathbf{\lambda}^T \mathbf{\Lambda}^{-1} \mathbf{\lambda}^{-1}$$

$$\begin{aligned} \text{Therefore, } \underline{\mathbf{I}}(s) &= (\mathbf{Y}(s)^{-1} + \mathbf{R}_v I)^{-1} \mathbf{E}\mu(s) \\ &= \left(\underbrace{(\mathbf{\Lambda}^{-1} + \mathbf{R}_v I)}_{\mathbf{\Lambda}_v^{-1}} + \eta \mathbf{1}\mathbf{1}^T \right)^{-1} \mathbf{E}\mu(s) \\ &= [\mathbf{\Lambda}_v - \eta \frac{\mathbf{\Lambda}_v \mathbf{1}\mathbf{1}^T \mathbf{\Lambda}_v}{1 + \eta \mathbf{1}^T \mathbf{\Lambda}_v \mathbf{1}}] \mathbf{E}\mu(s) \\ &= \mathbf{\Lambda}_v \mathbf{E}\mu(s) - \alpha \mathbf{\lambda}_v \mathbf{\lambda}_v^T \mathbf{E}\mu \end{aligned}$$

$$\text{where } \alpha^{-1} = Z_L^{-1} + \sum_m (r_m + R_{vm})^{-1}$$

$$\begin{aligned} \mathbf{\lambda}_v &= \left(\frac{1}{r_1 + R_{v1}} \quad \dots \quad \frac{1}{r_N + R_{vN}} \right)^T \\ \mathbf{\Lambda}_v &= \text{diag} \left(\frac{1}{r_1 + R_{v1}}, \dots, \frac{1}{r_N + R_{vN}} \right) \end{aligned}$$

Thus, it follows that for the k^{th} inverter in the steady state:

$$i_k(t) = I_k \sin(\omega_o t + \phi_k)$$

where:

$$I_k = \left(\sum_{m=1}^N \frac{E_m}{r_m + R_{vm}} \right) \frac{\gamma_k}{r_k + R_{vk}}$$

$$\gamma_k = \sqrt{(\beta_k - |\alpha| \cos \angle \alpha)^2 + |\alpha|^2 \sin^2 \angle \alpha}$$

$$\beta_k = E_k \left(\sum_{m=1}^N E_m (r_m + R_{vm})^{-1} \right)^{-1}$$

$$\cos \phi_k = (\beta_k - |\alpha(j\omega_o)| \cos \angle \alpha(j\omega_o)) / \gamma_k;$$

$$\sin \phi_k = (-|\alpha(j\omega_o)| \sin \angle \alpha(j\omega_o)) / \gamma_k;$$

By denoting $|\alpha| = |\alpha(j\omega_o)|$ and $\angle \alpha = \angle \alpha(j\omega_o)$,

$$\tan \phi_k = \frac{-|\alpha| \sin \angle \alpha}{\beta_k - |\alpha| \cos \angle \alpha(j\omega_o)} = \frac{\sin \angle \alpha}{\cos \angle \alpha - \frac{\beta_k}{|\alpha|}}$$

Now, from Theorem 1:

$$v_k(t) = V_{refk}(t) - R_{vk} i_k(t) = E_k \sin(\omega_o t + \psi_k)$$

$$\text{where } V_k = \sqrt{(E_k - R_{vk} I_k \cos \phi_k)^2 + (R_{vk} I_k \sin \phi_k)^2}$$

$$\text{and } \tan \psi_k = \frac{-R_{vk} I_k \sin \phi_k}{E_k - R_{vk} I_k \cos \phi_k} = \frac{\sin \phi_k}{\cos \phi_k - \frac{E_k}{R_{vk} I_k}}$$

Proof of Theorem 3: From the expression of $\tan \phi_k$ deduced in the previous proof.

Now, $\tan(\phi_k - \phi_j)$:

$$\begin{aligned}
&= \frac{\frac{\sin \angle \alpha}{\cos \angle \alpha - \beta_k/|\alpha|} - \frac{\sin \angle \alpha}{\cos \angle \alpha - \beta_j/|\alpha|}}{1 + \prod_{l=k,j} \left(\frac{\sin \angle \alpha}{\cos \angle \alpha - \beta_l/|\alpha|} \right)} \\
&= \frac{((\beta_k - \beta_j)/|\alpha|) \sin \angle \alpha}{\prod_{l=k,j} \left(\cos \angle \alpha - \beta_l/|\alpha| \right) + \sin^2 \angle \alpha} \\
&= \frac{(\delta_k - \delta_j) \sin \angle \alpha / (|\alpha| \boldsymbol{\lambda}_v^T (\mathbf{1} + \boldsymbol{\delta}))}{\prod_{l=k,j} \left(\cos \angle \alpha - \beta_l/|\alpha| \right) + \sin^2 \angle \alpha} \\
&= \frac{(\delta_k - \delta_j) \sin \angle \nu / (|\nu| (1 + \boldsymbol{\xi}^T \boldsymbol{\delta}))}{\prod_{l=k,j} \left(\cos \angle \nu - \frac{1 + \delta_l}{(1 + \boldsymbol{\xi}^T \boldsymbol{\delta}) |\nu|} \right) + \sin^2 \angle \nu} \\
&= \frac{\text{Im}(\nu) (\delta_k - \delta_j) / (1 + \boldsymbol{\xi}^T \boldsymbol{\delta})}{\prod_{l=k,j} \left(\text{Re}(\nu) - \frac{1 + \delta_l}{1 + \boldsymbol{\xi}^T \boldsymbol{\delta}} \right) + \text{Im}^2(\nu)}
\end{aligned}$$

where $\beta_k = E_k / \boldsymbol{\lambda}_v^T \mathbf{E} = E^* (1 + \delta_k) / \boldsymbol{\lambda}_v^T (1 + \boldsymbol{\delta}) E^*$, $\boldsymbol{\xi} = \boldsymbol{\lambda}_v / \boldsymbol{\lambda}_v^T \mathbf{1}$, $\nu := \alpha \boldsymbol{\lambda}_v^T \mathbf{1}$. Note that:

$$\text{Im}(\nu) = \text{Im}(\nu - 1) \approx -\text{Im}\left(1/Z_L \boldsymbol{\lambda}_v^T \mathbf{1}\right)$$

$$1 - \text{Re}(\nu) = \text{Re}\left(1/Z_L \boldsymbol{\lambda}_v^T \mathbf{1}\right)$$

Here, it is assumed that $|Z_L \boldsymbol{\lambda}_v^T \mathbf{1}| \gg 1$. Assuming that δ_k are small so that: $1 + \delta_k / (1 + \boldsymbol{\xi}^T \boldsymbol{\delta}) \approx 1$, it follows that

$$\begin{aligned}
\tan(\phi_k - \phi_j) &\approx -\boldsymbol{\lambda}_v^T \mathbf{1} |Z_L|^2 \text{Im}(1/Z_L) (\delta_k - \delta_j) \\
&= (\delta_k - \delta_j) \sum_{m=1}^N |Z_L| \sin \theta_L / (r_m + R_{vm}),
\end{aligned}$$

where $\cos \theta_L$ is the power factor of the load Z_L .

Proof of Corollary 3.1:

Note that

$$\nu = \frac{Z_L \boldsymbol{\lambda}_v^T \mathbf{1}}{1 + Z_L \boldsymbol{\lambda}_v^T \mathbf{1}} \Rightarrow 1 - \nu = \frac{1}{1 + Z_L \boldsymbol{\lambda}_v^T \mathbf{1}}.$$

Thus $\text{Img}(\nu) = \text{Img}(\nu - 1) = -\text{Img}(\frac{1}{1 + Z_L \boldsymbol{\lambda}_v^T \mathbf{1}}) \approx -\text{Img}(\frac{1}{Z_L \boldsymbol{\lambda}_v^T \mathbf{1}})$. Also $1 - \text{Re}(\nu) = \text{Re}\left(\frac{1}{Z_L \boldsymbol{\lambda}_v^T \mathbf{1}}\right)$. Here it is assumed that $|Z_L \boldsymbol{\lambda}_v^T \mathbf{1}| \gg 1$. Assuming that δ_k are small so that $\frac{1 + \delta_k}{(1 + \boldsymbol{\xi}^T \boldsymbol{\delta})} \approx 1$ it follows that

$$\begin{aligned} \tan(\phi_k - \phi_j) &\approx -\boldsymbol{\lambda}_v^T \mathbf{1} |Z_L|^2 \text{Img}\left(\frac{1}{Z_L}\right) (\delta_k - \delta_j) \\ &= \left(\sum_m \frac{|Z_L| \sin \theta_L}{r_m + Z_v} \right) (\delta_k - \delta_j), \end{aligned}$$

where $\cos \theta_L$ is the power factor of the load.

Chapter 3

Virtual Impedance Shaping for Low Voltage Microgrids

3.1 Introduction

AC Microgrids are becoming prevalent due to their economic and technical benefits compounded with the increased resiliency and reliability they offer [48]. However, the challenges of maintaining power quality in islanded AC microgrids require further investigation as non-linear loads are ubiquitous. Current operating standards for microgrids [49] require voltage and power quality to follow stringent limits even in the presence of such large number of non-linear loads for safe operation of the microgrid and its connected components.

Conventional droop-based approaches are not suitable for mitigating harmonic imbalances caused due to mismatches in harmonic current sharing among the various voltage source inverters (VSI) in a low-voltage (LV) microgrid with non-linear loads. These mismatches arise due to the sensitivity of the closed-loop output impedance at the VSI towards control methodology and line parameters. Virtual impedance based methods have shown promising applicability to shape the dynamic profile of the VSIs. Virtual impedance techniques have been employed for harmonic sharing and THD improvement [50], [51], [52], [53]. However, these approaches have limitations when applied to active (P), reactive (Q) and harmonic power sharing objectives. [54]

proposes a reactive power disturbance input in the conventional droop law to reduce reactive power sharing mismatch while affecting the active-power sharing accuracy and stability of the system frequency. [53] provide a signal injection method to mitigate power imbalances however, rendering additional complexities in employing this approach. Moreover, $P \sim Q$ decoupling characteristics change in LV AC microgrids with high R/X ratio. [55], [56] address the resistive output impedance but are not generalized to include complex impedances. [57] proposes an FFT-based harmonic extraction using a feed-forward loop that is highly dependent on parameter variation. [58] implements an inductive-capacitive virtual impedance but such approach can introduce unwanted resonances in the system. In [55], [56], [59] the equivalent output impedances are designed to be dominated by the virtual impedances to effectively reduce circulating currents and improving power sharing accuracy. However, line impedance mismatches are not considered. Secondary control approaches are considered in [60], [61] to improve P, Q sharing accuracy however, the centralized control strategies are complex and rely on availability of a communication network. [62] proposes a distributed apportioning method using voltage active power droop however harmonic power sharing is not considered.

In order to address these issues, a synchronous reference frame (SRF)-based harmonic extraction and shaping filter is proposed in this article for effective harmonic sharing between VSIs in an LV networks. SRF approaches show best performance under non-sinusoidal conditions if reactive-power sharing can be compensated by other means [63]. Thus, an isochronous Voltage-active power droop (VP-D) law is implemented to maintain active and reactive power sharing in the network. This implementation assumes the access to a timing signal in the microgrid for generating a frequency reference signal to the inverters such as a Global Positioning systems (GPS) receiver [64]. Advancements in GPS technology for providing timing signals and timing intervals at the accuracy of nanosecond timescales have become widely accessible and can be implemented for realizing such a microgrid. The SRF-based framework of [65], [66] is extended to achieve desired performance parameters in the system such as voltage regulation, P , Q , harmonic power sharing using nominal line parameters that can be estimated using various methods [67, 68].

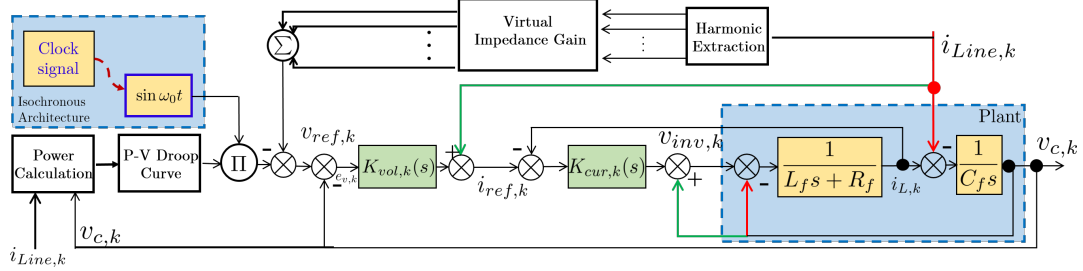


Figure 3.1: Virtual Impedance shaping with inner-outer controller and Voltage-Active Power Droop architecture

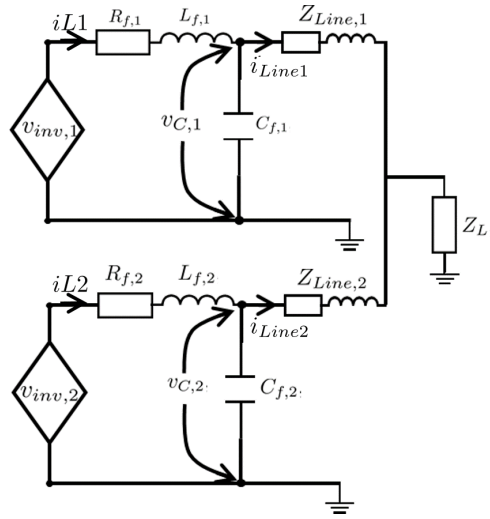


Figure 3.2: Two inverter setup with voltage source inverters, line parameters and common load.

3.2 System Description

3.2.1 Single Inverter System

The single inverter system comprises of a single-phase H-bridge inverter with an output LC-filter connected to a common load, Z_L , through a line impedance, $Z_{Line,k}$. The inverter is modeled as a controlled voltage source, $V_{inv,k}$. The dynamics of the k^{th} inverter employing average

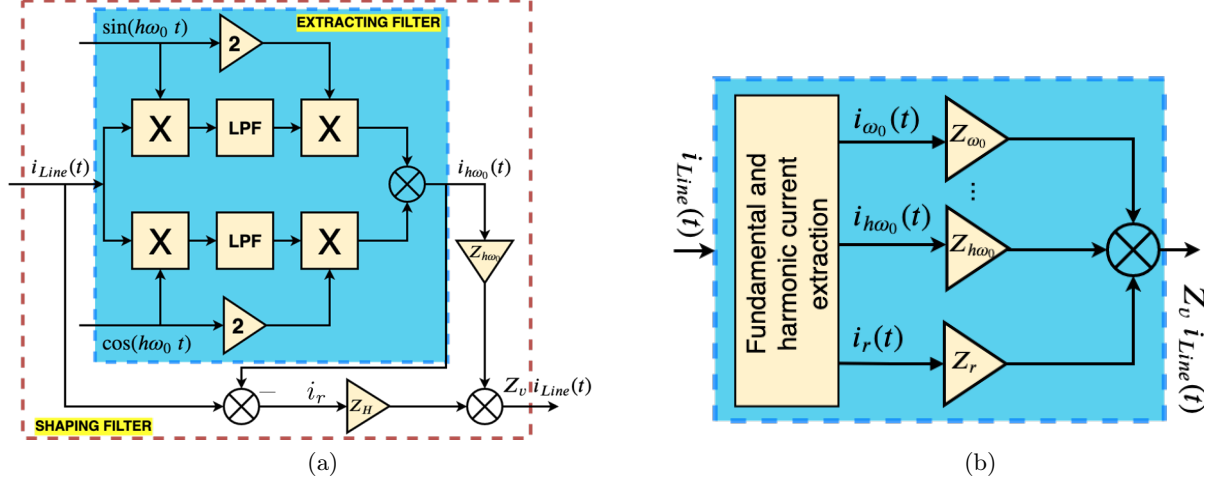


Figure 3.3: (a) Virtual impedance shaping and extracting filter of k^{th} inverter line current at $h\omega_0$ rad/s, (b) series of harmonic extracting filters for obtaining shaped reference voltage for k^{th} inverter.

model [36] are given as:

$$V_{c,k}(s) = G_{inv,k} V_{inv,k}(s) - G_{i,k} I_{Line,k}(s) \quad (3.1)$$

where $G_{inv,k} = \frac{1}{L_k C_K s^2 + R_k C_k s + 1}$ and

$$G_{i,k} = \frac{L_k s + R_k}{L_k C_K s^2 + R_k C_k s + 1}.$$

Here, $V_{c,k}(s)$ is the output voltage, $G_{inv,k}$ and $G_{i,k}$ are the transfer functions from the control input $V_{inv,k}$ to $V_{c,k}$ and $I_{Line,k}$ to $V_{c,k}$ respectively of the k^{th} VSI in laplace domain. The control design (shown in Fig. 1) is facilitated by a multi-loop controller with an outer voltage controller, $K_{vol,k}(s)$, based on the internal model principle design of a Proportional resonant controller structure [69] and an inner current loop, $K_{cur,k}$, to be a PI controller. The proposed

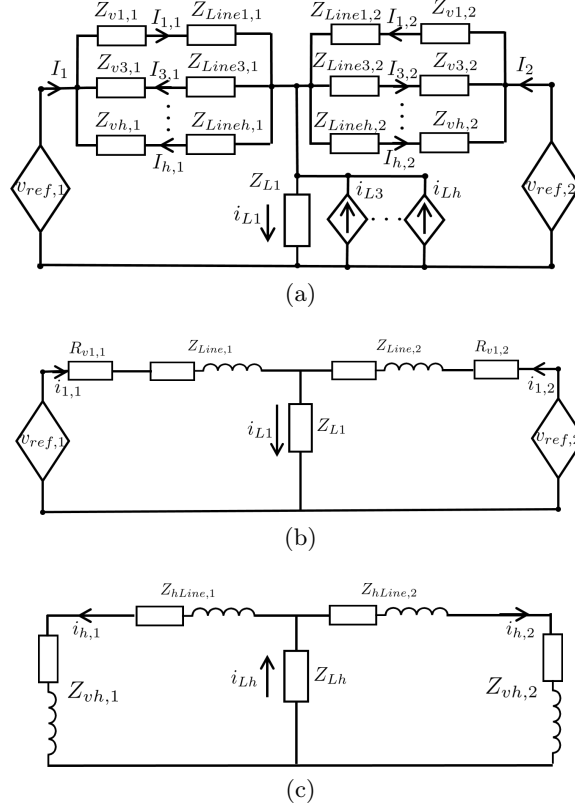


Figure 3.4: (a) Equivalent circuit of a two VSI interconnection , (b) equivalent circuit in fundamental domain, (c) equivalent circuit in harmonic domain.

control law of the inverter is given as:

$$V_{inv,k} = K_{ref,k} V_{ref,k}(s) - K_{v,k} V_{c,k}(s) - K_{ref,k} Z_{v,k} I_{Line,k}(s), \quad (3.2)$$

where $K_{ref,k} = K_{cur,k} K_{vol,k}$, $K_{v,k} = K_{ref,k} + K_{cur,k} C_k s - 1$. The two inverter microgrid setup is shown in Fig. 2.

3.2.2 Voltage-active power droop

An *outer voltage-active power droop law* (VP-D) is employed given as, $E_k = E^* - n_k(P_k - P_k^*)$, where E_k , E^* , n_k , P_k and P_k^* are inverter and reference output voltage magnitudes, droop

coefficient, measured output active power, and reference active power of the k^{th} inverter. The reference voltage to be tracked by each inverter is given by, $v_{ref,k} = E_k \sin \omega_0 t - \sum_h Z_{vhk} i_{Line,hk}$, where the time signal is obtained by an isochronous architecture to achieve accurate synchronization to a common clock. Here h denotes the harmonic number, $h = 1$ denotes the fundamental frequency component. Detailed analysis of the design of droop gains for such a system is provided in [70].

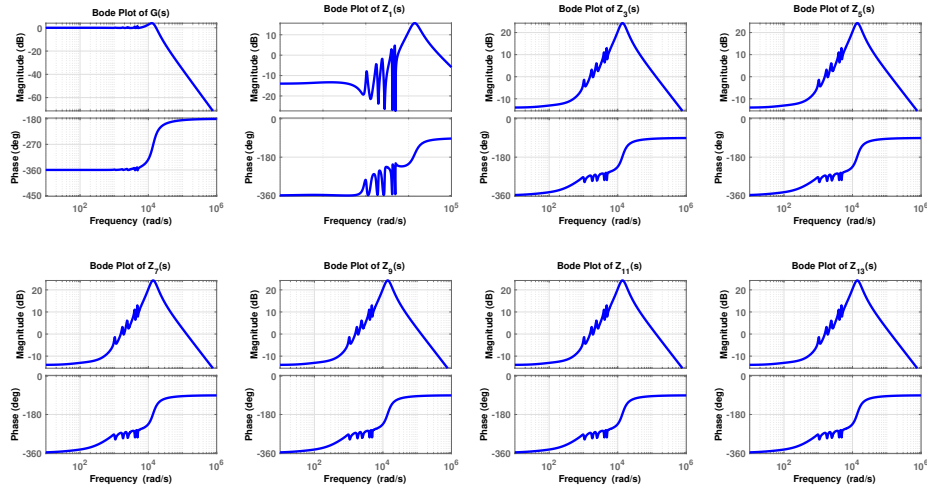


Figure 3.5: Bode plots of transfer function for voltage reference tracking $G(s)$ and virtual impedance designed for harmonic current sharing ($h = 1, 3, 5, 7, 9, 11, 13$) for the two-inverter setup.

3.2.3 Isochronous Architecture

An isochronous architecture is achieved by providing a common clock signal to all the inverters in the network. The reference voltage is provided to be $v_{ref,k} = E_k \sin \omega_0 t$. The advantages of such an architecture are i) the ability to operate the microgrid at a single frequency, thereby eliminating deviation of frequency that arises while using frequency-droop control, ii) simplified control of the inverters by implementing only VP-D control without explicitly controlling reactive power and, iii) enabling a ‘plug and play’ nature of the microgrid. Note that successful synchronization with the common clock and compliance with IEEE 1588-2008 standards [40], enabling accurate

and precise synchronization of the real-time clocks of the networked distributed devices to the common clock, is assumed throughout. It is imperative that such an architecture is robust to error in clock measurements or momentary loss of access to the common clock by one or more inverters in the system. Such a scenario can be mitigated by employing a corrective mechanism such as, a local PLL algorithm as proposed in [42].

3.2.4 Virtual Impedance Shaping and Extracting Filter

In order to obtain the harmonic components of the line current for each VSI, a direct component and a quadrature component are extracted in the synchronous reference frame. Fig. 3.3a shows the shaping and extracting filter that takes the k^{th} inverter line current, $i_{Line,k}(t)$ as input to extract the fundamental frequency component, $i_{\omega_0}(t)$ for ($h = 1$). The extracting filter receives sinusoidal modulation signals of frequency ω_0 rad/s.

Thus, the decomposition of the line current to obtain the h^{th} harmonic signal, is given as,

$$i_{h\omega_0}(t) = a_h \sin(h\omega_0 t) + b_h \cos(h\omega_0 t), \quad (3.3)$$

where $a_h = \frac{2}{T} \int_0^T i_{Line}(t) \sin(h\omega_0 t) dt$ and $b_h = \frac{2}{T} \int_0^T i_{Line}(t) \cos(h\omega_0 t) dt$, where T is the time-period of the periodic signal.

Remark: It is to be noted that the isochronous architecture allows for the local inverter controllers to have access to the common clock aiding in generation of modulation signals of appropriate frequency without additional complexities of implementing SRF-based phase-locked loops.

Selective harmonic components of the line current (see Fig. 3.3b), $i_{h\omega_0}(t)$ for $h = 3, 5, 7, \dots$, can be extracted by selecting respective harmonic modulation signals of frequency hf_0 Hz in the extracting filter in eq. (3.3). This approach is based on the extension of synchronous reference frame low pass filter to fundamental and harmonic frequency extraction for unbalanced nonlinear loads [65]. Virtual impedances are implemented as weighting functions to weigh the output of

the extracting filters to obtain a shaped output voltage reference, given as,

$$V_{ref,k} = E_k \sin \omega_0 t - \left(Z_{\omega_0} i_{\omega_0} + \sum_{h=3,5,7,\dots} Z_{h\omega_0} i_{h\omega_0} \right)$$

3.3 Analysis of Proposed Virtual Impedance Shaping Method

For k^{th} inverter, the Laplace transform of line current can be expressed as,

$$I_{Line,k}(s) = \sum_{h=1,3,5,\dots} I_{Line,hk}(s) \quad (3.4)$$

, where $h = 1$ denotes the fundamental frequency component.

substituting eq. (3.4) the control law given in eq. (3.1) and eq. (3.2), we get,

$$\begin{aligned} V_{c,k}(s) &= G_{inv,k}(s)V_{inv,k}(s) - G_{i,k}(s) \sum_{h=1,3,5,\dots} I_{Line,hk}(s) \\ &= G_{inv,k}(s) \left(K_{ref,k}(s)V_{ref,k}(s) - K_{v,k}(s)V_{c,k}(s) \right. \\ &\quad \left. - (K_{ref,k}(s)Z_{v,k} \sum_{h=1,3,5,\dots} I_{Line,hk}(s)) \right) \\ &\quad - G_{i,k}(s) \sum_{h=1,3,5,\dots} I_{Line,hk}(s) \\ &= G_{inv,k}(s) \left(K_{ref,k}(s)V_{ref,k}(s) - K_{v,k}(s)V_{c,k}(s) \right) \\ &\quad - \sum_{h=1,3,5,\dots} \left(G_{inv,k}(s)K_{ref,k}(s)Z_{vh,k}(s) \right. \\ &\quad \left. + G_{i,k}(s) \right) I_{Line,hk}(s) \end{aligned}$$

Hence, the inverter dynamics with the control law can be rewritten as,

$$V_{c,k} = G_k(s)V_{ref,k}(s) - \sum_{h=1,3,5,\dots} Z_{hk}(s)I_{Line,hk}(s), \quad (3.5)$$

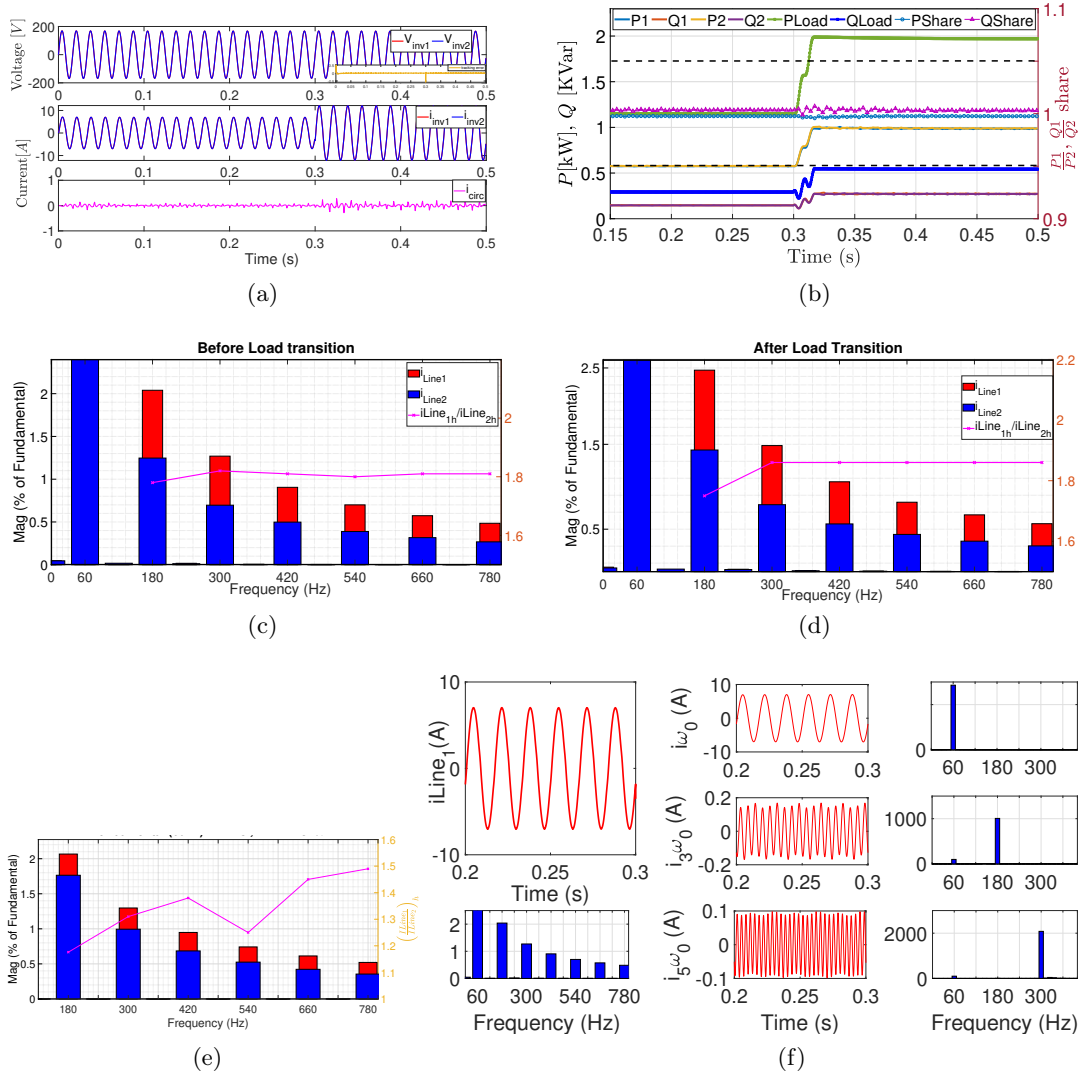


Figure 3.6: (a) Performance of proposed control law for voltage tracking and reduced circulating currents; (b) active, reactive power sharing accuracy; (c) harmonic current sharing before load transition for $t < 0.3s$; (d) harmonic current sharing after load transition at $t = 0.3s$; (e) harmonic current sharing between the two VSIs without virtual impedance shaping implemented; (f) Inverter 1 Line current with harmonic content (left); extracted fundamental (i_{ω_0}), third ($i_{3\omega_0}$) and fifth ($i_{5\omega_0}$) harmonics (middle); FFT analysis of the extracted signals (right).

where,

$$G_k(s) = \frac{G_{v_{inv},k}(s)K_{ref,k}(s)}{(1 + G_{v_{inv},k}(s)K_{v,k}(s))},$$

$$Z_{hk}(s) = \frac{(G_{v_{inv},k}(s)K_{ref,k}(s)Z_{vh,k} + G_{i,k})}{(1 + G_{v_{inv},k}(s)K_{v,k}(s))}.$$

The analysis of eq. (3.5) shows that the thevenin equivalent circuit of the VSI with the proposed control law and extracting filters can be decoupled into its constituent harmonics domain (See Fig. 3.4a) . Thus, the inverter interconnection can be analyzed in fundamental and harmonic circuits (as shown in Fig. 3.4b and Fig. 3.4c). In the harmonic domain, inverter output impedances are modeled as virtual impedances, $Z_{vh,k} = R_{v,k} + jh\omega_0 L_{v,k}$.

In order to achieve harmonic current sharing, the virtual impedance for h^{th} harmonics for inverter k can be designed

$$Z_{vh,k} + Z_{Line,hk} = Z_{tot,hk}, \quad (3.6)$$

for $h = 3, 5, \dots$ (Fig. 3.5) for a known nominal $Z_{tot,hk}$. Analysis of the harmonic domain also reveals that for the judicious choice of virtual impedances for given inverter k such that, $Z_{vh,k} = -Z_{Line,hk}$ results in generating a reference $v_{ref,k}$ for the isochronous VP-D-based control loop that allows for maximum mitigation of harmonic drops in output voltage at point of common coupling (PCC).

3.4 Results

3.4.1 Simulation-based validation

SIMULINK/MATLAB simulations are performed with the setup shown in Fig. 3.1 with the parameters given in Table 3.2. The two inverter system consists of VSIs of capacity 2.2kVA (1-ph, 120 V_{rms}) each implementing the proposed VP-D multi-loop controller to share active power of 1.1 kW (reactive power of 290 Var) with a load transition occurring at time $t = 0.3$ seconds to 2 kW (reactive power of 500 Var) symmetrically i.e. with an active power sharing

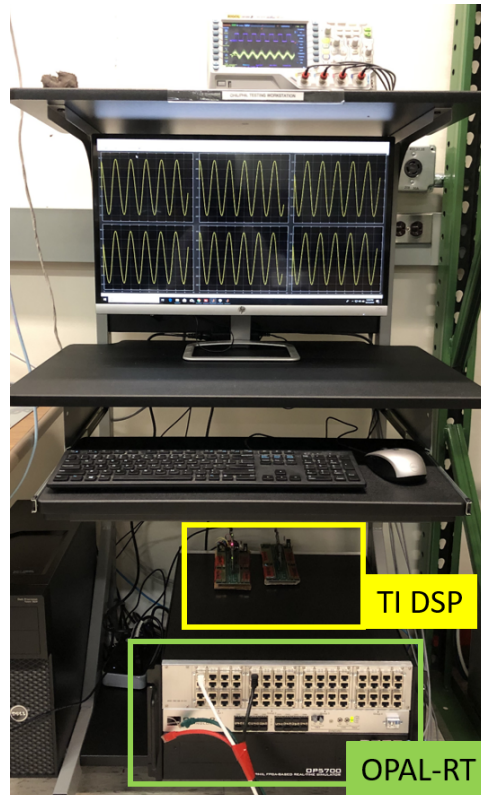


Figure 3.7: Two VSI setup for CHIL-based validation on OPAL-RT with TI DSP controllers.

ratio of 1 along with an increase in harmonic current demand of the load.

The performance of the proposed control law is shown in Fig. 6. Inverter output voltages and line currents are shown in Fig. 3.6a. It can be seen that the output voltages of both the inverters track the reference signal as shown by the tracking error along with the small circulating currents between the VSIs. Fig. 3.6b shows the active and reactive power sharing accuracy of the proposed architecture with virtual impedance design. The active power is shared by the two VSIs symmetrically as commanded by the outer droop law. It is important to note that the reactive power shared by both the VSIs are also symmetrical even when reactive power is not explicitly controlled in the proposed VP-D architecture. The non-linearities in the common load, Z_L , are modeled to have a total demand distortion (TDD) of 5% in compliance with IEEE standards for allowable harmonic currents in an electric power system [71]. TDD is considered here as it is a measure of the total harmonic current distortion against the full load demand of the power system. A 13^{th} order extracting filter is implemented for each voltage source inverter. Fig. 3.5 shows the bode plots of the complementary sensitivity transfer function $G(s)$ from $V_{ref}(s)$ to $V_c(s)$ as well as the output impedances Z_h for $h = 1$ to $h = 13$. The inductive component of the virtual impedance is realized by a second order generalized integrator (SOGI). The virtual impedances are designed to dominate line impedances and provide a harmonic current sharing ratio of $\left(\frac{I_{Line1}}{I_{Line1}}\right)_h = 1.82$. Results of this harmonic sharing before and after load transition at $t = 0.3$ are shown to meet the harmonic sharing ratio ($< \pm 5\%$) in Fig. 3.6c and Fig. 3.6d respectively. For comparison, harmonic current sharing of the two inverter setup without implementing virtual impedance shaping is shown in Fig. 3.6e. Finally, Fig. 3.6f shows the efficacy of the extracting filters where the output of the filters for the fundamental (relative to DC) and harmonic number 3 and 5 for line current of inverter 1.

An asymmetric power sharing case is shown with $Z_{Line2} = 0.05\Omega + jh\omega_0 0.17mH$. In this case the virtual impedances are designed to share the harmonic currents symmetrically and the active power asymmetrically with a sharing ratio of 1.3. An FFT analysis of the inverter line currents (Fig. 3.8b and Fig. 3.9b) shows the harmonic current sharing to be significantly improved as shown in Table 3.1. Again a reactive power sharing ratio of 1.1 is maintained with

the proposed VP-D architecture.

Fig. 3.10 shows a case where virtual impedances are designed to limit the circulating currents (and phase mismatches in line currents) between the two inverters. The virtual impedance shaping method resulted in reducing the phase lag between the output currents of the inverters, thereby reducing circulating current from 0.75 amps to 0.38 amps. THD of PCC voltage is also improved by 64% (from 1.01 to 0.36).

Table 3.1: Harmonic current sharing ratio for CHIL

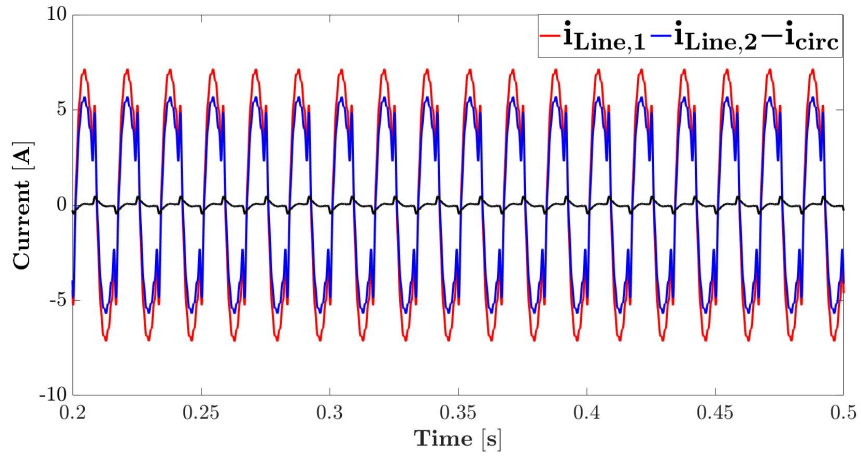
$\frac{I_{inv1,h}}{I_{inv2,h}}$	Without Virtual Impedance shaping	With Virtual Impedance Shaping
h=3	0.35	1.02
h=5	1.41	1.06
h=7	1.39	1.03
h=9	0.41	1.03
h=11	0.41	1.045
h=13	0.41	1.00

3.4.2 CHIL validation

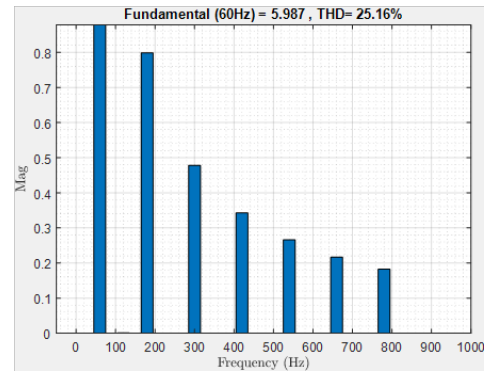
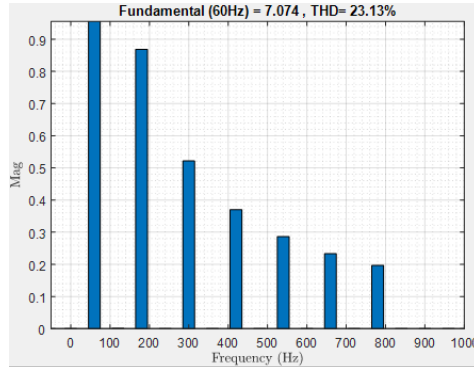
Controller-hardware-in-the-loop(CHIL)-based validations are performed for the two VSI-setup along with the parameters shown in Table 3.2 where the two line impedances are unequal. The CHIL setup (Fig. 3.7) consists of external TI DSP F28379D (delfino) micro-controller units to implement the proposed VIS method and demonstrate its applicability for real-world low cost embedded controllers. Discretized shaping and extracting filters are implemented and the results are shown in Fig. 3.11 through Fig. 3.13 with VIS implementation to achieve equal harmonic sharing (≈ 1.05). The CHIL simulation also validates accuracy of active and reactive power sharing to be close to 1 with the proposed VP-D control.

Table 3.2: Simulation Parameters

Parameters	Values
Nominal Frequency at PCC ($f_o = \frac{\omega_o}{2\pi}$)	60 Hz
Rated Power of Inverter ₁	2.2 kVA
Rated Power of Inverter ₂	2.2 kVA
Extracting Filter Order	13
Parameters for MATLAB/SIMULINK simulations	
Nominal Voltage at PCC (E^*)	$120\sqrt{2}$ V
R-L Load	2 kW / 1 kVAR (5 % TDD)
Z_{Line1}	$0.19\Omega + jh\omega_0 0.1\text{mH}$
Z_{Line2}	$0.2\Omega + jh\omega_0 0.17\text{mH}$
Parameters for CHIL experiments	
Nominal Voltage at PCC (E^*)	$240\sqrt{2}$ V
R-L Load	4 kW / 1 kVAR (5 % TDD)
Z_{Line1}	$0.02\Omega + jh\omega_0 0.05\text{mH}$
Z_{Line2}	$0.01\Omega + jh\omega_0 0.08\text{mH}$
Z_{Tot}	$0.04\Omega + jh\omega_0 0.1\text{mH}$



(a)



(b)

Figure 3.8: Two-inverter setup results without implementing virtual impedance loop; (a) Inverter line and circulating currents, (b) FFT analysis of Inv 1 and Inv 2 output currents.

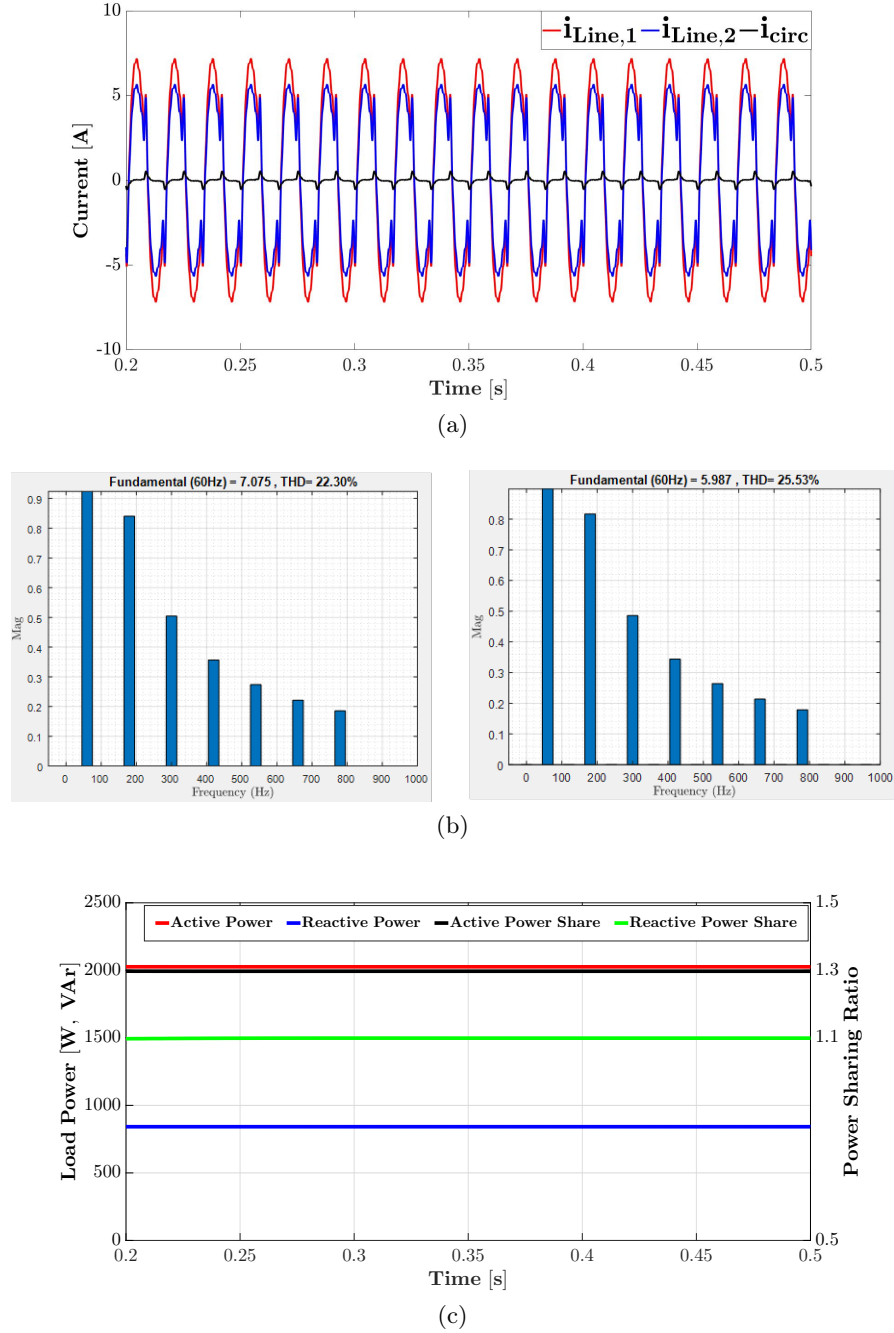


Figure 3.9: Two-inverter setup results with virtual impedance loop; (a) Inverter line and circulating currents, (b) FFT analysis of Inv 1 and Inv 2 output currents, (c) Active, reactive power sharing with VP-D .

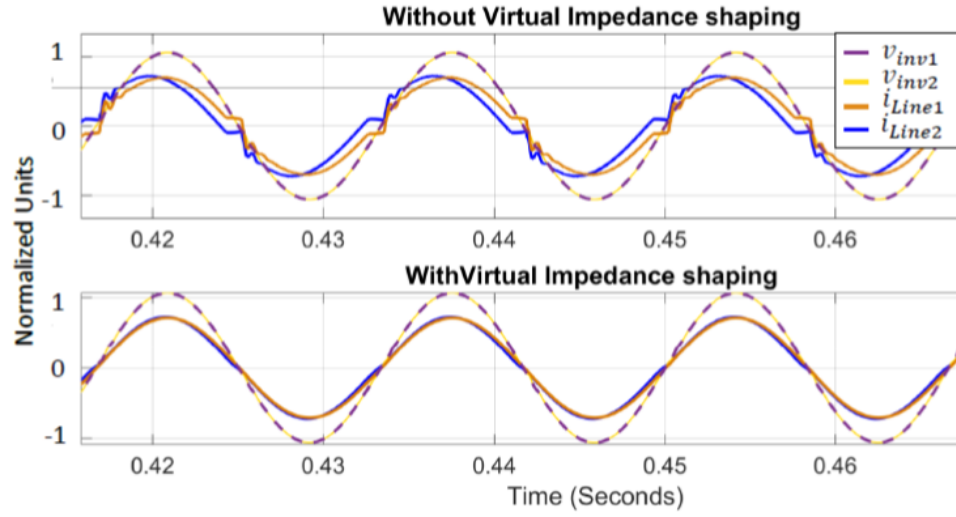


Figure 3.10: Inverter output voltages and line currents for improving circulating current with virtual impedance shaping

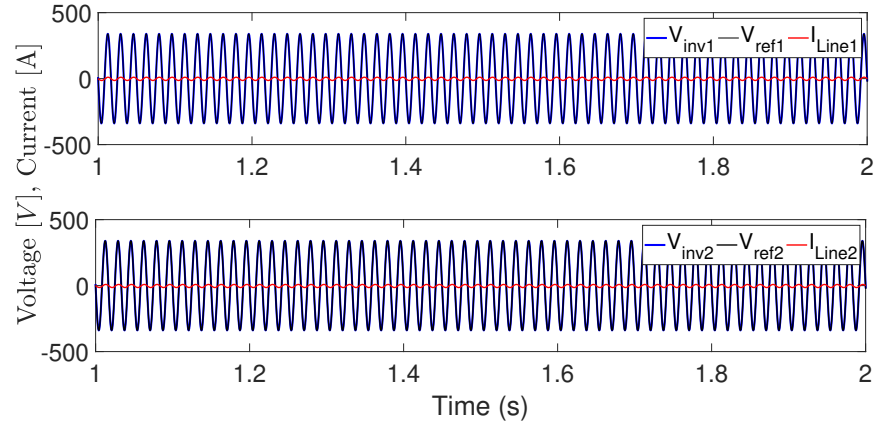


Figure 3.11: Performance of the proposed control of VIS units in CHIL simulations.

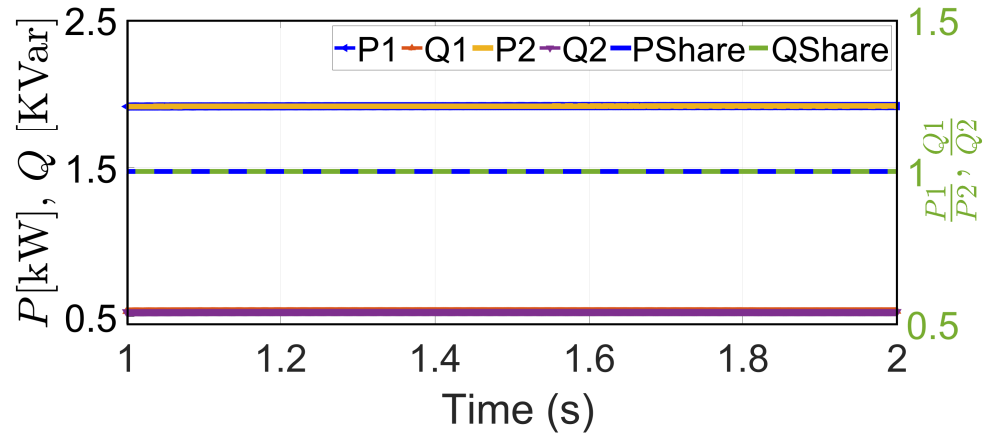


Figure 3.12: Active, reactive power sharing accuracy with VP-D in CHIL simulations.

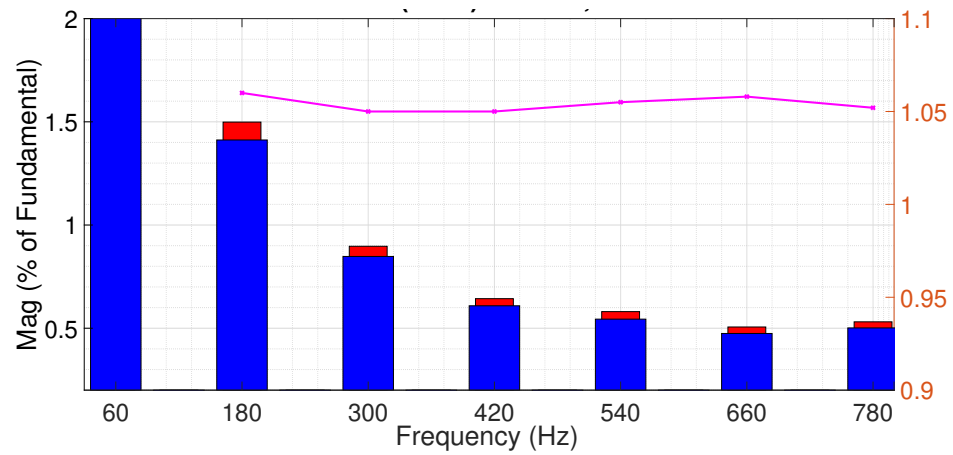


Figure 3.13: Harmonic sharing performance in CHIL simulations for the two VSI system.

Chapter 4

Distributed Finite Time Termination of Ratio Consensus for Averaging

Coordination of multi-agent systems has found applications across multiple disciplines like economics, neuroscience, social sciences and control of large scale systems [72]. One of the well studied problems in the coordination of multi-agent systems is the consensus problem [73] where all the agents in the network strive to agree on certain variables of interest by sharing information with their neighbors. Applications of the consensus framework include parallel computers [74], distributed coordination of mobile autonomous agents [75], distributed data fusion in sensor networks [76], and large scale power networks [77]. here, the size of the systems presents severe restrictions in terms of computational resources, local communications and sensing capabilities [73]. Due to these limitations coordination of multiple agents in large scale networked system calls for distributed algorithms rather than centralized algorithms.

A special case of consensus is the averaging protocol which computes the average of the initial values of variables of interest through local communication [78, 79]. Under the assumptions of connectedness and aperiodicity of the stochastic weight matrix, in the average consensus

protocol, every node *asymptotically* attains a value which is the average of the initial conditions of all agents in the network. The asymptotic nature of convergence of the average consensus protocol is a problem for real-time applications, where the consensus value is used for a subsequent purpose. Moreover, the nodes keep updating their states and communicating forever and hence, power, communication and computational resources are wasted that can be prohibitive in resource constrained applications. One such example being large sensor networks where the power to each node is limited. In such situations, it is necessary for each node to detect without a central authority if convergence is achieved within a specified error margin and terminate its updates and communication in finite time.

Manitara et al propose a scheme based on event-triggered strategies for finite-time termination of average consensus in [80], but this scheme requires that the communication network be undirected and does not provide a mechanism for each node to independently determine when approximate consensus is reached. Another scheme proposed in [81] requires the computation of a minimal polynomial by each node to determine the finite-time stopping criterion which makes it computationally expensive. The problem of terminating the average consensus in finite-time in the presence of time-varying but bounded delays is presented in [82]. It relies on computing and storing Hankel matrices at every iteration, which can be computationally expensive if the size of matrix is large; here the size depends on the network size as well as the number of iterations needed for convergence. The authors of [83] propose a mechanism for approximate consensus where edge weights are adjusted as algorithm proceeds thereby activating/deactivating links. The method in [83] relies on obtaining pairwise comparison between its own value and the values of each of its neighbors, and, decides whether to transmit values to neighbors or not at every iteration. The work in this article is based on an entirely different approach, first presented in [2] on distributed finite time stopping criterion for average consensus, which has lowered complexity and needs lesser computational resources. In [2], each node while running the average consensus protocol also simultaneously runs maximum consensus and minimum consensus protocols, thereby enabling each node to detect the occurrence of convergence within a specified tolerance in finite time in a distributed manner. While this algorithm is computationally inexpensive, it is restricted to communication weight matrix being doubly

stochastic. It is difficult to realize a doubly stochastic weight matrix in a distributed manner in a directed graph [83]. A solution to this problem was proposed in [84] where distributed averaging is achieved using ratio consensus under the assumption that the communication weight matrix is column stochastic. The assumption of column stochastic weight matrix allows for the flexibility to choose weights in a distributed manner; a simple choice of weights for a node is the inverse of the number of out-neighbors of a node (that is, the number of nodes to which the state of the node under consideration is an input) [84]. Thus, any agent has to only know its own out-degree to assign the weights to its outgoing links. Hence, column stochastic matrices along with the ratio consensus protocol form an important class for distributed averaging of initial values of the agents. [11] combines the ideas proposed in [2] and [84] for finite-time resource allocation (frequency regulation specifically) in a network of islanded AC microgrids thereby demonstrating the practical utility of these algorithms. However, [11] does not provide provable guarantees for the same.

4.1 Definitions

In this section we present basic notions of graph theory and linear algebra which are essential for the subsequent developments. Detailed description of graph theory and linear algebra notions are available in [85] and [86] respectively.

Definition 1 (*Cardinality of a set*) Let A be a set. The cardinality of a set A denoted by $|A|$ is a measure of the number of elements of the set A .

Definition 2 (*Directed and Undirected Graph*) A directed graph G is a pair (V, E) where V is a set of vertices or nodes and E is a set of edges, which are ordered subsets of two distinct elements of V . If an edge from $j \in V$ to $i \in V$ exists then it is denoted as $(i, j) \in E$. An undirected graph G is a pair (V, E) where V is a set of vertices or nodes and E is a set of edges such that for every pair of distinct nodes $i \in V$ and $j \in V$, if $(i, j) \in E$ then $(j, i) \in E$.

Definition 3 (*Path*) In an undirected graph, a path between nodes $i \in V$ and $j \in V$ is a sequence of distinct edges of G of the form $(i, k_1), (k_1, k_2), \dots, (k_{m-1}, k_m), (k_m, j)$ and $(j, k_m), (k_m, k_{m-1}), \dots, (k_2, k_1), (k_1, i)$.

In a directed graph, a directed path from node i to j exists if there is a sequence of distinct directed edges of G of the form $(k_1, i), (k_2, k_1), \dots, (j, k_m)$.

Definition 4 (*Connected and Strongly Connected Graph*) An undirected graph is connected if it has a path between each pair of distinct nodes i and j . A directed graph is strongly connected if it has a directed path between each pair of distinct nodes i and j .

Definition 5 (*Neighbor Set*) Given an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ and a node $j \in \mathcal{V}$, the neighbor set of j is defined as $\mathcal{N}_j := \{i | (j, i) \in \mathcal{E} \text{ or } (i, j) \in \mathcal{E}\}$. In the context of this paper, information exchange is possible between node j and members of $\mathcal{N}_j$. We assume that information can flow in both directions along an undirected edge.

Definition 6 (*In-neighbor Set and Out-neighbor Set*) In a graph $\mathcal{G} := (\mathcal{V}, \mathcal{E})$, the in-neighbor set, $\mathcal{N}_j^-$, and out-neighbor set, $\mathcal{N}_j^+$, of node $j \in \mathcal{V}$, are given as $\mathcal{N}_j^- := \{i | (j, i) \in \mathcal{E}, i \neq j\}$ and $\mathcal{N}_j^+ := \{i | (j, i) \in \mathcal{E}, i \neq j\}$ respectively. The cardinalities of the in-neighbor set and out-neighbor set of a node $i \in \mathcal{V}$ are called the in-degree (D_i^-) and out-degree of the node (D_i^+) respectively.

Definition 7 (*Column Stochastic Matrix*) A real $n \times n$ matrix $A = [a_{ij}]$ is called a column stochastic matrix if $1 \geq a_{ij} \geq 0$ for $1 \leq i, j \leq n$ and $\sum_{i=1}^n a_{ij} = 1$ for $1 \leq j \leq n$.

Definition 8 (*Irreducible Matrix*) A $N \times N$ matrix A is said to be irreducible if for any $i, j \in \{1, \dots, N\}$, there exist $m \in \mathbb{N}$ such that $(A^m)(i, j) > 0$, that is, it is possible to reach any state from any other state in a finite number of hops.

Definition 9 (*Primitive Matrix*) A non negative matrix A is primitive if it is irreducible and has only one eigenvalue of maximum modulus.

Definition 10 (*Diameter of a Graph*) The diameter of a graph is the longest shortest path between any two nodes in the network.

4.2 Averaging using Ratio Consensus

In this section, the key result from [84], which enables reaching average consensus using a ratio of two states maintained by each node is summarized. Consider a directed graph $G = \{V, E\}$ of N nodes. Each node $i \in V$ maintains two states at time k , denoted by $x_i(k)$ (referred as numerator state of node i) and $y_i(k)$ (referred as denominator state of node i). Node i updates its state at the $(k+1)^{th}$ iteration according to the following policy:

$$x_i(k+1) = p_{ii}x_i(k) + \sum_{j \in N_i} p_{ij}x_j(k), \text{ and,} \quad (4.1)$$

$$y_i(k+1) = p_{ii}y_i(k) + \sum_{j \in N_i} p_{ij}y_j(k). \quad (4.2)$$

Lemma 4.2.1 *Suppose that $y_i(0) = 1$ for all $i \in V$ and P be a primitive column stochastic matrix with $P(i, i) > 0$. Then, $y_i(k) > 0$ for all $k \in \mathbb{N}, i \in V$.*

Proof: Clearly, $(P^k)(i, i) \geq (P^{k-1})(i, i)P(i, i), k \in \mathbb{N}$. It follows that, $P^k(i, i) \geq (P(i, i))^k > 0$. Thus, from $y(k) = P^k y(0)$, it is seen that,

$$\begin{aligned} y_i(k) &= \sum_{j \in N_i \cup i} (P^k)(i, j)y_j(0) \\ &\geq (P^k)(i, i)y_i(0) \\ &\geq (P(i, i))^k y_i(0) > 0, \text{ for all } k \in \mathbb{N}, i \in V. \end{aligned}$$

□ The above lemma enables us to ensure that the ratio $x_i(k)/y_i(k)$ is well defined for all $k \in \mathbb{N}, i \in V$ and is used in the next theorem.

Theorem 4 *Suppose the weight matrix P with $P(i, j) = p_{ij}$ associated with the directed graph G is primitive and column stochastic with $P(i, i) > 0$ for all $i \in V$. Let the initial conditions for the numerator states be $\{x_1(0), x_2(0), \dots, x_N(0)\}$ and for the denominator states be $y_i(0) = 1$ for all $i \in V$. Then the ratio $\frac{x_i(k)}{y_i(k)}$ asymptotically converges to $\frac{\sum_{i=1}^N x_i(0)}{N}$ for all $i = 1, \dots, N$,*

that is, the ratio of the numerator and denominator states converge to the average of the initial conditions (referred as ratio consensus).

Proof: See [84] for proof. □

4.3 Distributed Finite Time Termination

In this section, first results based on the update rules (4.1) and (4.2) are established followed by the definitions and convergence of Max-Min consensus algorithms. Subsequently, a finite-time termination criterion for average consensus is developed based on these results. Let us consider the maximum value of the ratio of consensus protocols given by (4.1) and (4.2) over all nodes at any time instant k be given by,

$$M(k) := \max_{i \in V} \frac{x_i(k)}{y_i(k)}, y_j(k) \neq 0, j \in V \quad (4.3)$$

$$m(k) := \min_{i \in V} \frac{x_i(k)}{y_i(k)}, y_j(k) \neq 0, j \in V \quad (4.4)$$

Lemma 4.3.1 *Consider the update rule (4.1), (4.2) with the assumptions of Theorem 4. Then for all time instants $k' \geq k$ and for all $i \in V$,*

$$\frac{x_i(k')}{y_i(k')} \leq M(k) \quad (4.5)$$

and

$$\frac{x_i(k')}{y_i(k')} \geq m(k). \quad (4.6)$$

Proof: We prove the theorem using induction. By definition of $M(k)$, for $k' = k$, the proof is trivial. For $k' = k + 1$,

$$\begin{aligned} \frac{x_i(k+1)}{y_i(k+1)} &= \frac{\sum_{j \in N_i \cup \{i\}} p_{ij} x_j(k)}{\sum_{j \in N_i \cup \{i\}} p_{ij} y_j(k)} \\ &= \frac{p_{ii} x_i(k) + \sum_{j \in N_i} p_{ij} x_j(k)}{p_{ii} y_i(k) + \sum_{j \in N_i} p_{ij} y_j(k)} \\ &= \frac{p_{ii} \frac{x_i(k)}{y_i(k)} + \frac{1}{y_i(k)} \sum_{j \in N_i} p_{ij} x_j(k)}{p_{ii} + \frac{1}{y_i(k)} \sum_{j \in N_i} p_{ij} y_j(k)}. \end{aligned}$$

Using (4.3), it follows that,

$$\begin{aligned} \frac{x_i(k+1)}{y_i(k+1)} &\leq \frac{p_{ii} M(k) + \frac{1}{y_i(k)} \sum_{j \in N_i} p_{ij} M(k) y_j(k)}{p_{ii} + \frac{1}{y_i(k)} \sum_{j \in N_i} p_{ij} y_j(k)} \\ &\leq \frac{x_i(k+1)}{y_i(k+1)} \leq M(k). \end{aligned}$$

Suppose it is asserted that for $k' = k + l, l \geq 1$, $\frac{x_i(k+l)}{y_i(k+l)} \leq M(k)$ for all $i \in V$. Then we have,

$$\begin{aligned} \frac{x_i(k+l+1)}{y_i(k+l+1)} &= \frac{\sum_{j \in N_i \cup \{i\}} p_{ij} x_j(k+l)}{\sum_{j \in N_i \cup \{i\}} p_{ij} y_j(k+l)} \\ &= \frac{p_{ii} \frac{x_i(k+l)}{y_i(k+l)} + \frac{1}{y_i(k+l)} \sum_{j \in N_i} p_{ij} x_j(k+l)}{p_{ii} + \frac{1}{y_i(k+l)} \sum_{j \in N_i} p_{ij} y_j(k+l)}. \end{aligned}$$

It follows from the inductive assumption that,

$$\begin{aligned} \frac{x_i(k+l+1)}{y_i(k+l+1)} &\leq \frac{p_{ii} M(k) + \frac{1}{y_i(k+l)} \sum_{j \in N_i} p_{ij} M(k) y_j(k+l)}{p_{ii} + \frac{1}{y_i(k+l)} \sum_{j \in N_i} p_{ij} y_j(k+l)} \\ &= M(k). \end{aligned}$$

This implies that $\frac{x_i(k+l+1)}{y_i(k+l+1)} \leq M(k)$ for all $i \in V$. This completes the proof for (4.5). The proof of (4.6) is similar and is left to the reader. $\square$

Lemma 4.3.2 Consider the update rule (4.1) and (4.2) with the assumptions of Theorem 4 where the initial time instant is k . Let $M(k) := \max_{j \in V} \frac{x_j(k)}{y_j(k)}$ and $m(k) := \min_{j \in V} \frac{x_j(k)}{y_j(k)}$. Let i be a

node such that $\frac{x_i(k')}{y_i(k')} < M(k)$ and let j be a node such that $\frac{x_j(k')}{y_j(k')} > m(k)$ for some time instant $k' \geq k$. Then for all time instants $k'' \geq k'$, $\frac{x_i(k'')}{y_i(k'')} < M(k)$ and $\frac{x_j(k'')}{y_j(k'')} > m(k)$.

Proof: The proof is based on induction. It follows from the hypothesis that for time instant $k'' = k'$, $\frac{x_i(k')}{y_i(k')} < M(k)$.

Suppose the assertion is that for some integer $q \geq 1$ and for $k'' = k' + q$, $\frac{x_i(k'+q)}{y_i(k'+q)} < M(k)$. Then we have,

$$\begin{aligned} \frac{x_i(k' + q + 1)}{y_i(k' + q + 1)} &= \frac{\sum_{j \in N_i \cup \{i\}} p_{ij} x_j(k' + q)}{\sum_{j \in N_i \cup \{i\}} p_{ij} y_j(k' + q)} \\ &= \frac{p_{ii} x_i(k' + q) + \sum_{j \in N_i} p_{ij} x_j(k' + q)}{p_{ii} y_i(k' + q) + \sum_{j \in N_i} p_{ij} y_j(k' + q)} \\ &= \frac{p_{ii} \frac{x_i(k'+q)}{y_i(k'+q)} + \frac{1}{y_i(k'+q)} \sum_{j \in N_i} p_{ij} x_j(k' + q)}{p_{ii} + \frac{1}{y_i(k'+q)} \sum_{j \in N_i} p_{ij} y_j(k' + q)}. \end{aligned}$$

Using the inductive assertion it follows that,

$$\frac{x_i(k' + q + 1)}{y_i(k' + q + 1)} < \frac{p_{ii} M(k) + \frac{1}{y_i(k'+q)} \sum_{j \in N_i} p_{ij} x_j(k' + q)}{p_{ii} + \frac{1}{y_i(k'+q)} \sum_{j \in N_i} p_{ij} y_j(k' + q)}.$$

Using Lemma 4.1, it follows that,

$$\begin{aligned} &\frac{x_i(k' + q + 1)}{y_i(k' + q + 1)} \\ &< \frac{p_{ii} M(k) + \frac{1}{y_i(k'+q)} \sum_{j \in N_i} p_{ij} y_j(k' + q) M(k)}{p_{ii} + \frac{1}{y_i(k'+q)} \sum_{j \in N_i} p_{ij} y_j(k' + q)} \\ &= M(k). \end{aligned}$$

This completes the proof. The other inequality can be proven similarly and is left to the reader.

□

Remark 4.1 Note that $\frac{x_i(k')}{y_i(k')} \leq M(k)$, for $k' \geq k$ follows from Lemma 4.1 Lemma 4.2 emphasizes the strict inequality $\frac{x_i(k'')}{y_i(k'')} < M(k)$ for $k'' > k'$ if at any time instant k' , the inequality $\frac{x_i(k')}{y_i(k')} < M(k)$ is satisfied. Similar remark holds for the min value as well.

Lemma 4.3.3 Consider the update rule (4.1) and (4.2) with the assumptions of Theorem 4 with initial time k such that $m(k) = \min_{j \in V} \frac{x_j(k)}{y_j(k)} < \max_{j \in V} \frac{x_j(k)}{y_j(k)} = M(k)$. Then for all $k' \geq k + D$ and for all $i \in V$,

$$\frac{x_i(k')}{y_i(k')} < M(k), \text{ and,} \quad (4.7)$$

$$\frac{x_i(k')}{y_i(k')} > m(k). \quad (4.8)$$

Proof: From the hypothesis that $m(k) < M(k)$ it follows that there exists a node $i \in V$ such that $\frac{x_i(k)}{y_i(k)} < M(k)$. Consider an arbitrary node $j \in V$. Since the graph is strongly connected, there exists a directed path from node i to node j in G . Let this path be denoted as $(m_1, i), (m_2, m_1), \dots, (j, m_{d_j-1})$. Then,

$$\begin{aligned} \frac{x_{m_1}(k+1)}{y_{m_1}(k+1)} &= \frac{p_{m_1 i} x_i(k) + \sum_{l \in N_{m_1}} p_{m_1 l} x_l(k)}{p_{m_1 i} y_i(k) + \sum_{l \in N_{m_1}} p_{m_1 l} y_l(k)} \\ &= \frac{p_{m_1 i} \frac{x_i(k)}{y_i(k)} + \frac{1}{y_i(k)} \sum_{l \in N_{m_1}} p_{m_1 l} x_l(k)}{p_{m_1 i} + \frac{1}{y_i(k)} \sum_{l \in N_{m_1}} p_{m_1 l} y_l(k)} \\ &< \frac{p_{m_1 i} M(k) + \frac{1}{y_i(k)} \sum_{l \in N_{m_1}} p_{m_1 l} x_l(k)}{p_{m_1 i} + \frac{1}{y_i(k)} \sum_{l \in N_{m_1}} p_{m_1 l} y_l(k)}. \end{aligned}$$

By definition of $M(k)$, $\frac{x_l(k)}{y_l(k)} \leq M(k)$, thus $x_l(k) \leq y_l(k)M(k)$. It follows that,

$$\begin{aligned} \frac{x_{m_1}(k+1)}{y_{m_1}(k+1)} &< \frac{p_{m_1 i} M(k) + \frac{1}{y_i(k)} \sum_{l \in N_{m_1}} p_{m_1 l} y_l(k) M(k)}{p_{m_1 i} + \frac{1}{y_i(k)} \sum_{l \in N_{m_1}} p_{m_1 l} y_l(k)} \\ &= M(k). \end{aligned}$$

This implies that $\frac{x_{m_1}(k+1)}{y_{m_1}(k+1)} < M(k)$. Using Lemma 4.2 it follows that $\frac{x_{m_1}(k'_{m_1})}{y_{m_1}(k'_{m_1})} < M(k)$ if $k'_{m_1} \geq k + 1$. Similarly, it follows that if $k' \geq k + 2$, then $\frac{x_{m_2}(k')}{y_{m_2}(k')} < M(k)$ and that for all $k' \geq k + d_j$, $\frac{x_j(k')}{y_j(k')} < M(k)$. Note that since $D \geq d_j$, it follows that $k + D \geq k + d_j$. The condition $k' \geq k + D$ is independent of the index j and where the node j was chosen arbitrarily. Thus it can be concluded that $\frac{x_i(k')}{y_i(k')} < M(k)$ for all $k' > k + D$ and for all $i \in V$. This completes the proof. The other inequality can be proven similarly and is left to the reader. $\square$

The following theorem shows that the maximum (minimum) of the ratio of the nodal states under (4.1) and (4.2) strictly decreases (increases) after finite time.

Theorem 5 Consider the update rule (4.1) and (4.2) with the assumptions of Theorem 4 with the initial ratio vector being $r(nD) := [\frac{x_1(nD)}{y_1(nD)} \frac{x_2(nD)}{y_2(nD)} \dots \frac{x_N(nD)}{y_N(nD)}]$ such that $\min r(nD) < \max r(nD)$, where, $n = 0, 1, 2, \dots$. Then, $M((n+1)D) < M(nD)$ and $m((n+1)D) > m(nD)$.

Proof: Using $k = nD$ in Lemma 4.3, it follows that for $k' \geq (n+1)D$, $\frac{x_i(k')}{y_i(k')} < M(nD)$ for all $i \in V$. This implies that $\max_{i \in V} \frac{x_i((n+1)D)}{y_i((n+1)D)} < M(nD)$. Thus, $M((n+1)D) < M(nD)$. This completes the proof of the first inequality. The other inequality can be proved similarly and is left to the reader. $\square$

Theorem 6 Consider the update rule (4.1), (4.2) with the assumptions of Theorem 4. Then, $\lim_{n \rightarrow \infty} M(nD) = \frac{\sum_{j=1}^N x_j(0)}{N}$ and $\lim_{n \rightarrow \infty} m(nD) = \frac{\sum_{j=1}^N x_j(0)}{N}$.

Proof: It follows from Theorem 4 that, $\lim_{k \rightarrow \infty} \frac{x_j(k)}{y_j(k)} = \frac{\sum_{j=1}^N x_j(0)}{N}$ for all $j \in V$. This implies that for any $j \in V$ and for any given $\epsilon > 0$, there exists K such that for all $k \geq K$, $|\frac{x_j(k)}{y_j(k)} - \frac{\sum_{j=1}^N x_j(0)}{N}| < \epsilon$. This implies that for all $j \in V$, $-\epsilon < \frac{x_j(k)}{y_j(k)} - \frac{\sum_{j=1}^N x_j(0)}{N} < \epsilon$. It follows that $-\epsilon < \max_{j \in V} \frac{x_j(k)}{y_j(k)} - \frac{\sum_{j=1}^N x_j(0)}{N} < \epsilon$. Thus, there exists K such that $k \geq K$ implies that $|M(k) - \frac{\sum_{j=1}^N x_j(0)}{N}| < \epsilon$. Similarly, $|m(k) - \frac{\sum_{j=1}^N x_j(0)}{N}| < \epsilon$. This implies that $\lim_{k \rightarrow \infty} M(k) = \frac{\sum_{j=1}^N x_j(0)}{N}$ and $\lim_{k \rightarrow \infty} m(k) = \frac{\sum_{j=1}^N x_j(0)}{N}$. Since, $\{M(nD)\}_{n=0,1,2,\dots}$ and $\{m(nD)\}_{n=0,1,2,\dots}$ are sub-sequences of convergent sequences, they converge to the same limit. Hence, it follows that $\lim_{n \rightarrow \infty} M(nD) = \frac{\sum_{j=1}^N x_j(0)}{N}$ and $\lim_{n \rightarrow \infty} m(nD) = \frac{\sum_{j=1}^N x_j(0)}{N}$. $\square$

Corollary 6.1 Consider the update rule (4.1), (4.2) with the assumptions of Theorem 4. Then, $\lim_{n \rightarrow \infty} M(nD) - m(nD) = 0$.

Proof: The proof directly follows from Theorem 4.2. $\square$ The following subsection introduces Maximum/ Minimum consensus protocols and the preceding results are used to design a finite time stopping criterion for ratio consensus given by (4.1) and (4.2).

4.3.1 Maximum Consensus Protocol

The Maximum Consensus Protocol denoted by MXP computes the maximum of the given initial node conditions $z(0) = [z_1(0) \ z_2(0) \ \dots \ z_N(0)]^T$ in a distributed manner. It takes $z(0)$ as an input and generates a sequence of node values based on the following update rule for node i ,

$$z_i(k+1) = \max_{j \in N_i \cup \{i\}} z_j(k). \quad (4.9)$$

Lemma 4.3.4 *Maximum consensus protocol given by (4.9) converges to $\max_{j \in V} z_j(0)$ in finite time $k \leq D$.*

Proof: See [2] for the proof. □

4.3.2 Minimum Consensus Protocol

The Minimum Consensus Protocol denoted by MXP computes the minimum of the given initial node conditions $w(0) = [w_1(0) \ w_2(0) \ \dots \ w_N(0)]^T$ in a distributed manner. It takes $w(0)$ as an input and generates a sequence of node values based on the following update rule for node i ,

$$w_i(k+1) = \min_{j \in N_i \cup \{i\}} w_j(k). \quad (4.10)$$

Lemma 4.3.5 *Minimum consensus protocol given by (4.10) converges to $\min_{j \in V} w_j(0)$ in finite time $k \leq D$.*

Proof: See [2] for the proof. □

4.3.3 Distributed Finite Time Termination Algorithm

In this section, we propose an algorithm using the Max-Min Consensus for stopping the ratio consensus protocol in finite time based on a user specified threshold ρ . This algorithm first appeared in [11] as an extension of the Max-Min consensus based finite time termination of averaging consensus [2]; however, analytical guarantees were not provided. The initial conditions for the MXP and MNP protocols are set as the initial ratios held by the nodes. The MXP and

MNP protocols are re-initialized at $k = nD$, where $n = 1, 2, \dots$, with $z(nD) = \frac{x(nD)}{y(nD)}$ and $w(nD) = \frac{x(nD)}{y(nD)}$ respectively. Define $\bar{\alpha}_i(nD) := \max z(nD) = M(nD)$, $\underline{\alpha}_i(nD) = \min w(nD) = m(nD)$ and $\beta_i(nD) := \bar{\alpha}_i - \underline{\alpha}_i = M(nD) - m(nD)$.

Algorithm 1: Finite-time termination of ratio consensus (at each node $i \in V$)

Input:
 $| \quad x_j(0), y_j(0) = 1, j \in N_i \cup i, \rho ; \quad \quad \quad // \text{ Initial condition}$

Initialize:
 $| \quad k := 0;$
 $| \quad z_i := x_i(0)/y_i(0);$
 $| \quad w_i := x_i(0)/y_i(0);$
 $| \quad n := 1;$

Repeat:
 $| \quad /* \text{ ratio consensus updates of node } i \text{ given by (4.1) and (4.2) } */$
 $| \quad x_i(k+1) := p_{ii}x_i(k) + \sum_{j \in N_i} p_{ij}x_j(k);$
 $| \quad y_i(k+1) := p_{ii}y_i(k) + \sum_{j \in N_i} p_{ij}y_j(k);$
 $| \quad /* \text{ maximum and minimum consensus updates of node } i \in V \text{ given by (4.9) and (4.10) respectively } */$
 $| \quad z_i := \max_{i \in N_i \cup \{i\}} z_i;$
 $| \quad w_i := \min_{i \in N_i \cup \{i\}} w_i;$
 $| \quad \text{if } k = nD \text{ then}$
 $| \quad | \quad \bar{\alpha}_i := z_i;$
 $| \quad | \quad \underline{\alpha}_i := w_i;$
 $| \quad | \quad \beta_i := \bar{\alpha}_i - \underline{\alpha}_i;$
 $| \quad | \quad \text{if } \beta_i < \rho \text{ then}$
 $| \quad | \quad | \quad \text{break ;} \quad \quad \quad // \text{ stop } x_i, y_i, z_i \text{ and } w_i \text{ updates}$
 $| \quad | \quad \text{else}$
 $| \quad | \quad | \quad \text{set } z_i = x_i(nD)/y_i(nD);$
 $| \quad | \quad | \quad w_i = x_i(nD)/y_i(nD);$
 $| \quad | \quad | \quad n = n + 1;$
 $| \quad \text{end}$
 $| \quad k = k + 1;$

Theorem 7 *Algorithm 1 converges in finite-time.*

Proof: It follows from *Corollary 4.1* that $M(nD) - m(nD) \rightarrow 0$ as $n \rightarrow \infty$. Thus, for any given $\rho > 0$, there exists an integer $n \geq t(\rho)$ such that $|M(nD) - m(nD)| < \rho$ for all nodes in the graph $G = \{V, E\}$.

Remark 7.1 *It is shown in [87] that no finite time distributed averaging protocol is possible without the knowledge of any global network parameter if each node only knows its history of estimates. The only global parameter needed for this algorithm is the knowledge of graph diameter. However, it should be noted that each node does not need to know the actual diameter but some upper bound of the diameter. One of the many algorithms in literature which can accomplish the task of finding diameters is provided in [88]. Each node can first run this protocol to determine the diameter in a distributed manner. In many applications, an upper bound on the diameter is readily available.*

4.4 Simulation and Experimental Results

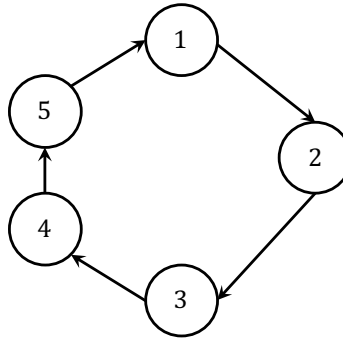


Figure 4.1: A 5 node ring network. Note that self loops are not shown.

In this section we present simulation and experimental results validating the algorithm in the previous section.

4.4.1 Simulation Results

Consider the 5 node directed ring network shown in Fig. 4.1 with the weight matrix,

$$P = \begin{bmatrix} 1/2 & 0 & 0 & 0 & 1/2 \\ 1/2 & 1/2 & 0 & 0 & 0 \\ 0 & 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 0 & 1/2 & 1/2 \end{bmatrix},$$

and the initial condition of the numerator states, $x(0) = [10 \ 20 \ 30 \ 40 \ 0]$. The application of Algorithm 1 with $\rho = 1$ and $D = 5$ results in distributed finite time termination of computations performed by the agents in 20 iterations as shown in Fig. 4.2. Observe that the ratio of the nodal states are close to the average of the numerator initial conditions.

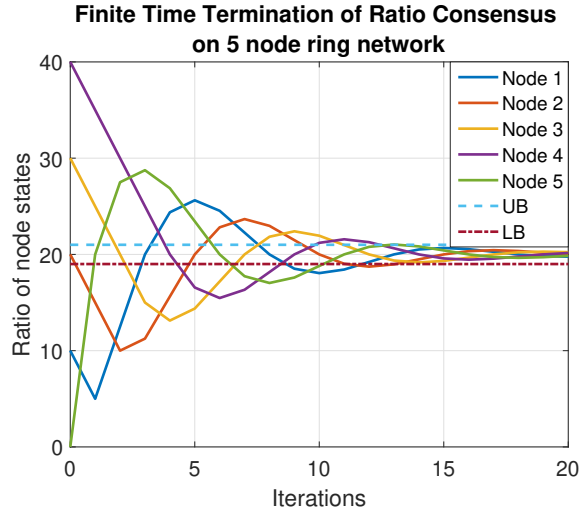


Figure 4.2: Finite time termination of ratio consensus on the 5 node ring network in 20 iterations with $\rho = 1$. Here UB and LB denote the margin of $\rho = 1$ from the average of the numerator initial condition.

Consider the 5 node undirected network in Fig. 4.3 undergoing the ratio consensus protocol

with the initial condition for the numerator states, $x(0) = [10, 20, 30, 40, 0]$ and weight matrix,

$$P = \begin{bmatrix} 1/4 & 1/4 & 1/3 & 1/3 & 0 \\ 1/4 & 1/4 & 1/3 & 0 & 1/3 \\ 1/4 & 1/4 & 1/3 & 0 & 0 \\ 1/4 & 0 & 0 & 1/3 & 1/3 \\ 0 & 1/4 & 0 & 1/3 & 1/3 \end{bmatrix}.$$

The application of Algorithm 1 with $\rho = 1$ and $D = 3$ results in distributed finite time termination of computations performed by the agents in 7 iterations as shown in Fig. 4.2. Observe that the ratio of the nodal states are close to the average of the numerator initial conditions.

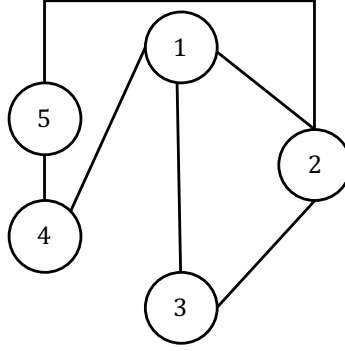


Figure 4.3: 5 node undirected network undergoing ratio consensus. Note that self loops are not shown.

4.4.2 Experimental Results

The undirected network shown in Fig. 4.3 is implemented using 5 Raspberry Pi devices [89] as shown in Fig. 4.5. Each of the Raspberry Pis communicates with its neighbors through wifi. The details of the experimental setup can be found in [90]. The ratio consensus equations (4.1) and (4.2) are updated every 1 second. The weight matrix P and the numerator initial condition are same as in the previous case, while, the threshold ρ is chosen as 0.01. In this case, Algorithm 1 terminates in 27 iterations, which is more than that of Fig. 4.4 due to a tighter threshold.

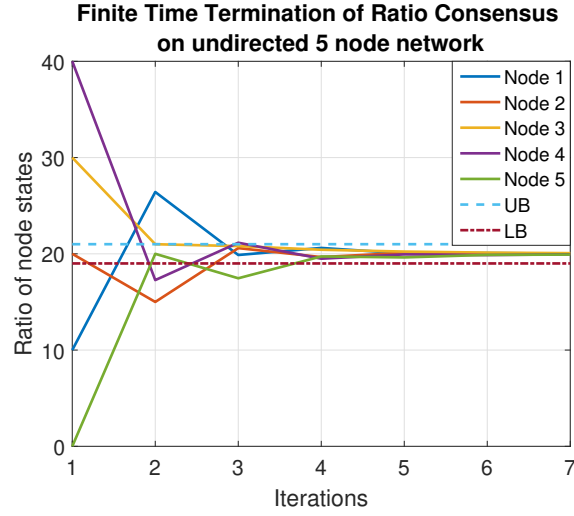


Figure 4.4: Finite time termination of ratio consensus on the 5 node undirected network in 7 iterations with $\rho = 1$. Here UB and LB denote the margin of $\rho = 1$ from the average of the numerator initial condition.



Figure 4.5: A network of 5 Raspberry Pi's interacting according to the ratio consensus protocol with the interaction network being the undirected network of Fig. 4.3. The communication between a device with its neighbor is through wifi.

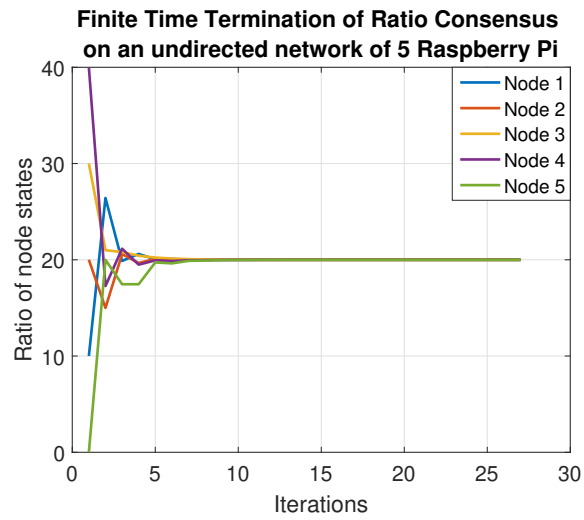


Figure 4.6: Finite time termination of ratio consensus on the 5 node network of Raspberry Pi's with the underlying graph as shown Fig. 4.3 and $\rho = 0.01$. Ratio consensus terminated close to the average of the numerator initial values in 27 iterations.

Chapter 5

Distributed Power Apportioning with Early Dispatch for Ancillary Services in Renewable Grids

5.1 Introduction

Dispatch methodologies for deploying ancillary services for reliable operation of the modern grid are changing significantly with penetration of a large number of smaller DERs scattered throughout the network (termed as *decentralization*). The addition of renewable energy sources (RES) such as, wind and photovoltaic (PV) generation, with increased generation variability necessitates increased flexibility of dispatch operations [91,92]. As a result, providing essential reliability services, such as, secondary frequency response (SFR), which corrects for imbalances between total generation and load in the system to restore frequency to its nominal value needs to emphasize DERs and RESs as a focus. An increase in power electronics-interfaced DERs, energy storage systems (ESS), and flexible loads interacting with the grid have shown promise in providing SFR, (as an example, consider the Electric Reliability Council of Texas's (ERCOT's)

responsive reserve services (RRS)), thereby demonstrating the viability of DERs for such applications [93,94]. A tighter integration of distribution system operators (DSO) with aggregators, independent power producers and a large number of residential/commercial units with DER assets ('prosumers') is exigent.

To achieve SFR, centralized, decentralized as well as distributed frameworks are proposed in the literature where a large number of DERs need to be coordinated on a fast timescale. Earlier works, such as [95], focused on implementing a centralized control approach for coordinating DERs, wherein a secondary centralized controller at the distribution level collects states of DERs via a communication network and sends dispatch commands to local actuators. One such centralized approach for providing aggregation services from DERs is a third party aggregator, *Next Kraftwerke*, [96] operating in the Belgian power system market. *Next Kraftwerke* combines decentralized generation and consumption assets of various prosumers that engage as balance responsible parties (BRPs). *Next Kraftwerke* operates the pool of assets from BRPs in a coordinated way to offer balancing services such as secondary frequency response to the Belgian transmission system operator *Elia*. However, in order to enable the virtual power plant (VPP) aggregation services at each BRP, an expensive remote communication and control system is required from each BRP (via Next Kraftwerke's Next Box) to a centralized server that makes optimized decisions on providing the balancing services. Moreover, BRPs have to share their generation capacities with the organization at all times. Centralized approaches, thus, often require additional communication networks from every DER asset that the central aggregator can access to meet SFR requirements satisfactorily. Moreover, centralized control approaches have added challenges of reduced resiliency due to the vulnerability of a single communication link from BRPs to the central aggregator where the failure of the central aggregator adversely affects the capability of aggregated DERs to provide the required SFR service.

Decentralized approaches for secondary frequency control regulate the system frequency without relying on any centralized communication between DERs. Due to measurements available only at the local DER, such methods are limited to – i) local proportional droop controllers with decentralized integral control and ii) decentralized transfer of contingency reserves [97]. However in practice, the former approaches suffer from poor robustness [98] whereas the latter

approaches rely on enabling loads with the function of high-quality contingency reserve and do not consider generation-based dispatch [99].

In order to mitigate these challenges, distributed control approaches are proposed with DERs [100] whereby DERs can communicate local states in the network with their neighboring DERs only, imposed by a communication graph.

The advantages of distributed approaches include coordination using only local computations, plug-and-play capability, resiliency to link failures, robustness of the system to computational node failures as well as more privacy to the users. A gather-broadcast method was presented in [101], and [102] developed a distributed average integral method; however, these methods are often sensitive to gain coefficients and can result in slow convergence to dispatch outputs. Liu et al [103, 104] have adopted a distributed approach for current sharing in DC microgrids where the secondary controller of each DER requires discrete-time information from neighboring DERs for achieving higher accuracy of current sharing and voltage regulation. Reference [105] presented a distributed SFR approach but did not consider RESs and relies on heuristics to achieve dispatch requests. Distributed *consensus*-based algorithms form a primary approach for many applications where distributed decision making is needed; in this article, achieving consensus between agents on decision variables, forms a main thrust for realizing fast distributed SFR. Major challenges faced by consensus-based distributed approaches include: (i) inherent delays in communication channels that has significant impact on accuracy [106] and (ii) the asymptotic nature of convergence of the algorithms entails in principle that the result is known only with an infinite horizon. In this regard, [84] presents a ratio-consensus protocol in which the nodes converge asymptotically to the ratio of the sum of the initial value of the two states maintained by each agent (or node). The DERs' power reference commands are computed based on the ratio consensus algorithm using DER generation capacities as states. However, delays present in communication channels are not addressed. Reference [107] extends this approach when the network topology is dynamic and communication delays are present. However, these works guarantee convergence only asymptotically which makes them unsuitable for real-time applications. However, for coordination for SFR, the result of the distributed algorithm will determine the power dispatch command of each individual DER. Thus there is

a need for a stopping criterion at each DER, which can be utilized to terminate the consensus algorithm and for determining DER dispatch decisions.

As alluded to earlier, a challenge with distributed algorithms that exchange information with neighbors and update their state multiple times over many iterations is the need to ascertain when to stop iterating and use the decision parameter for a subsequent action, such as determining how much power a DER needs to dispatch. Here, if the detection of convergence can be achieved (within a pre-specified tolerance value) by the nodes distributedly, algorithm run-times longer than necessary can be avoided, making it possible to employ low computational footprint, low cost devices.

To circumvent the issue of asymptotic convergence, the authors in [81] have presented a finite-time algorithm where each node can determine the final consensus value using the minimal polynomial associated with the weight matrix in each state update iteration. However, to calculate the coefficients of the minimal polynomial each node has to run N (total number of agents) different linear iterations each for at least $N + 1$ time-steps. Based on the above article, [108] achieves finite time consensus based on the network observability matrix. Reference [109] has proposed algorithms to compute the consensus value using the minimal number of observations of an arbitrary chosen node in a network. However, these methods rely on computing the rank of a given matrix and that of its kernel which make them computationally expensive. Moreover, the algorithm presented in [109] assumes no delays are present in the communication channels. Limitations of high computational footprint and large storage requirements at each node, render the approaches ([81, 108, 109]) unsuitable for applications, such as fast distributed SFR, where coordination with a fast response time is required.

Reference [11] proposed a distributed finite time termination of ratio consensus, which built on [84] and [2], for frequency regulation in a network of islanded ac microgrids, but it did not consider communication delays. Distributed finite-time termination of ratio consensus in presence of bounded delays is presented in our earlier work [110]. Reference [110] presents a distributed finite time stopping criteria and its convergence analysis in presence of bounded delays by proposing two additional protocols (Maximum Consensus Protocol and Minimum Consensus Protocol) that makes the ratio consensus protocol with bounded delays applicable

for practical applications. It is important to note that the methodology developed in [110] is not directly applicable to the task of aggregating DERs at timescales relevant to secondary frequency response. Indeed significant modifications, including *early dispatch* with related *brown start*, as presented in Section IV are needed to be able to meet the proposed response times. Toward these goals we have now added analysis of the ‘brown start’ mechanism in this article. In the article, analytical results that delineate the effects of brown start and other modifications needed for SFR timescales are provided. In addition, unlike earlier works the validation resulting from the interactions of the distributed finite-time layer with the underlying power controller layer is considered.

All of the previous works reported above, while proposing methods to coordinate and control a large number multi agent systems such as DER aggregation, are not instantiated to meet the objective of achieving demand and generation balance via SFR commands to participate in the ancillary service market and obtain response within SFR timescales. Moreover, these approaches are not generalized to include and prioritize renewable energy sources available in the network with variability and validations are limited only to simulation-based studies.

Thus, an effective framework for distributed aggregation based SFR is currently absent. We remark that in [84], distributed apportioning was achieved for a few DERs. Here, the algorithms used did not address the issues of non-ideal nature of communication, nor any guarantees on the finite-time distributed stopping criterion are utilized. Without addressing these issues, the consensus based strategies remain inapplicable for practical sized distributed DER aggregation goals. Indeed, to the best of the authors’ knowledge, no prior work or framework has demonstrated a provably guaranteed distributed method, with instantiation of a large scale coordination to achieve DER based SFR. This article develops a scalable DSO-centric framework towards coordinating large numbers (1000+) of DERs to provide support to system operators via aggregators in dispatching SFR. The framework meets the SFR ancillary demand (global objective) of the system operator by aggregating *distribution-level* DERs in a distributed manner while respecting local capacity constraints of each DER. Coordinated response and control of aggregated DERs are achieved at SFR timescales: initial response times of less than 5 seconds and a ramp to response set point within 1 minute are demonstrated (motivated by Electric

Reliability Council of Texas [ERCOT] regulation reserve services[RRS]). ERCOT uses dispatch instruction signals to deploy regulation reserve services with a command update rate of 4 sec (requirement R8 of NERC standard BAL-005) and expected ramp time of one minute following a deployment instruction, to update the ancillary service (AS) schedule for RRS for generation and load resources [111]. Readers are guided to [112] for a survey on details of deployment of various SFR services present in North American electricity markets.

In addition, the California ISO has identified the following provisions, imposed by the imminent high-DER transmission-distribution coordination, that need to be met [113]:

- 1) *DSO/Aggregator level*: Ability to deploy mechanisms to modify DER behavior via commands to elicit responsive energy consumption targets to be held to a certain level over a period of time as well as requiring these to be regulated up or down every few seconds-minutes as a response to a system signal.
- 2) *Participating DERs*: Ability to participate in the ancillary service market in a non-discriminatory, flexible and secure manner.

The proposed framework will address these technical challenges, establish analytical guarantees and validate on a real-world system to demonstrate its viability for obtaining fast coordinated responses (< 5 s).

Moreover, the distributed power apportioning framework comes with guarantees on reaching desired and feasible dispatch decisions in finite-time, even when the communication suffers from uncertainties such as delays. The developed framework's efficacy is demonstrated using low-cost *Raspberry Pi* (Rpi) devices (with local communication and computational intelligence capabilities) interfacing with an underlying power controller layer. The framework here is robust to the presence of bounded delays in the communication channels. Further, early dispatch and brown-start mechanisms to enable participating DERs to respond to an SFR signal at timescales faster than the state-of-the-art distributed approaches are developed. We now summarize the major contributions of this article:

- 1) A distributed framework for coordinated power apportioning in the presence of communication delays that preserves the privacy of private values (capacity, output power) of the participating DERs.

- 2) An extension of distributed stopping criteria [2] where each DER in the network can detect convergence within a tolerance independently which is subsequently employed to devise a novel *early dispatch* mechanism. This allows the network to achieve SFR faster than state-of-the-art distributed approaches (initial response time of less than 5 s and ramp time of 1 minute). Moreover, the states communicated with neighbor DERs do not compromise private values such as total demand, capacity, or current generation, thus, allowing for a privacy-preserving and non-discriminatory participation of DERs in the network.
- 3) A brown-start approach whereby DERs, already participating in SFR, smoothly transition to new states in order to meet new system operator commands in the presence of changes in generation capacities. We provide a bound on error in the converged ratio state due to brown start mechanism for any DER node i in the network, to the finite time converged state that increases only linearly in the change in flexibility in each node and the tolerance value
- 4) A large scale validation employing 40+ physical hardware DERs and 250 nodes on a real-world distribution system model. Experimental results show that, with the apportioning protocol, dispatch objectives are met within the targeted metrics. To the best of our knowledge, distributed DER coordination and control at such a scale is not demonstrated earlier with a large-scale validation.

Readers are guided to 4.1 for the definitions of terminologies from graph theory which are used for subsequent developments.

The proposed distributed finite time termination framework for aggregating DERs, demonstrates full activation times of 1 minute with initial response time of 5 seconds is achieved for experimental setup with power hardware and real-world communication networks. Moreover, these response times are achieved within the performance requirement standards required in most North American and European electricity markets (sometimes even faster than the full activation times for existing centralized methodologies). Our proposed distributed framework achieves these performance metrics for a large network system of DERs. The proposed framework is uniquely placed where we can achieve these fast activation times while adding the advantages of robustness and distributed computation which are the limitations of a centralized architecture.

Note: More details on this work can be found in the published articles [114].

5.2 System Description

We consider DERs (such as a PV array and battery ESS), each interfaced with DC-AC inverter connected to local ac loads and a grid connection and to other units through a point of common coupling (PCC) in a microgrid or to the grid through an aggregator (see Fig. 5.1). A detailed description of graph theory and linear algebra notions that are used in the subsequent development are available in [85].

5.2.1 Communication Network of DER Units

In order to facilitate exchange of information to arrive at viable power commands for meeting ancillary demand services of the grid, we consider a network of DER units as a multi-agent system (MAS) with agents/nodes interacting with their neighbors, over a communication infrastructure (*Communication Layer* in Fig. 5.1). We consider a graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where vertices $\mathcal{V} = \{1, 2, \dots, N\}$ denote the DER nodes in the network, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges representing the communication topology overlay on the underlying power system infrastructure. DER units communicate with each other as dictated by the network topology where each node communicates with its neighbors using multiple allowable communication modalities (wired, wireless, hybrid). Communication can be bidirectional or directional. In the network representation (see Fig. 5.1), a bidirectional channel is represented using a bi-directed edge and a directed channel with a directed edge. The underlying (Rpi-based) communication channel is uncertain suffering from delays.

Assumption 4 $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ is connected.

Assumption 5 For any node pair $i, j \in \mathcal{V}$ and $(i, j) \in \mathcal{E}$, the delay on the edge from node j to node i , denoted as τ_{ij} , satisfies: $\tau_{ij} \leq \bar{\tau} < \infty$, where $\bar{\tau} > 0$.

Assumption 5 reflects the *partial synchrony* condition for a real-world communication network in distributed computing. This framework also adheres to communication protocols that guarantee no packet loss, such as Transmission Control Protocol-based websockets protocols.

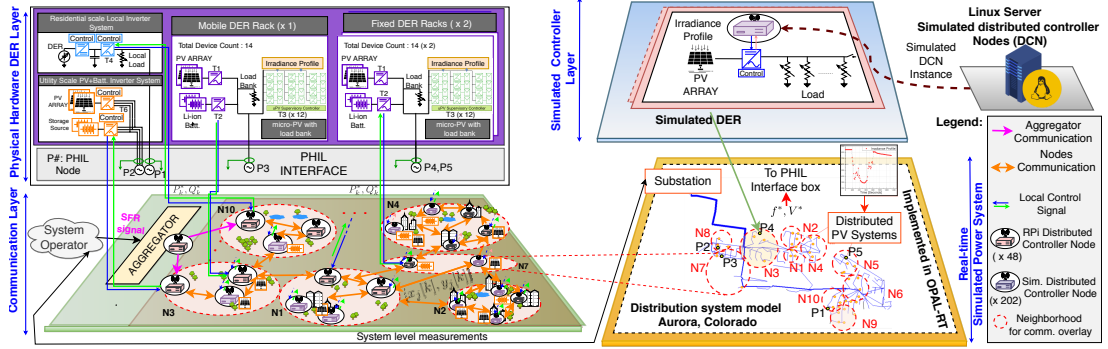


Figure 5.1: Schematic diagram of the system under study, consisting of residential, utility-scale DER units; real-world distribution system model from Aurora, CO, USA with interfaces to controllers for simulated DER units; medium-high power hardware-in-the-loop and aggregator interfacing the distribution system to the rest of the grid.

We consider, an *aggregator* to be an entity interfacing with the DSO and responsible for accumulating DERs available at the distribution level to provide a grid ancillary power command, ρ_d , to the network of DERs. Here, the aggregator can communicate the command ρ_d only to nodes in its communication neighborhood; the aggregator is assumed to have $l \geq 1$ neighbors. The demand signal considered here is the SFR signal where the following specifications have to be met (i) initial response time of less than 5 s, (ii) ramp response time to set point within 1 minute, (iii) maintaining the desired SFR command for a maximum time period of at least 30 minutes or as required by the system operator.

5.2.2 Problem Formulation

An aggregator, upon receiving a system operator's SFR signal (can be commanded manually or automatically with less than 6 s refresh rates) at time instants $t_0 < t_1 < t_2 < \dots$, sends a power command, $\rho_d(t_m)$, at time instant t_m , where $m \in \{0, 1, 2, \dots\}$, to one or more aggregator-facing DER in the network. The N participating DERs with a given graph topology, $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, need to collectively meet the aggregator command, $\rho_d(t_m)$, while communicating only to their neighboring (neighbors determined by the communication network layer) DER units and respecting their individual resource (generation) constraints. Let $\pi_i^{max}(t_m)$ and $\pi_i^{min}(t_m)$ be the maximum and minimum generation capacities of the DERs at the i^{th} DER, respectively, at

instant t_m . Let $\pi_i^*(t_m)$ denote the steady-state reference power command to be supplied by the i^{th} DER in response to an aggregator command, $\rho_d(t_m)$. The resource apportioning problem can be formulated mathematically as: for any time instant t_m determine;

$$\begin{aligned} & \left\{ \pi_i^*(t_m) \right\}_{i=1}^N \\ \text{such that } & \sum_{i \in \mathcal{V}} \pi_i^*(t_m) = \rho_d(t_m) \\ & \pi_i^{min}(t_m) \leq \pi_i^*(t_m) \leq \pi_i^{max}(t_m) \text{ for all } i \in \mathcal{V}. \end{aligned} \quad (5.1)$$

The viability of the resource apportioning problem holds when $\sum_{i=1}^N \pi_i^{min}(t) \leq \rho_d(t) \leq \sum_{i=1}^N \pi_i^{max}(t)$; otherwise for $\rho_d(t) \geq \sum_{i=1}^N \pi_i^{max}(t)$, we set $\pi_i^*(t) = \pi_i^{max}(t)$ for all $i \in \mathcal{V}$.

Let us define the flexibility of the i^{th} DER at time instant t_m to be $\pi_i^f(t_m) := \pi_i^{max}(t_m) - \pi_i^{min}(t_m)$. Then, one solution [62] for the apportioning problem (5.1) is given as:

$$\pi_i^*(t_m) = \pi_i^{min}(t_m) + \alpha(t_m) \pi_i^f(t_m), \quad (5.2)$$

where $\alpha(t_m) := \frac{\rho_d(t_m) - \sum_{i=1}^N \pi_i^{min}(t_m)}{\pi^f(t_m)}$ is the ‘sharing factor’ and $\pi^f(t_m) := \sum_{i=1}^N \pi_i^f(t_m)$ is the total flexibility across the network. This solution satisfies all the constraints of (5.1), as $\sum_{i=1}^N \pi_i^*(t_m) = \rho_d(t_m)$ and $\pi_i^{min}(t_m) \leq \pi_i^*(t_m) \leq \pi_i^{max}(t_m)$ for all $i \in \mathcal{V}$. This solution also provides a fair way of apportioning where generation flexibility (π_i^f) of each DER is scaled with a fix scaling and added to its minimum generation capacity.

Remark 7.2 Note that if $\alpha(t_m)$ is available to the DER then it can determine its dispatch command as $\pi_i^*(t_m)$; the above solution guarantees feasibility of local constraints while meeting ancillary service demand, ρ_d . In Section 5.4, we provide a distributed method to determine $\alpha(t_m)$ within any pre-specified tolerance.

We remark that the resource apportioning problem (5.1) needs to be solved faster than the time interval between t_{m-1} and t_m when a new dispatch command is placed. In real-world power networks the maximum time to respond required by the system operator is in the order

of minutes [112]. The approximate solution to (5.1) is found based on a finite-time termination criteria using a user-selected tolerance parameter which provides some flexibility wherein faster convergence results with a larger tolerance.

5.3 Apportioning Using Distributed Averaging

The resource apportioning problem with constraints can be solved using average consensus protocols. Here, we develop the extension of distributed averaging protocol ([2, 107, 110]) toward solving the resource apportioning problem for the asymptotic case.

Assumption 6 *Let p_{ij} denote the weight on information coming from node j to node i . The weight matrix $P(i, j) = p_{ij} \geq 0$ associated with $\mathcal{G}$ is primitive (it is irreducible and has only one eigenvalue of maximum modulus) and column stochastic (all elements of every column of the matrix sum to one).*

Assumption 7 *Any node $i \in \mathcal{V}$ in $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ has access to its own value at any instant k without any delay.*

Moreover, we consider the underlying power network to be viable (i.e., the underlying distribution power network has power capacity larger than the dispatched power commands from all DERs).

Definition 11 *(In-neighbor Set and Out-neighbor Set) In a graph $\mathcal{G} := (\mathcal{V}, \mathcal{E})$, the in-neighbor set, $\mathcal{N}_j^-$, and out-neighbor set, $\mathcal{N}_j^+$, of node $j \in \mathcal{V}$, are given as $\mathcal{N}_j^- := \{i | (j, i) \in \mathcal{E}, i \neq j\}$ and $\mathcal{N}_j^+ := \{i | (j, i) \in \mathcal{E}, i \neq j\}$, respectively. The cardinality of $\mathcal{N}_j^+$ of a node $i \in \mathcal{V}$ is called the out-degree of the node denoted by D_i^+ .*

5.3.1 Power Apportioning Protocol

Let the DER communication network be represented by a graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$. Let $\mathcal{N}_d$ denote the set of $l = |\mathcal{N}_d|$ nodes directly communicating with the aggregator, referred to as *command circulating nodes*. Upon receiving a DSO signal for the dispatch of DER as part of the SFR at any time instant t_m , the aggregator communicates the ancillary service request, $\rho_d(t_m)$, to the

network of DERs by communicating to the command circulating nodes in $\mathcal{N}_d$. At each time instant t_m , each node i initializes two states $[r_i(0), s_i(0)]^T$ where:

$$r_i(0) = \begin{cases} \frac{\rho_d(t_m)}{l} - \pi_i^{min}(t_m), & \text{if } i \in \mathcal{N}_d, \\ -\pi_i^{min}(t_m), & \text{if } i \notin \mathcal{N}_d, \end{cases} \quad (5.3)$$

$$s_i(0) = \pi_i^{max} - \pi_i^{min}, \text{ for all } i \in \mathcal{V}. \quad (5.4)$$

Each node i executes the following state updates:

$$r_i(k+1) = p_{ii}r_i(k) + \sum_{j \in \mathcal{N}_i^-} p_{ij}r_j(k - \tau_{ij}), \quad (5.5)$$

$$s_i(k+1) = p_{ii}s_i(k) + \sum_{j \in \mathcal{N}_i^-} p_{ij}s_j(k - \tau_{ij}), \quad (5.6)$$

where, $\pi_i^{max}(t_m)$ and $\pi_i^{min}(t_m)$ denote the maximum and minimum power capacity of the i^{th} DER at time t_m respectively, and τ_{ij} is the delay in communicating data from node j to node i . We remark that, $s_i(0) > 0$ for all $i \in \mathcal{V}$ as $\pi_i^{max}(t_m) > \pi_i^{min}(t_m)$.

Remark 7.3 (*Distributed Synthesis*) The weight matrix being column stochastic enables the p_{ij} to be chosen in a purely distributed manner. A simple scheme assuring column stochasticity is i^{th} node sets the weights $p_{ji} = 1/(D_i^+ + 1)$ for all $j \in \{\mathcal{N}_i^+ \cup i\}$ and communicates to node j , $\sigma_{ji}(k) = p_{ji}r_i(k)$. Each node j executes $r_j(k+1) = p_{jj}r_j(k) + \sum_i \sigma_{ji}(k - \tau_{ji})$ thus realizing (5.5) and (5.6).

Theorem 8 Under Assumptions 4 – 7, the power apportioning protocol (5.3) – (5.6) converges asymptotically, with

$$\lim_{k \rightarrow \infty} \frac{r_i(k)}{s_i(k)} \rightarrow \alpha(t_m), \quad (5.7)$$

for each node $i \in \mathcal{V}$.

Proof: The result can be obtained directly by noting that for a given time instant t_m :

$$\sum_{i=1}^N r_i(0) = \sum_{i \in \mathcal{N}_d} \left(\frac{\rho_d(t_m)}{l} - \pi_i^{min}(t_m) \right) + \sum_{i \notin \mathcal{N}_d} (-\pi_i^{min}(t_m)) \text{ and } \sum_{i=1}^N s_i(0) = \sum_{i=1}^N (\pi_i^{max}(t_m) - \pi_i^{min}(t_m)) \text{ and applying the convergence of ratio consensus (Lemma 1 of [107]).} \quad \square$$

5.3.2 RES Prioritization

In order to demonstrate the dispatchability of available RES, RES prioritization scheme is proposed with an objective to dispatch spinning RES-based DERs as SFR before ESS for long-term reserve requirements. Here, in the power apportioning protocol we set $\pi_i^{min}(t_m) = \pi_i^{max}(t_m) - \epsilon_{RES}$, where $\epsilon_{RES} > 0$ is a small number. This enforces all available RES capacities at the initialization of the protocol are used, allowing for RES prioritization; the effectiveness as validated experimentally.

Section 5.3.1 provides a distributed protocol to allocate resources to meet the demand $\rho_d(t_m)$ by a DER network; however, it is clear that this protocol is not amenable to accommodating aggregator commands at subsequent time instants $t_{m+1}, t_{m+2}, \dots$ as obtaining $\alpha(t_m)$ at each DER is asymptotic, where the desired dispatch, $\pi_i^*(t_m)$, of the i^{th} DER is determined only in the limit $k \rightarrow \infty$. In order to mitigate this issue, we now propose the distributed finite-time termination protocol.

5.4 Distributed Finite-Time Termination

In this section, a stopping criterion for terminating the ratio consensus algorithm in finite-time based on a specified tolerance, ϵ is developed. Consider the maximum and minimum value of the ratio of consensus protocols (5.5)–(5.6) over all nodes within a time horizon $\bar{\tau}$ from any time instant k , given as:

$$M(k) := \max_{\substack{j \in V \\ n = \{0, 1, 2, \dots, \bar{\tau}\}}} \frac{r_j(k-n)}{s_j(k-n)}, \quad s_j(k-n) \neq 0, j \in V \quad (5.8)$$

$$m(k) := \min_{\substack{j \in V \\ n = \{0, 1, 2, \dots, \bar{\tau}\}}} \frac{r_j(k-n)}{s_j(k-n)}, \quad s_j(k-n) \neq 0, j \in V \quad (5.9)$$

Under assumptions 4–7, it is shown in [110] that the global maximum (minimum) in the network is decreasing (increasing). Of particular interest are the maximum and minimum values over an “epoch” which is equal to $T(D, \bar{\tau}) := D(1 + \bar{\tau}) + \bar{\tau}$, which captures the upper bound on number of iterations required for any node to communicate to any other node in the network.

It can be shown that the global maximum and minimum sampled at every epoch form strictly monotonic sequences. Furthermore, these sequences converge asymptotically to the consensus value; the ratio of the sum of initial states. We now introduce the maximum consensus and minimum consensus protocols that determine the global maximum and minimum in finite number of iterations.

5.4.1 Maximum and Minimum Consensus Protocols

The maximum consensus protocol (MXP) works as follows. Consider at time instant $\ell\bar{\tau}$, the value held by the i^{th} node is $z_i(\ell\bar{\tau})$ with $z_i(0)$ being the initial value at $\ell = 0$. The i^{th} node holds this value for $\bar{\tau}$ units of time and thus; $z_i(\ell\bar{\tau} + k) = z_i(\ell\bar{\tau})$, for $k = 1, 2, \dots, \bar{\tau} - 1$. As $\bar{\tau}$ is the upper bound on delay, in $\bar{\tau}$ units of time node i receives $z_j(\ell\bar{\tau})$ for all $j \in \mathcal{N}_i^-$ and thus it can execute the update:

$$z_i(\ell\bar{\tau} + \bar{\tau}) = \max_{j \in \mathcal{N}_i^- \cup \{i\}} z_j(\ell\bar{\tau}), \quad (5.10)$$

Similarly, the minimum consensus protocol (MNP) computes the minimum of the given initial node conditions $w(0) := [w_1(0) \ w_2(0) \dots w_n(0)]^T$ in a distributed manner. At time instant $\ell\bar{\tau}$, i^{th} node executes the update:

$$w_i(\ell\bar{\tau} + \bar{\tau}) = \min_{j \in \mathcal{N}_i^- \cup \{i\}} w_j(\ell\bar{\tau}), \quad (5.11a)$$

$$w_i(\ell\bar{\tau} + k) = w_i(\ell\bar{\tau}), \text{ for } k = 1, 2, \dots, \bar{\tau} - 1. \quad (5.11b)$$

Remark 8.1 *MXP and MNP converge to the maximum and minimum of the initial conditions, respectively, within an epoch $T(D, \bar{\tau})$ [110]. Thus, any node running MXP and MNP (initialized at instant k) can determine the global maximum, $M(k)$, and minimum, $m(k)$ distributedly in $D(1 + \bar{\tau}) + \bar{\tau}$.*

The MXP and MNP protocols at each node $i \in V$ are re-initialized at every epoch; here, for every $k = \theta(D(1 + \bar{\tau}) + \bar{\tau}) = \theta T$, where $\theta = 1, 2, \dots$, we initialize $z_i(k) = \frac{r_i(k)}{s_i(k)}$ and $w_i(k) = \frac{r_i(k)}{s_i(k)}$

(with $r_i(k)$ and $s_i(k)$ governed by (5.5) and (5.6)). In an epoch, each node i can determine the maximum and minimum over the entire network. Here each node executes the MXP and MNP algorithms to determine $M(\theta T) = \max_j z_j(\theta T)$ and $m(\theta T) = \min_j w_j(\theta T)$ with an epoch T iterations. It follows from the monotonicity and convergence property of MXP/MNP that $M(\theta T) \searrow \frac{\sum_{j=1}^N r_j(0)}{\sum_{j=1}^N s_j(0)}$ and $m(\theta T) \nearrow \frac{\sum_{j=1}^N r_j(0)}{\sum_{j=1}^N s_j(0)}$. Thus given any tolerance ϵ , there exists a θ_0 such that if $\theta \geq \theta_0$, $M(\theta T) - m(\theta T) \leq \epsilon$ where $M(\theta T) - m(\theta T)$ can be determined by every node in finite time. Thus, the consensus value can be determined with a tolerance ϵ in finite number of iterations. [110] establishes rigorously that given a threshold $\epsilon > 0$, Algorithm 2 terminates in a finite number of iterations.

Let $\alpha_i(t_m, \theta) := \frac{r_i(\theta T)}{s_i(\theta T)}$ be the ‘sharing factor’ at i^{th} DER at epoch θ , then

$$|\alpha_i(t_m, \theta_0) - \alpha(t_m)| \leq \epsilon. \quad (5.12)$$

That is, the ‘sharing factor’ $\alpha_i(t_m, \theta)$ at i^{th} DER converges to the global ‘sharing factor’ $\alpha(t_m)$. Here, θ_0 is the stopping epoch. Similarly, if $\pi_i^*(t_m, \theta_0) := \pi_i^{min}(t_m) + \alpha_i(t_m, \theta_0)\pi_i^f(t_m)$ is the reference power command for i^{th} DER at epoch θ_0 , then

$$|\pi_i^*(t_m, \theta_0) - \pi_i^*(t_m)| \leq \epsilon \pi_i^f(t_m). \quad (5.13)$$

Equations (5.12) and (5.13) use the finite-time termination of Algorithm 2 to prove that the distributed dispatch at each DER is within chosen tolerance of centralized dispatch.

Remark 8.2 *The only global parameters needed by each node in order to implement the stopping criteria using (5.5) and (5.6) are upper bounds on both the maximum delay, $\bar{\tau}$, and diameter of the network, D , which can be enforced by design.*

5.4.2 Early Dispatch Mechanism

For achieving an initial response time of < 5 s for DERs participating in SFR support, we propose an early dispatch mechanism. In this mechanism, we use the ratio value from the most recent epoch to obtain reference power commands for each DER. Here, the i^{th} DER in response

to an ancillary demand of $\rho_d(t_m)$ for time t_m , updates its dispatch every epoch θ according to:

$$\pi_i^*(t_m, \theta) = \pi_i^{min}(t_m) + \alpha_i(t_m, \theta)\pi_i^f(t_m) \quad (5.14)$$

(see Algorithm 2). For practical implementation, early dispatch is implemented for $\theta > 3$; for the first three epochs ($\theta = 1, 2, 3$), the dispatch $\pi_i^*(t_m, \theta)$ remains unaltered as $\pi_i^*(t_{m-1})$ (within the tolerance, ϵ). Here, once the node i , satisfies the stopping criterion (using MXP and MNP), the early dispatch will become the steady-state dispatch where for large enough epoch number θ_0 , that is $\pi_i^*(t_m, \theta_0) \approx \pi_i^*(t_m)$ (see equation 5.13), for all $i \in \mathcal{V}$. Moreover, $\alpha_i(t_m, \theta_0)$ is guaranteed to stay within the global maximum and minimum ratios (computed via the MXP and MNP protocols) for the DER units running the power apportioning protocol and due to the monotonicity property of the ratio consensus protocol (which ensures that with every iterate early dispatch mechanism improves its performance).

5.4.3 Black Start: Power Demand as new Input to the network

In order to apply the proposed framework to provide SFR as ancillary service to the grid, the DER network should be able to respond to subsequent aggregator demand, $\rho_d(t_m)$ at time instants t_m for $m = 0, 1, 2, \dots$. In order to achieve this, the command circulating nodes in the DER network can be provided with the aggregator demands, $\rho_d(t_m)$, at any given time instant t_m , and initialized according to (5.3) and (5.4). Upon convergence, within a specified tolerance, the DER network can accept the next aggregator demand, $\rho_d(t_{m+1})$ at the next time instant t_{m+1} . However, utilizing such an approach results in reinitialization of the ratio states of all the DER nodes in the network as given by (5.3) and (5.4). This can result in significant deviations in the total aggregated power delivered by the DERs in the power system network which can result in instability of the power system. Although the power deviations depend on the changes in the aggregator demands at subsequent instants, the DER aggregation framework needs to be robust to any aggregator demand. One approach to deal with this shortcoming is to ‘*delay the dispatch*’ commands from DERs until the aggregated power delivered in the network is within an acceptable range. However, such an approach requires additional measurements of total power delivered by the entire network at all instants. Moreover, delaying the dispatch commands often

results in falling short of the initial response conditions of < 5 seconds as required for providing SFR as ancillary service to the grid. Note that the black start mechanism here refers to the black start of the proposed algorithm rather than the black start of the actual power resources in the network.

In order to mitigate this issue, we next propose a brown start mechanism.

5.4.4 Brown Start: Changes in Power Demand as Input

In this section, we propose a brown-start mechanism where command circulating nodes require as input, the change in the requested power command from the previous time instant ($\Delta\rho_d(t_m)$) as the aggregator signal where for $m > 0$:

$$\Delta\rho_d(t_m) = \rho_d(t_m) - \rho_d(t_{m-1}), \quad (5.15)$$

with $\Delta\rho_d(t_0) := \rho_d(t_0)$. Furthermore, the initialization at all the agents is provided as an *update* of the dispatch values of the earlier ancillary service request. Thus at time t_m , with total ancillary service request of $\rho_d(t_m)$, the agents initialize according to: $r_i(0) :=$

$$\begin{cases} \Delta\rho_d(t_m)/l - \pi_i^{min}(t_m) + \pi_i^*(t_{m-1}), & \text{if } i \in \mathcal{N}_d, \\ -\pi_i^{min}(t_m) + \pi_i^*(t_{m-1}), & \text{if } i \notin \mathcal{N}_d \end{cases} \quad (5.16)$$

Here, let us define the steady state value of the ratio states at each time step as $\alpha_\Delta(t_m)$ and converged value at every node i as $\alpha_{i,\Delta}(\theta_0, t_m)$. Next theorem bounds the error between $\alpha_\Delta(t_m)$ and $\alpha(t_m)$.

Theorem 9 *Let the first state $r_i(0)$ for the Power Apportioning Protocol be initialized by (5.16). Further, let the change in each nodes flexibility be bounded by C , that is $\frac{\pi^f(t_m)}{\pi^f(t_{m+1})} < C$ for $m > 0$. Then,*

$$|\alpha_\Delta(t_m) - \alpha(t_m)| < mC\epsilon. \quad (5.17)$$

Proof: We have,

$$\begin{aligned}
\alpha_\Delta(t_m) &= \frac{1}{\pi^f(t_m)} \left[\sum_{i \in \mathcal{N}_d} \left(\frac{\Delta \rho_d(t_m)}{l} - \pi_i^{min}(t_m) \right. \right. \\
&\quad \left. \left. + \pi_{i,\Delta}^*(t_{m-1}) \right) + \sum_{i \notin \mathcal{N}_d} \left(-\pi_i^{min}(t_m) + \pi_{i,\Delta}^*(t_{m-1}) \right) \right] \\
&= \frac{1}{\pi^f(t_m)} \left[\left(\rho_d(t_m) - \rho_d(t_{m-1}) \right) + \right. \\
&\quad \left. \sum_{i=1}^N \left(\pi_{i,\Delta}^*(t_{m-1}) - \pi_i^{min}(t_m) \right) \right] \\
&= \frac{\rho_d(t_m) - \sum_{i=1}^N \pi_i^{min}(t_m)}{\pi^f(t_m)} + \frac{1}{\pi^f(t_m)} \left[-\rho_d(t_{m-1}) \right. \\
&\quad \left. + \sum_{i=1}^N \left(\pi_i^{min}(t_{m-1}) + \alpha_{i,\Delta}(t_{m-1}, \theta_0) \pi_i^f(t_{m-1}) \right) \right] \\
&= \alpha(t_m) - \frac{\pi^f(t_{m-1})}{\pi^f(t_m)} \frac{\rho_d(t_{m-1}) - \sum_{i=1}^N \pi_i^{min}(t_{m-1})}{\pi^f(t_{m-1})} \\
&\quad + \frac{1}{\pi^f(t_m)} \sum_{i=1}^N \alpha_{i,\Delta}(t_{m-1}, \theta_0) \pi_i^f(t_{m-1})
\end{aligned}$$

As $|\alpha_{i,\Delta}(t_{m-1}, \theta_0) - \alpha_\Delta(t_{m-1})| < \epsilon$ (from (5.12)), we can write $\alpha_{i,\Delta}(t_{m-1}, \theta_0) = \alpha(t_{m-1}) + \psi_{i,m}$, where $|\psi_{i,m}| < \epsilon$ for $i = 1 \dots N$ and $m > 0$. Then,

$$\begin{aligned}
\alpha_\Delta(t_m) - \alpha(t_m) &= -\frac{\pi^f(t_{m-1})}{\pi^f(t_m)} \alpha(t_{m-1}) \\
&\quad + \frac{1}{\pi^f(t_m)} \sum_{i=1}^N \left(\alpha_\Delta(t_{m-1}) + \psi_{i,m} \right) \pi_i^f(t_{m-1}) \\
\alpha_\Delta(t_m) - \alpha(t_m) &= \frac{\pi^f(t_{m-1})}{\pi^f(t_m)} \left(\alpha_\Delta(t_{m-1}) - \alpha(t_{m-1}) \right) \\
&\quad + \frac{1}{\pi^f(t_m)} \sum_{i=1}^N \psi_{i,m} \pi_i^f(t_{m-1})
\end{aligned}$$

Expanding $\alpha_\Delta(t_{m-1}) - \alpha(t_{m-1})$ till $m = 1$ gives us,

$$\begin{aligned}\alpha_\Delta(t_m) - \alpha(t_m) &= \frac{\pi^f(t_0)}{\pi^f(t_m)} \left(\alpha_\Delta(t_0) - \alpha(t_0) \right) \\ &\quad + \sum_{q=1}^m \left(\frac{1}{\pi^f(t_q)} \sum_{i=1}^N \psi_{i,q} \pi_i^f(t_{q-1}) \right)\end{aligned}$$

As $\alpha_\Delta(t_0) = \alpha(t_0)$, we have

$$\begin{aligned}|\alpha_\Delta(t_m) - \alpha(t_m)| &= \left| \sum_{q=1}^m \left(\frac{1}{\pi^f(t_q)} \sum_{i=1}^N \psi_{i,q} \pi_i^f(t_{q-1}) \right) \right| \\ &< \sum_{q=1}^m \left| \frac{\pi^f(t_{q-1})}{\pi^f(t_q)} \right| \epsilon \\ &< mC\epsilon\end{aligned}$$

□

Thus, with the new initialization of r_i by (5.16), instead of (5.3), the error between $\alpha_\Delta(t_m)$ and $\alpha(t_m)$ increases only linearly and can be managed. From (5.13), the error in power dispatch for brown start will also increase only linearly.

The advantage of the brown start mechanism above is that the dispatch maintains continuity whereby the new dispatch gracefully adapts to the new values from dispatch commands from the recent past instead of (5.3) which will involve abruptly changing values that do not incorporate the recent dispatch states over the power network. Moreover, typically the change in dispatch $\Delta\rho_d(t_m)$ is small, and thus convergence to steady state is faster.

5.4.5 Insufficient DER Capacity for SFR Dispatches

Typically in electricity markets, enough generation reserves and controllable load resources are committed such that certain reliability performance standards are always met; however, since in the proposed distributed framework the aggregator/DSO does not have access to real-time capacities of participating DERs, it is imperative that during insufficient capacity periods, i.e., when $\sum_{i=1}^N \pi_i^{max}(t_m) < \rho_d(t_m)$, the maximum generation capacity be commanded in order to minimize the error between the commanded and dispatched output power. Algorithm 2 in such a

Algorithm 2: Distributed finite-time termination of resource apportioning in the presence of communication delays (at each node $i \in V$)

Repeat: At each time instant t_m , ($m \in \{0, 1, 2, \dots\}$)

Input:
 $\pi_i^{\min}(t_m), \pi_i^{\max}(t_m), \epsilon, \bar{\tau}, D$

Initialize:
if $m = 0$ // Black start
then
Aggregator input: $\frac{\rho_d(t_m)}{l}$ if $i \in \mathcal{N}_d$ // Initialize $r_i(0)$ as in (5.3)
else
// Brown start
Aggregator input: $\frac{\Delta \rho_d(t_m)}{l}$ if $i \in \mathcal{N}_d$ // Initialize $r_i(0)$ as in (5.16)
end
 $s_i(0) = \pi_i^{\max}(t_m) - \pi_i^{\min}(t_m);$
 $z_i := r_i(0)/s_i(0), w_i := r_i(0)/s_i(0);$
 $k := 0, \gamma := 1, \theta := 1;$

Repeat:
/* ratio consensus updates of node i given by (5.5), (5.6) */
 $r_i(k+1) := p_{ii}r_i(k) + \sum_{j \in N_i^-} p_{ij}r_j(k - \tau_{ij});$
 $s_i(k+1) := p_{ii}s_i(k) + \sum_{j \in N_i^-} p_{ij}s_j(k - \tau_{ij});$
if $k+1 = \gamma(\bar{\tau}+1)$ **then**
/* maximum and minimum consensus updates given by (5.10(a)), (5.11(a)) for node i */
 $z_i := \max_{j \in N_i^- \cup \{i\}} z_j, w_i := \min_{j \in N_i^- \cup \{i\}} w_j;$
 $\gamma := \gamma + 1$
end
emit: $r_i(k+1), s_i(k+1), w_i$ and z_i
if $k+1 = \theta(D(1+\bar{\tau}) + \bar{\tau})$ **then**
if $z_i - w_i < \epsilon$ **then**
 $r_i^* = r_i(k+1);$
 $s_i^* = s_i(k+1);$
break ; // stop r_i, s_i, w_i and z_i updates
else
 $z_i := r_i(\theta(D(1+\bar{\tau}) + \bar{\tau})/s_i(\theta(D(1+\bar{\tau}) + \bar{\tau})));$
 $w_i := r_i(\theta(D(1+\bar{\tau}) + \bar{\tau})/s_i(\theta(D(1+\bar{\tau}) + \bar{\tau})));$
 $\theta := \theta + 1;$
 $\pi_i^*(t_m, \theta) := \pi_i^{\min}(t_m) +$
 $z_i(\pi_i^{\max}(t_m) - \pi_i^{\min}(t_m));$
// Early Dispatch Mechanism
end
end
 $k = k + 1;$
 $\pi_i^*(t_m) := \pi_i^{\min}(t_m) + \frac{r_i^*}{s_i^*}(\pi_i^{\max}(t_m) - \pi_i^{\min}(t_m))$ // final power reference
command for node $i \in V$

scenario will command DER units to distributedly dispatch output power, $\pi_i^*(t_m) = \pi_i^{\max}(t_m)$.

This is further validated in Test Case II in Section 5.7.

5.5 Numerical Simulations

In this section, we provide numerical results for the performance validation of the proposed framework. We consider the effect of parameters such as graph diameter, D ; tolerance parameter, ϵ ; difference between black start and brown start mechanisms as well as the comparison of the distributed framework to provide response to an aggregator demand with and without early dispatch. Finally, we will compare the result of using a finite time distributed stopping against a decentralized local stopping criteria at each node. These results show the efficacy and viability of implementing such a system in a real world network as demonstrated in the power-hardware-in-the-loop experimental results followed in Section 5.7.1.

For the numerical simulations, we consider a 25 node communication network with diameter, $D = 24$, to demonstrate the performance of the distributed finite time stopping criteria over the decentralized local stopping criteria. Furthermore, we consider a 48 node communication network with diameter, $D = 15$ (as shown in Fig. 5.4(b)) as the communication network of a 48 DER node network interfacing with an aggregator. The topology of the 48 node DER network

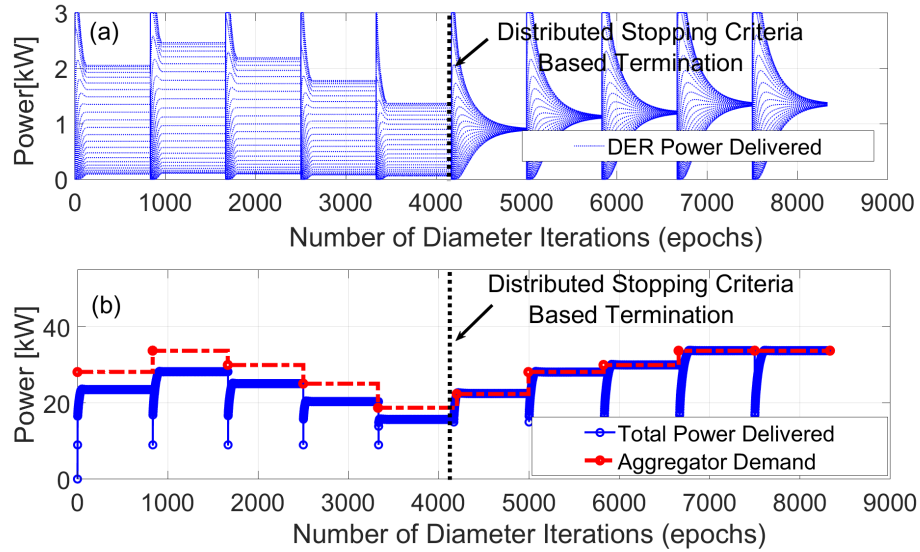


Figure 5.2: a) Power delivered by participating DERs with transition from decentralized local stopping criteria to distributed finite-time stopping criteria at 4165 epochs, (b) aggregator demand received and total power delivered by the 25 DER network.

is reconfigured to have diameter, $D = 5$ (Fig. 5.4(d)), to compare the effect of communication network diameter in the proposed framework. Two nodes are selected as the neighboring nodes of the aggregator. Unless stated otherwise, the minimum and maximum capacity for each DER in the network is assigned randomly from a uniform distribution and the aggregator demand is set such that the apportioning problem is feasible according to (5.1). The aggregator demands are received after convergence of the previous convergence cycle and a bounded time delay of 100 ms is considered as the communication channel between any two nodes in the network.

Distributed finite time stopping criteria vs. decentralized local stopping criteria

We first compare the distributed finite time stopping criteria proposed in Section 5.4 with a decentralized local stopping criteria at each node described next. In the decentralized stopping criteria each node, upon receiving an aggregator demand at $t = t_m$, updates its states according

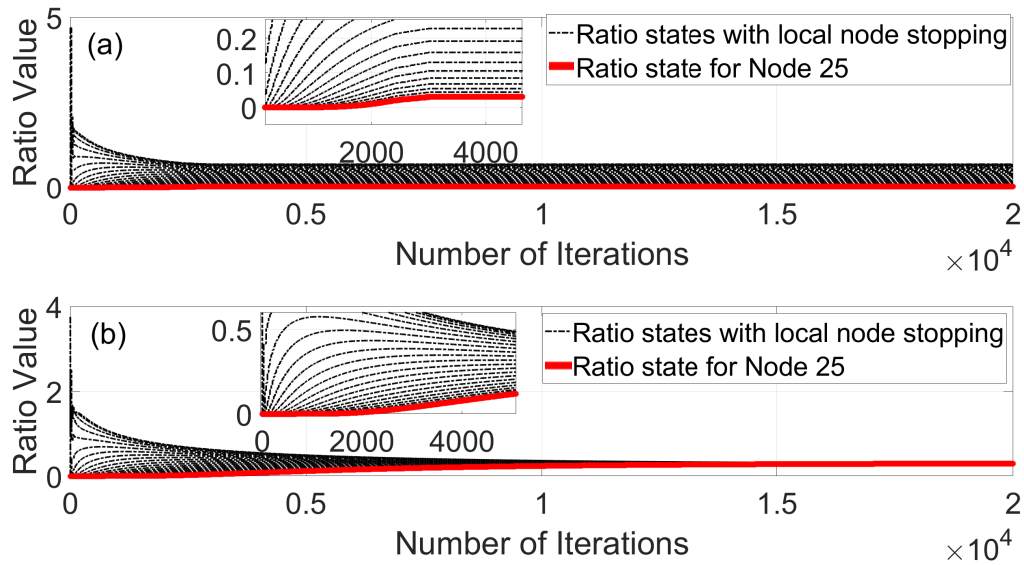


Figure 5.3: Ratio state updates for the 25 node chain network with (a) local node stopping criteria; (b) distributed finite-time stopping criteria.

to (5.5) and (5.6). Each participating node, i , then waits for a fixed number of iterations, $\tilde{T}$, and determines the relative change in its ratio state i.e. $\Delta_{\alpha,i} := |\alpha_i(\theta, t_m) - \alpha_i(\theta - 1, t_m)|$. Any given node, i , thus, stops its updates, if for a given tolerance, ϵ , and a fixed iteration count, $T > \tilde{T}$, the following holds:

$$\Delta_{\alpha,i} \leq \epsilon, \text{ for all } i \in \mathcal{V}. \quad (5.18)$$

Fig. 5.2 shows the results for numerical validation of utilizing decentralized local stopping criteria at each node for the first five aggregator demands (upto 4165 epochs). Each aggregator demand lasts for 20,000 iterations (or 833 epochs). The network is chosen to be a chain network with 25 DER nodes (with $D = 24$), $\tilde{T} = 2400$ iterations (100 epochs) and $\epsilon = 1 \times 10^{-4}$ for all nodes. This network is chosen as the node updates are slow due to the larger diameter of the network and similar capacities of neighboring DER nodes. It is evident that the decentralized stopping criteria results in the aggregator demand (red dotted line) to be not met by the DER dispatch power as shown in Fig. 5.2(b) (see the initial 4165 epochs). After 4165 epochs, the distributed stopping criteria is utilized for terminating nodes upon detecting convergence within a specified tolerance bound, $\epsilon = 1 \times 10^{-4}$. Five subsequent aggregator demands are provided to the DER network. The aggregator demands are met by the DER network as evident in Fig. 5.2(b). The individual power output provided by each DER is shown in Fig. 5.2(a). The DER node ratio updates for the entire convergence cycle for epochs 0 to 833 and epochs 4165 to 5000 are shown in Fig. 5.3(a) and Fig. 5.3(b) respectively. It can be observed that, the distributed stopping criteria allows the ratio state of each node to be within ϵ -tolerance of all the other nodes in the network which is not the case for the decentralized local stopping criteria. Note that, the brown start mechanism of Section 5.4.4 is not implemented for this scenario for fair comparison between the two stopping criteria.

Since in this framework one need not know the topology of the DER node network, it is difficult to decide upon a fixed number of epochs after which to implement a decentralized local node-based criteria for stopping. In contrast, the proposed distributed finite-time stopping criteria always guarantees that each node in the network stops only when the ratio state for every other node in the network is ϵ -close to the ratio state of the node. Also note that the

scenario presented here suffers from similar limitations when using ratio states from neighboring nodes to determine distributed stopping. Thus, the minimum and maximum protocols proposed are imperative for implementing a successful distributed finite-time stopping criteria.

Effect of graph diameter (D)

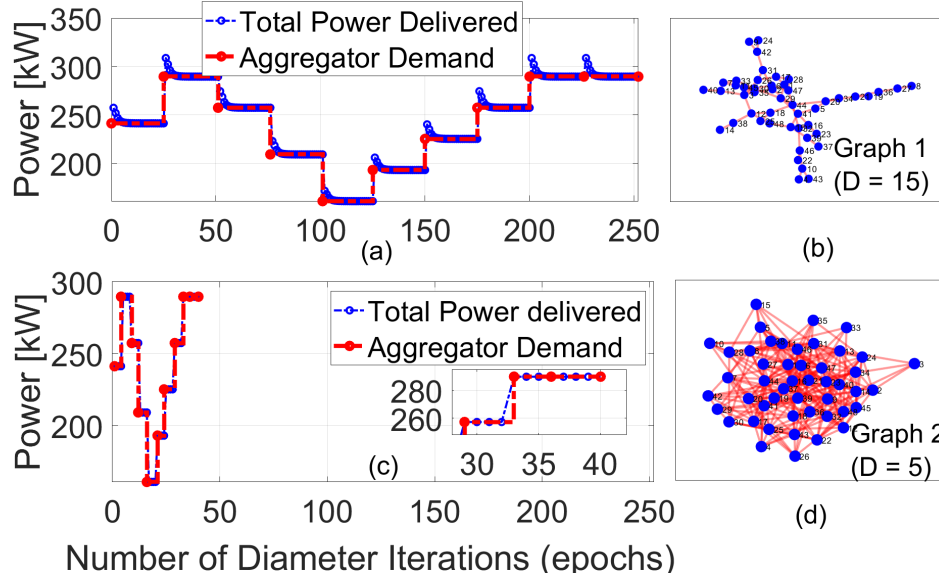


Figure 5.4: (a) Total aggregated power delivered to aggregator demand for the 48 DER network with (a) Graph 1, (c) Graph 2 ; DER communication network (b) Graph 1: 48 node DER network with diameter, $D = 15$, (d) Graph 2: 48 node DER network with diameter, $D = 5$.

Here, we compare the performance of the proposed framework with distributed stopping criteria based on the graph diameter, D . We consider a 48 node communication network, graph-1 with diameter, $D = 15$ and another denser topology defined on the same vertex set (graph-2) with diameter, $D = 5$ as shown in Fig. 5.4(b) and Fig. 5.4(d) respectively. Note that by design of the numerical simulations in order to demonstrate the effect of the parameter, new aggregator demands (marked as red circles) are provided to the aggregator facing nodes only after the network has converged (within tolerance, ϵ). As analyzed in [115,116], the rate of convergence of distributed consensus algorithms depends on the second maximal eigenvalue of

the weight matrix, $[p_{ij}]$, thus the proposed algorithm converges faster for the graph with a smaller diameter (graph-2) as evident from Fig. 5.4(a) and Fig. 5.4(c). This results in a faster secondary response to an aggregator demand and thus can be utilized as a design guideline for aggregating DER nodes where a faster secondary frequency response is desired. For example, the secondary operating reserves in Ireland with response time requirements of 15-90 seconds or Finland's automatic-frequency restoration reserves with response time requirements of 120 seconds in the European Network of Transmission System Operators for Electricity (ENTSO-E) markets.

Effect of tolerance parameter (ϵ)

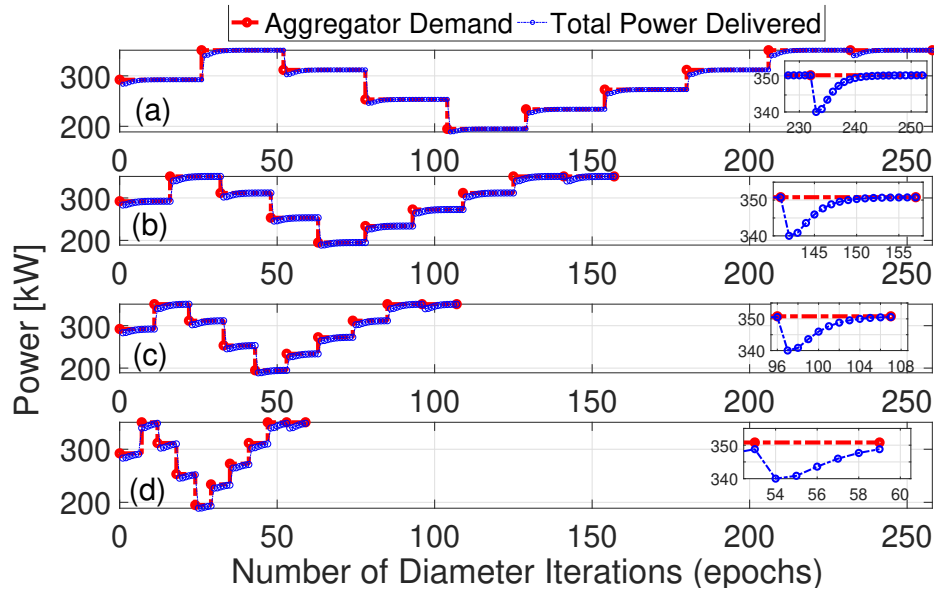


Figure 5.5: Total aggregated power delivered to aggregator demand for the 48 DER network with tolerance value (a) 1×10^{-5} , (b) 1×10^{-3} , (c) 1×10^{-2} and (d) 1×10^{-1} .

The effect of the tolerance parameter, ϵ , as a design criteria on the performance and response of the proposed framework is shown in Fig. 5.5. The 48 node network is considered again and tolerance values 1×10^{-5} , 1×10^{-3} , 1×10^{-2} and 1×10^{-1} are chosen and the response of the DER network towards aggregator demands are shown in Fig. 5.5(a), (b), (c) and (d) respectively. The error in the converged total power delivered to aggregator demand is shown in Table 5.1

Table 5.1: Effect of tolerance parameter on performance

Tolerance, ϵ	Error in meeting Aggregator demand, ($\rho_d = 350.8 kW$)	Number of epochs to convergence
1×10^{-5}	0.0028%	258
1×10^{-3}	0.02%	157
1×10^{-2}	0.11%	107
1×10^{-1}	0.57%	59

for the final aggregator demand, $\rho_d = 380.8 kW$.

It is evident that there is a clear trade-off between accuracy of achieving the aggregator demand and the number of iterations required to meet the global objective where a smaller tolerance value achieves higher accuracy of meeting the aggregator demand (thereby tighter regulation of the system frequency). On the other hand, a larger tolerance allows for faster response but lower accuracy in reaching the demand and thus a larger deviation in frequency.

For numerical simulations and experimental validation, we consider a tolerance of 1×10^{-2} as the DER network is observed to achieve the aggregator demand within $\pm 0.2\%$ while meeting the secondary frequency response objectives.

Black start mechanism vs. brown start mechanism

Here, we compare the performance of the proposed algorithm while implementing the black start and the brown start mechanism of Section 5.4.3 and 5.4.4 respectively. The 48 node network is considered again. The results for six subsequent aggregator commands are shown in Fig. 5.6(a). It is evident that reinitialization of ratio states in the network results in a significant deviation in the initial power dispatched by the network whereas the brown start mechanism developed in this article has a better transient response. Fig. 5.6(b) shows the case when ρ_d is increased from 816 kW to 916 kW ($\Delta\rho_d = 100$) kW at $t = 800$ seconds and kept constant at 916 kW ($\Delta\rho_d = 0$) at $t = 1000$ seconds up to $t = 1200$ seconds. The deviation in the total power delivered by the DER network is $\sim 25\%$ when using the black start mechanism whereas that of the brown start mechanism is $< 0.5\%$. As discussed in Section 5.4.3, a ‘*delay the dispatch*’ strategy can

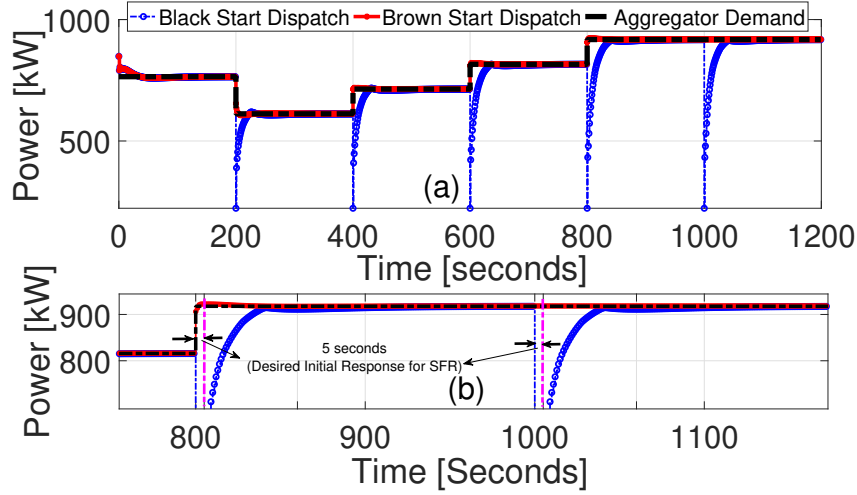


Figure 5.6: (a) Comparison of black start and brown start dispatch response for varying SFR commands (b) zoomed response at $t = 1000$ with and without black start when the SFR command, ρ_d , does not change.

be utilized until the total power delivered is within an acceptable range (for example, $< \pm 2\%$). However, this strategy can result in failing to meet the < 5 second initial response requirement of SFR as an ancillary service as shown in the zoomed plots in Fig. 5.6(b) which is > 5 seconds when using black start mechanism whereas the initial response is around 1 second for the brown start mechanism (within the desired performance requirements). These results demonstrate the superior performance of the brown start mechanism while providing SFR as an ancillary service to the grid while meeting the desired performance standards.

With and without early dispatch

This section provides the comparison of the proposed framework with early dispatch and without early dispatch (that is dispatch upon convergence using the 48 DER node network). As discussed in Section 5.4.2, while implementing early dispatch mechanism, each DER implements (5.14) to update its dispatch at every epoch θ . For $\theta < 3$, the dispatch command for the i^{th} DER, $\pi_i^*(t_m, \theta)$, remains fixed at $\pi_i^*(t_{m-1})$ (within the tolerance, ϵ). In contrast, the DER network without early dispatch waits for the convergence cycle to terminate in finite time where the ratio states of the nodes converge to the consensus value, within a specified tolerance, upon which

each DER then dispatches the new power command obtained. Until convergence, the dispatch command for the i^{th} DER, $\pi_i^*(t_m)$, remains fixed at $\pi_i^*(t_{m-1})$.

Fig. 5.7(a) shows the power reference updates for each DER implementing early dispatch (blue solid lines) and without implementing early dispatch (green dots) in response to ten subsequent SFR commands received by the DER network. The tolerance parameter, ϵ , is chosen to be 1×10^{-4} . As alluded to earlier, for the case without early dispatch, the reference power commands from the proposed algorithm are available only after the convergence in the network is detected (shown in green dots), whereas while implementing early dispatch, the reference power commands for all the participating DERs are available after $\theta > 3$ (shown in blue lines). It is clear that when the DERs do not have any reference commands available to them in the no early dispatch case, the previous power reference commands are implemented. This is discussed next. Fig. 5.7(b) presents the total power delivered with (blue line) and without (green dashed line) the early dispatch mechanism along with the aggregator demands. It is evident that the early dispatch mechanism allows the DER network to respond to the aggregator demand within 5 seconds and achieve SFR within full activation time of 1 minute whereas without the early

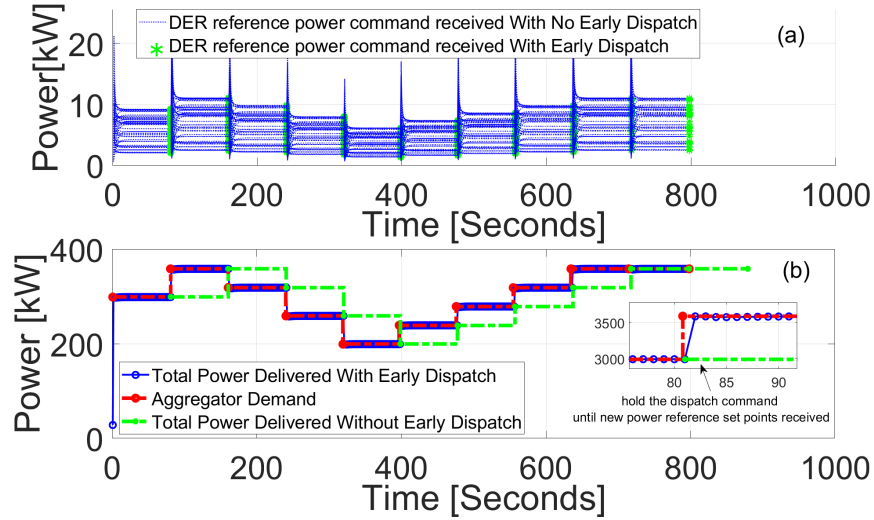


Figure 5.7: (a) Power reference commands obtained by participating DERs with and without early dispatch mechanism, (b) Total aggregated power delivered to aggregator demand for the 48 DER network with and without Early Dispatch.

dispatch mechanism, the < 5 second initial response can not be met (as shown in the zoomed figure in Fig. 5.7(b)). Moreover, in this scenario, without early dispatch mechanism, the full activation time is ~ 80 seconds that violates the desired performance requirement of 1 minute for SFR as an ancillary service for real-world DER networks. Note again that by design of the numerical simulations, new aggregator demands (marked as red circles) are provided to the aggregator facing nodes only after the network has converged (within tolerance, ϵ). This is further validated in the PHIL setup experimentally in the next section.

Effect of delay: comparison of Algorithm 1 with [2]

Here, we compare the conventional minimum/maximum consensus protocol presented in our earlier work [2] to the proposed distributed finite time algorithm to handle communication delays. We consider the 25 node DER network with diameter, $D = 24$. The maximum delay in the communication channel between any two nodes, $\bar{\tau}$, is simulated to be 100 ms.

The results of implementing the algorithm in [2] and the proposed algorithm (Algorithm 1) are shown in Fig. 5.8. It can be clearly observed that when only the diameter of the network to reset the maximum and minimum protocols results in convergence to an incorrect ratio state whereas using $D(1 + \bar{\tau}) + \bar{\tau}$ results in convergence to the correct ratio state (as evident from meeting the aggregator demand in Fig. 5.8).

The convergence of ratio states for the aggregator demand at 7500 diameter iterations are shown in Fig. 5.9. The error in meeting the aggregator demand is around 24%. This can result in causing the frequency deviations to be larger and thereby adversely affecting the power system network. This shows the efficacy and robustness of the proposed algorithm towards inherent communication delays while meeting SFR demands. Next, we present the HIL-based validation of the proposed methodology.

5.6 Experimental Configuration

A novel PHIL experimental configuration is to validate the distributed protocol containing 40+ physical hardware generation devices, 48 physical distributed controller nodes (DCNs), 202

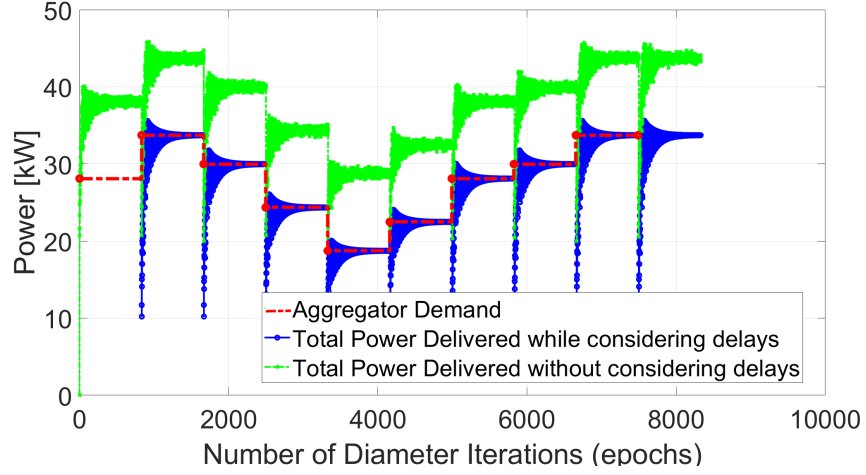


Figure 5.8: Total aggregated power delivered to aggregator demand for the 25 DER network ($D = 24$) with the algorithm of [2] (green) and Algorithm 1 (blue) in the presence of communication delays.

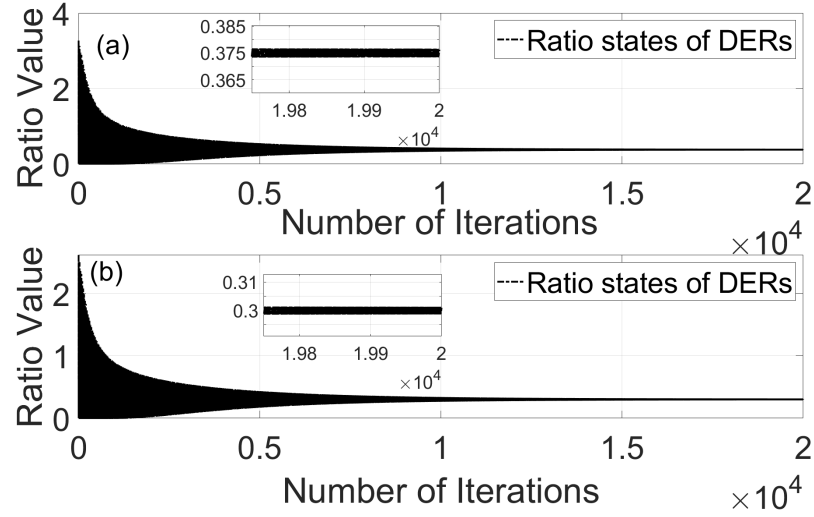


Figure 5.9: Ratio states for the aggregator demand at 7500 diameter iterations for (a) Algorithm in [2] converges incorrectly to 0.375, (b) Algorithm 1 converges correctly to 0.3 (ratio of the true average of the initial states).

simulated DCNs, and a real-time power system model. The components of the experimental configuration (Fig. 5.1) are described next.

5.6.1 Real-Time Distribution System Model (RT-DSM) Layer

The underlying RT-DSM used to represent the power network is based on a distribution network model from Aurora, Colorado, USA, which represents around 2000 customers and is augmented in this study to have 50.2% distributed PV RES penetration (both residential and commercial DER units —total PV 8.06 MVA, battery ESS (BESS) capacity of 1.203 MW). The network has a peak load of 7.1 MVA and has 5.285 MVA of controllable DERs. Irradiance profile data from Aurora, CO region and load profile data measured from the Aurora distribution system model are used. The RT-DSM was executed on a 12-core OPAL-RT OP5707 using ePhasorSim with a time step of 10 ms. The capacities of participating DERs are shown in Table 5.2. PHIL interfaces, including power amplifiers and feedback sensors for measurement, are provided to interconnect the physical hardware under test of Fig. 5.1 at five independent PCCs, P_1 - P_5 , in the RT-DSM.

5.6.2 Simulated Controller (SC) Layer

The simulated controller layer further consists of: 1) simulated local DER controllers (simLCs) that provide control signals to distributed PV inverters in the RT-DSM. simLC provides real-time measurements of minimum and maximum DER capacity to its DCN interface; 2) simulated distributed controller nodes (simDCNs) form the communication interfaces to simLC and provide power reference commands (P_k^* , Q_k^* , see Fig. 5.1) based on the power apportioning protocol.

5.6.3 Communication Layer

Rpi-based DCNs, with bounded time delays, are deployed to communicate with phDERs (described later) and simLCs. A communication topology is generated with the diameter and average node degree corresponding to the distribution network with 250 nodes overlayed on DCNs (simulated and Rpi-based) to represent the weakly-connected neighborhoods of RT-DSM as shown in Fig. 5.10. The WebSocket-based communication protocol between DCNs is implemented over Ethernet.

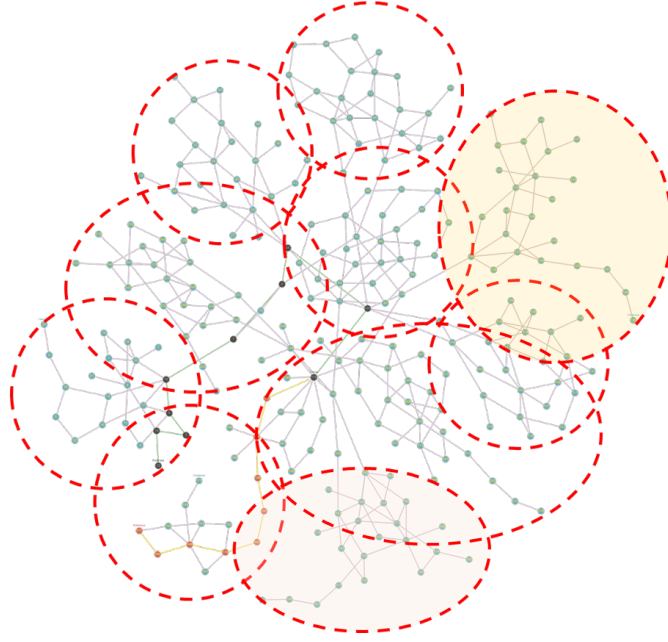


Figure 5.10: Communication graph ($D = 19$) for the DER network for PHIL experiments with neighborhoods.

5.6.4 Physical Hardware DER System (phDER) Layer

- 1) Residential-scale DER: This 2-kVA custom-built DER (Fig. 5.11 (a)) is a Type-4 Inverter connected to a PV simulator, residential loads, and is interfaced at P1 - 240-V PCC.
- 2) Utility-scale DER (*Type-6 Inverter*): This custom-built DER consists of a BESS (100 kW three-phase, Lithium-ion based 32.8 kWh) and a commercial PV inverter (100 kW, three-phase) (Fig. 5.11 (b)) coordinated via an integrated controller and interfaced at the three-phase PCC P2.
- 3) DER racks: Each DER rack (Fig. 5.11 (c),(d)) consists of two commercial-off-the-shelf residential PV string inverters (Type 1, 3/5 kVA and Type 2 3.8 kVA), a BESS inverter (Type 5, 5 kW) and three racks of 12 PV microinverters (Type 3, 320 W each). Each DER rack is capable of receiving a power reference command via MODBUS protocol, and the racks are interfaced at 240 V PCCs P3 - P5 as shown in Table 5.2. Note that the physical DER units are scaled through PHIL interfaces to emulate large-scale hardware devices in RT-DSM (Table 5.2).



Figure 5.11: Hardware under test for PHIL experiments.

5.7 Results

5.7.1 Experimental power-hardware-in-the-loop results

The developed PHIL system is used for demonstrating the capability of the finite-time power apportioning protocol toward providing SFR as an ancillary service to the grid. Each phDER is facilitated with a Rpi-based module for communication with other network DER units. For validation, $\bar{\tau}$ of 50 ms for Test Case I and 20 ms for Test Case II are chosen based on typical round trip times observed. The communication topology of Fig. 5.10 is chosen.

Test Case I: Black Start: DER Aggregation and Early Dispatch with Fixed Capacities

We consider the scenario where the SFR commanded by the aggregator is ρ_d (instead of $\Delta\rho_d$) and participating DERs have fixed capacities. The RT-DSM follows a net-load profile where the feeder consumption supplied by the bulk power system is 200 kW (Fig. 5.12 (a))

Table 5.2: DER ratings for experiments

Type	Inverters			Device Count		PHIL Interface point
	Capacity (kW)					
	physical		sim.	phys.	sim.	
	actual	scaled				
Type 1	3/5/5	100/75 /375	3-6 /100	2/1/1	155/7	P3-P5
Type 2	3.8	375	100	1	10	P3-P5
Type 3	0.32 each	6.25/8.33 /0.25	-	12/12/12	-	P3-P5
Type 4	2	2	-	1	-	P1
Type 5	5/5	5/100	7-10	1/2	65	P3-P5
Type 6	100	1000	-	1	-	P2
Total		2,410 (MW)	2,864 (MW)	46	237	

and DER units collectively provide 225 kW. A fixed irradiance of 800 W/m^2 is considered for Test Case I. Distributed PV inverters in the network have headroom available to provide SFR. At $t = 27 \text{ s}$, the aggregator sends a new total power command, $\rho_d = 500 \text{ kW}$, to achieve a change in feeder net active power consumption by $\Delta\rho_d = -275 \text{ kW}$ (see Fig. 5.12 (b)). Two command circulating nodes receive command signals, $\rho_d/2$, each and initiate the power apportioning algorithm with early dispatch. The tolerance, ϵ , is set to 0.01 for all test cases. Fig. 5.14 shows the reference power command updates of the three Rpi-DCNs associated with Mobile DER Rack 1 and corresponding DER output power. Clearly, the initial response of the DERs is obtained within 2 s of receiving the aggregator command by the command circulating nodes, and the finite-time criteria is met within 50 s for the distribution network under validation. Fig. 5.13 shows the power reference updates and the total active power reference for all the 250 participating DERs. These results establish that the desired SFR command is met and the response times achieved by the simulated and phDER units satisfy the initial response time requirement of 5 seconds and ramp time to response of 1 minute.

Figure 5.12(a), compares the case with and without DER' dispatch to meet active power in the feeder. The response of the feeder with respect to the increase in aggregated DER' output is presented in Fig. 5.12(b). The red colored line here corresponds to the change in dispatch

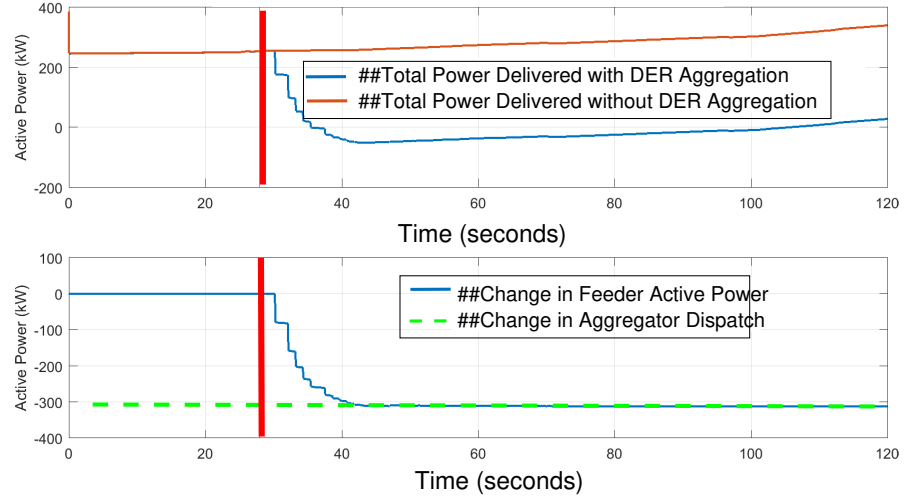


Figure 5.12: Active power consumed by distribution network with and without DER aggregation for Test Case-I.

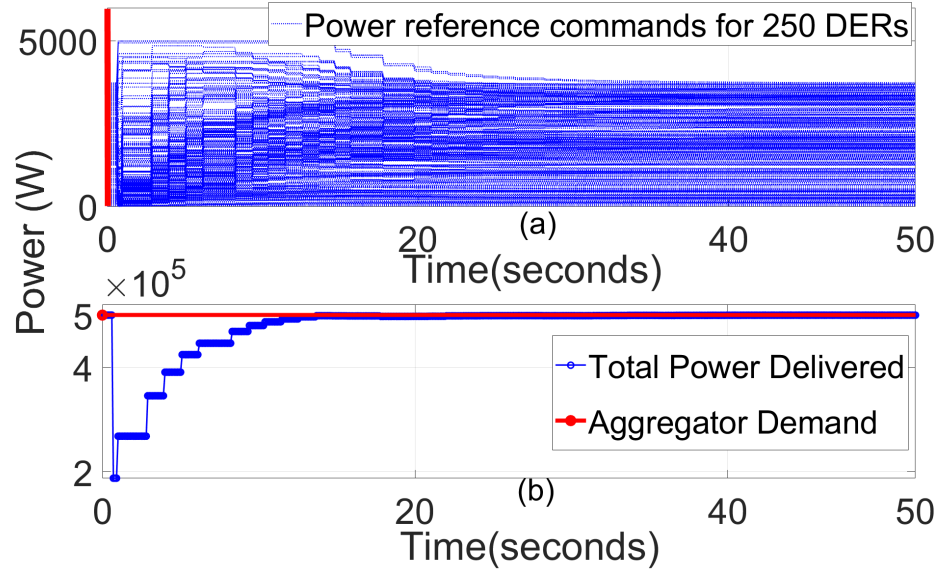


Figure 5.13: (a) Reference power command updates for 250 DER units in the network, (b) Total reference power command from the DER network with $\Delta\rho_d = 275\text{ kW}$ and $\rho_d = 500\text{ kW}$.

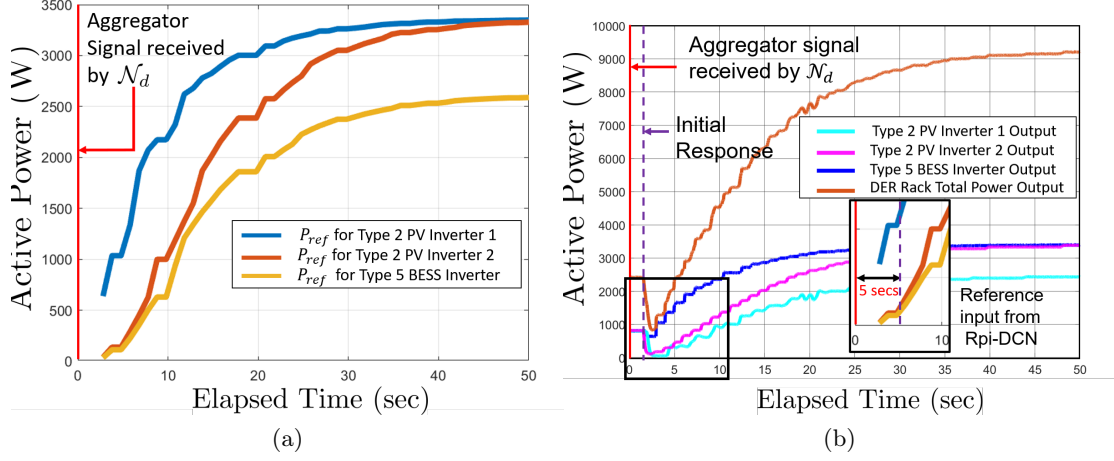


Figure 5.14: (a) Power reference commands at Rpi-DCNs, (b) dispatched output power for DER Rack 1 inverters.

command received by command circulation nodes, $\mathcal{N}_d$. The secondary feeder voltages remain within $\pm 1\%$ of nominal.

Test Case II: Brown Start: DER Aggregation with Early Dispatch for Time-Varying Capacities and Aggregator command

This test case emphasizes SFR due to continuous dispatches and variability in PV generation capability, requiring adjustments of minimum and maximum capacities of network DERs. In this scenario, the brown-start mechanism is implemented to achieve power apportioning due to changes in the system operator's command and/or changes in generation capacities of participating PV and BESS DERs. Here, each PV inverter follows an irradiance profile sampled every 30 seconds. Initially, at $t = 0$ s, $\mathcal{N}_d = 2$ (here Mobile DER Rack 1: Type 1 and Type 2 inverters) receive the initial aggregator command $\rho_d(t_0) = 3.62$ MW and within 5 s all the units in the network respond initially to meet the net command. A $\Delta\rho_d(t)$ of -0.82 MW at $t = 900$ s and +1.01 MW at $t = 1080$ s are provided as aggregator commands to the network via $\mathcal{N}_d$ (see Fig. 5.15). At time periods, $0 < t < 900$ s, $900 < t < 1080$ s, and $t > 1080$ s, the brown-start and early dispatch mechanisms allow DERs to be dispatched to achieve feasible solutions to (5.1) in the presence of variable solar irradiance and discharging/charging BESS DERs. The ramp response time (within $\pm 5\%$ of the steady-state value) to meet these commands by the aggregate

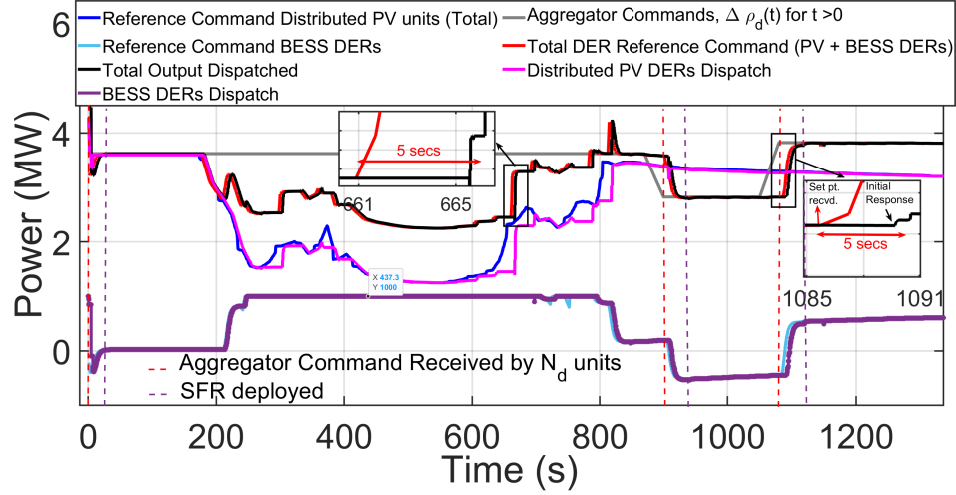


Figure 5.15: Reference power commands obtained at DCNs and output power dispatched for Test Case II.

DER network in the RT-DSM is $\sim 21 - 44$ s (to within SFR dispatch timescales) as shown by the purple dotted lines in Fig. 5.15. Fig. 5.16 shows the commanded power from physical DERs and simulated DERs in RT-DSM as well as the output response of selected physical hardware and simulated devices in the system. From $0 \leq t \leq 180$ s, because the total demand can be solely met by DERs with RES prioritization, due to the high irradiance that allows the distributed PV units to operate at their full capacity, the BESS units are not dispatched. For $180 < t < 800$ s, however, because of the decrease in irradiance, the net output power of all distributed PV units decreases, causing the algorithm to command BESS units in the network to dispatch their cumulative maximum capacity of ~ 1200 kW in order to minimize the difference in commanded and generated power. Similarly, at $t = 800$ s, when RES generation output was higher, BESS dispatch was adjusted back to meet the deficit. At $t = 900$ s, because the RES generation is higher than demanded $\rho_d(t)$, surplus PV generation is used to charge the BESS units. Finally, at $t = 1080$ s, both RES and BESS units are dispatched to meet and sustain the changed power commands of the aggregator with a ramp response time of less than 50 s. The secondary-side feeder voltages deviations were observed be $< \pm 5\%$ (within ANSI C84.1).

Note that the fast and coordinated dispatch of (simulated and physical hardware) BESS

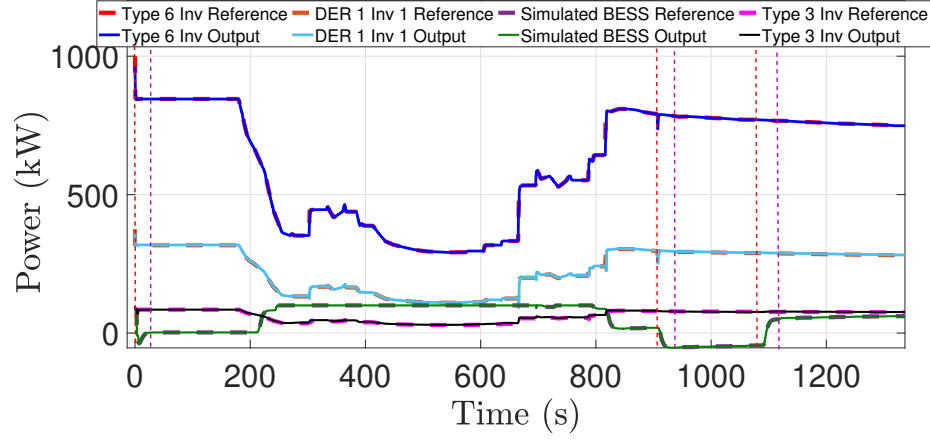


Figure 5.16: Reference Power command from Rpi-DCNs/sim-DCNs and scaled dispatched power from (selected) phDER.

units was achieved distributedly, demonstrating the efficacy of the proposed finite time criteria to provide AS in the face of RES variability, for example, ERCOT energy markets for real-time and day-ahead market dispatch decisions. This scenario is designed specifically to demonstrate the point that the best performance from the coordinated aggregated DER units is achieved in the presence of variability and can support the grid through coordinated dispatch of BESS units. We envisage that this framework along with existing methodologies, such as day-ahead market mechanisms for committing resources with variability [117] and demand response methods, can provide the technological leap required for providing proposed ancillary services at such fast timescales.

Chapter 6

Distributed Detection of Malicious Attacks on Consensus Algorithms with Applications in Power Networks

Consensus-based distributed algorithms are well suited for coordination among agents in a cyber-physical system. These distributed schemes, however, suffer from their vulnerability to cyber attacks that are aimed at manipulating data and control flow. In this article, we present a novel distributed method for detecting the presence of such intrusions for a distributed multi-agent system following ratio consensus. We employ a Max-Min protocol to develop low cost, easy to implement detection strategies where each participating node detects the intrusion independently, eliminating the need for a trusted certifying agent in the network. The effectiveness of the detection method is demonstrated by numerical simulations on a 1000 node network to demonstrate the efficacy and simplicity of implementation.

6.1 Introduction

Distributed coordination and decision making has enabled new paradigms in cyber-physical systems that include sensor networks, autonomous vehicle systems and power networks. A key strategy often employed is achieving consensus among the agents in a network by sharing local information with its neighbors [79, 118]. The applicability of Consensus-based distributed coordination methods often depends on the termination of the consensus algorithm in finite-time (see [2, 83] for static graphs and [119] for dynamic topology networks) which allows for the inferences reached to be used by the agents to perform subsequent important operations; for example, to provide ancillary services by local agents in a power grid (see [120] and [62]). Moreover, the implementation of such methods in a cyber-physical system might necessitate a prior centralized coordination step. In many situations, such centralized coordination is not feasible. To this end, a ratio consensus algorithm that eliminates the need of a centralized framework is presented in [84, 121]. Such distributed decision making schemes depend heavily on trust and credibility of the participating agents. Each agent gathers information by relying on its neighbors to share their information honestly. When factoring this type of information exchange into a model, each agent's lack of honesty can skew the resulting decision (consensus value). It is imperative to increase the resilience of multi-agent systems towards such malicious or manipulative behaviour. To address the cyber-security issues in distributed algorithms, a number of different approaches have appeared in the literature. One such method encompasses trust based models for multi-agent systems [122]. Such trust-based approaches need repeated interactions between the agents and need to gather a large amount of information from different agents in the network to formulate notions of cooperation or rejection of transactions. Reference [123] formulates an intrusion detection system based on network traffic analysis whereas authors in [124], [125] assert the need of creating an r -robust graph for a resilient consensus. Thus, most proposed methods involve techniques that impose additional constraints on the network that are difficult to generalize.

This article presents a distributed algorithm for detection of malicious nodes in a cyber-physical system. We use the monotonic properties of global maximum and minimum over the network following the consensus algorithm, to detect intrusions. The proposed algorithm has many

benefits over the existing counter parts in the literature [122–126]. Firstly, the scheme proposed eliminates the need of a unique identifier (as required in [126]) for each node in the network which allows each node to individually detect the intrusion. The results developed here enable a truly distributed way of detection as an agent utilizes information only from its neighbors. The only global information needed is an upper bound on the graph diameter D (defined in Section 6.2). Furthermore, it obviates the need for any centralized planning beforehand. Secondly, the scheme proposed has a small memory requirement; specifically, each node needs to store D values for each of its neighbors (unlike the memory requirement which increases as a polynomial function of the number of nodes N in [126]). Also, the computations involved are simple mathematical operations like multiplication of scalar values, in contrast to techniques using matrix inversions and rank calculations which can easily become intractable as the size of the network increases. This makes the proposed algorithm suitable for *ad-hoc* networks where the agents have less computational power and storage capacity. Unlike other works (see [123], [126]) the proposed algorithm does not need a learning stage where the network parameters used for intrusion detection are estimated before the actual detection protocol can be deployed, thus, facilitating an on-line implementation in cyber-physical systems like smart micro-grids. Also, the proposed scheme is topology independent unlike schemes where the graph topology is the determining factor for tolerance towards malicious agents [125]. With the above benefits, this algorithm can be applied to a multitude of cyber-physical systems.

6.1.1 Definitions

Readers are guided to 4.1 for the definitions of terminologies from graph theory which are used for subsequent developments.

Definition 12 (*Epoch*) *An epoch is defined as any D consecutive state update iterations i.e. k^{th} epoch defines the discrete-time interval $[kD + 1, (k + 1)D]$.*

Definition 13 (*In-Degree*) *In-degree of a node i is the number of elements $|N_i^-|$, where $N_i^- = \{j : (i, j) \in E\}$.*

Definition 14 (*Out-Degree*) *The out-degree of a node i is the number of elements $|N_i^+|$, where*

$$N_i^+ = \{j : (j, i) \in E\}.$$

6.2 Preliminaries

6.2.1 Problem Formulation

Let q denote the intruder node violating the update rules (1a), (1b) such that, given $\rho > 0$, for all $i \in V$,

$$\left| \lim_{k \rightarrow \infty} \frac{x_i(k)}{y_i(k)} - \frac{\sum_{j=1}^N x_j(0)}{\sum_{j=1}^N y_j(0)} \right| > \rho, \quad (6.1)$$

$$\lim_{k \rightarrow \infty} \left(\frac{x_i(k)}{y_i(k)} - \frac{x_q(k)}{y_q(k)} \right) = 0. \quad (6.2)$$

In other words, the intruder's aim is to steer the final convergence value of the ratio consensus away from the true convergence value (ratio of sum of initial states) to a new desired value and achieve consensus among all the nodes in the network at the desired value.

Objective: *Given update rules (1a), (1b) for all $i \in V$ and an intruder q , the goal is to detect any intrusions of the form (6.1) and (6.2) in the network in a distributed manner.*

Next, we substantiate the motivations of intruders and the need for a distributed detection algorithm through a distributed power allocation framework. The proposed detection algorithm, however, is applicable for detecting malicious attacks in other consensus-based applications such as distributed optimization [127], [128], finite time consensus algorithms in higher dimensions [129, 130], movement coordination in autonomous vehicles, task allocation in unmanned aerial vehicles (UAVs) for rescue and search operations.

The following subsections summarize the ratio consensus protocol and distributed apportioning algorithm developed in previous chapters.

6.2.2 Ratio Consensus Protocol

Consider a directed graph $G = \{V, E\}$ containing N nodes. Each node $i \in V$ maintains two states at time k , denoted by $x_i(k)$ (referred as numerator state of node i) and $y_i(k)$ (referred as denominator state of node i). Node i updates its state at the $(k+1)^{th}$ discrete iteration according to the following update law:

$$x_i(k+1) = p_{ii}x_i(k) + \sum_{j \in N_i^-} p_{ij}x_j(k), \quad (6.3a)$$

$$y_i(k+1) = p_{ii}y_i(k) + \sum_{j \in N_i^-} p_{ij}y_j(k), \quad (6.3b)$$

where N_i^- is the set of in-neighbors of node i . We next present a result from [84] which establishes the convergence of the ratio $x_i(k)/y_i(k)$.

Theorem 10 *Suppose the weight matrix P with $P(i, j) = p_{ij}$ associated with the directed graph G is primitive and column stochastic with $P(i, i) > 0$ for all $i \in V$. Let $\{x_1(0), x_2(0), \dots, x_N(0)\}$, $y_i(0) = 1$ be the initial conditions for numerator and denominator states respectively for all $i \in V$. Then, the ratio $\frac{x_i(k)}{y_i(k)}$ asymptotically converges to $\alpha := \frac{1}{N} \sum_{i=1}^N x_i(0)$ for all $i = 1, \dots, N$ (referred to as ratio consensus).*

Lemma 6.2.1 *With the assumptions of Theorem 10, sum of state values of x and y are conserved, i.e. for any k ,*

$$\sum_{i=1}^N x_i(k) = \sum_{i=1}^N x_i(0), \quad \sum_{i=1}^N y_i(k) = \sum_{i=1}^N y_i(0),$$

for updates according to (6.3a) and (6.3b).

Proof: Update equation (6.3a) can also be written as

$$x_i(k+1) = \sum_{j=1}^N p_{ij}x_j(k)$$

$$\begin{aligned}
\text{Then, } \sum_{i=1}^N x_i(k+1) &= \sum_{i=1}^N \left[\sum_{j=1}^N p_{ij} x_j(k) \right] \\
&= \sum_{j=1}^N \left[\sum_{i=1}^N p_{ij} x_j(k) \right] = \sum_{j=1}^N x_j(k) \left[\sum_{i=1}^N p_{ij} \right]
\end{aligned}$$

As P is column-stochastic, we have $\sum_{i=1}^N p_{ij} = 1$, thus,

$$\sum_{i=1}^N x_i(k+1) = \sum_{j=1}^N x_j(k)$$

Result can be easily derived now by using induction. Same result can be shown for y . $\square$

6.3 Framework for distributed apportioning in power networks

The ratio consensus protocol discussed above is used for distributed coordination of Distributed Generation (DG) units to meet the ancillary service demand. We first summarize below the distributed averaging protocol [7, 84, 110] Here the ratio consensus algorithm is used to compute the power reference commands for the DG units based on their generation capacities. Let ρ_d represent the total amount of power requested by an aggregator of the smart micro-grid to be supplied by the N DG units (see Fig. 6.1(a)). Let π_j^* represent the power supplied by DG j while respecting its own generation capacity and load constraints. π_j^{max} , π_j^{min} represent the maximum and minimum power the j^{th} DG unit can supply. Thus,

$$\sum_{j=1}^N \pi_j^* = \rho_d, \text{ and } \pi_j^{min} \leq \pi_j^* \leq \pi_j^{max} \text{ for all } j \in V. \quad (6.5)$$

We refer to the problem of determining π_j^* respecting the above constraints as the resource apportioning problem where (6.5) gives the feasibility criteria for all participating units. A fair way of apportioning DG units to supply the demanded power is given by [7]:

$$\pi_j^* = \pi_j^{min} + r(\pi_j^{max} - \pi_j^{min}) \text{ for all } j \in V, \quad (6.6)$$

$$\text{where, } r = \frac{\rho_d - \sum_{j=1}^N \pi_j^{\min}}{\sum_{j=1}^N (\pi_j^{\max} - \pi_j^{\min})}. \quad (6.7)$$

The power apportioning algorithm [7] provides a distributed way to calculate r when a single aggregator can communicate to at least one node in the network.

6.4 Cyber-secured distributed network framework

In this section, we propose a cyber-secured framework for networks running distributed consensus algorithms. We first present the max-min protocols to obtain global maximum and minimum values of the network and use the monotonicity property of these protocols [121] to devise the proposed distributed intruder detection algorithm. Define the maximum and minimum value of the ratio of numerator and denominator states given by (6.3a) and (6.3b) over all nodes at any time instant k as,

$$M(k) := \max_{i \in V} \frac{x_i(k)}{y_i(k)}, y_j(k) \neq 0, j \in V, \quad (6.8a)$$

$$m(k) := \min_{i \in V} \frac{x_i(k)}{y_i(k)}, y_j(k) \neq 0, j \in V. \quad (6.8b)$$

Lemma 6.4.1 *For all time instants $k' \geq k$ and for all $i \in V$,*

$$\frac{x_i(k')}{y_i(k')} \leq M(k), \quad \frac{x_i(k')}{y_i(k')} \geq m(k). \quad (6.9)$$

Proof: See [121] for proof. □

The following theorem shows that the maximum (or minimum) value $M(k)$ (or $m(k)$) strictly decreases (or increases) after finite time.

Theorem 11 *Define the initial ratio vector as $r(nD) := \left[\frac{x_1(nD)}{y_1(nD)} \quad \frac{x_2(nD)}{y_2(nD)} \quad \dots \quad \frac{x_N(nD)}{y_N(nD)} \right]$ such that $\min(r(nD)) < \max(r(nD))$, where, $n = 0, 1, 2, \dots$. Then,*

$$M((n+1)D) < M(nD) \text{ and } m((n+1)D) > m(nD)$$

Proof: See [121] for proof. $\square$

6.4.1 Maximum-Minimum (MXP-MNP) Consensus Protocol

The Maximum and Minimum Consensus Protocol referred as MXP and MNP protocol computes the maximum and minimum of the given initial node values $v(0) = [v_1(0) \ v_2(0) \ \dots \ v_N(0)]^T$ in a distributed manner. It takes $v(0)$ as an input and generates a sequence of node values based on the following updates for any node $i \in V$,

$$z_i(k+1) = \max_{j \in N_i^- \cup i} z_j(k), \quad w_i(k+1) = \min_{j \in N_i^- \cup i} w_j(k), \quad (6.10)$$

where $z_i(0) = w_i(0) = v_i(0)$.

Lemma 6.4.2 *The estimates $z_i(k)$ and $w_i(k)$ in (6.10) converges to $\max_{j \in V} z_j(0)$ and $\min_{j \in V} w_j(0)$ respectively in finite time $k \leq D$ for all $i \in V$.*

Proof: See [2] for the proof. $\square$

The MXP and MNP protocols are reset after every epoch and the initial conditions are set as the initial ratios held by the nodes, that is, $z_i(nD) = x_i(nD)/y_i(nD)$ and $w_i(nD) = x_i(nD)/y_i(nD)$ respectively for all $i \in V$. Notice that in the above scheme the global maximum and minimum values of the ratios at the beginning of an epoch are available to each node after next D iterations (as one iteration of the Max-Min algorithm is completed in D iterations) [2]. In other words $MXP(k) = M((k-1)D)$ and $MNP(k) = m((k-1)D)$.

6.4.2 Intruder Detection Algorithm

Based on the above discussion we now present a detection algorithm. The algorithm requires each node in the network to store and access the last D state updates of all of its neighbors. We define the values stored by a node i as: $x_{ij}[kD : (k+1)D]$ and $y_{ij}[kD : (k+1)D]$ for all $j \in N_i^-$. This storage is maintained in a *StorageBuffer* at each node. If each node in the network has access to last D state updates of all of its neighbors, δ_m cannot be chosen successfully by intruder node i with the assumptions of **A1** and **A2** (following the attack strategy (6.14) and

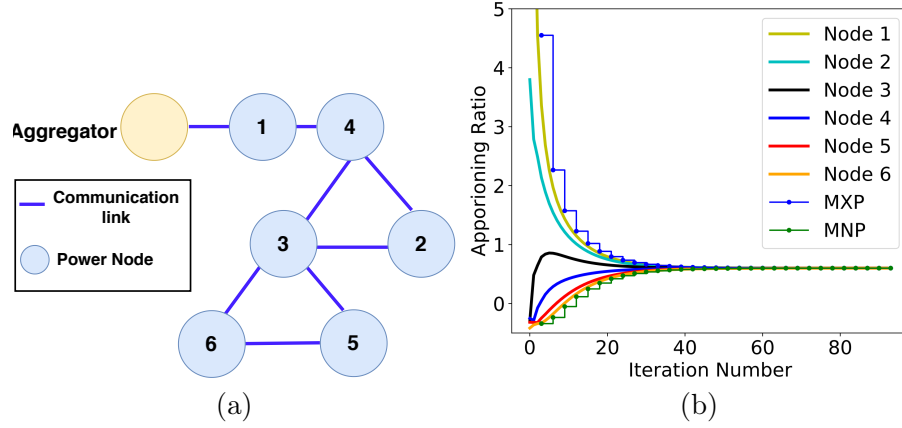


Figure 6.1: (a) Communication topology of power network with aggregator ($D = 3$) and node 1 as the demand circulation node, (b) state value updates with MXP and MNP values.

(6.15)) as it can be detected due to one of the following violations:

$$\frac{x_{ji}[kD : (k+1)D]}{y_{ji}[kD : (k+1)D]} \leq MXP(k+1) \text{ or,} \quad (6.11)$$

$$\frac{x_{ji}[kD : (k+1)D]}{y_{ji}[kD : (k+1)D]} \geq MNP(k+1), \quad (6.12)$$

for all nodes $j \in N_i^+$. Such a detection will happen simultaneously at all the neighboring nodes of i followed by the generation of an *AttackFlag* signal that is emitted by each node to its neighbors. Within the next D iteration of the algorithm the *AttackFlag* signal regarding intrusion is propagated to all the nodes in the network which results in the interruption of the consensus protocol and further actions can be taken to make the network secure. The complete detection algorithm is presented in Algorithm 3.

Remark 11.1 *The proposed detection algorithm requires each node i to store $|N_i^-| \times D$ values thus, can be implemented in low-cost devices such as Raspberry-pi units.*

6.5 Analysis of attack strategies

In this section, we describe various attack models for intruders. We classify the intrusion attacks based on two criteria: (1) the impact they have on the power network and (2) the kind of strategy used by these intruders. Based on the impact caused, intrusions are categorized as vandalism attacks and manipulative attacks. The vandalism attacks directed towards the power network are used to disrupt the proper functioning of the power network. Such attacks can be easily detected by monitoring physical quantities like voltage and current profile of the power network [131] and are not within the scope of this paper. Manipulation attacks on the other

Algorithm 3: Distributed intruder detection protocol (at each node $i \in V$)

Repeat:

Input: D ; // Upper bound on graph Diameter

Initialize:

| $k = 1; l = 1; AttackFlag_i = False; StorageBuffer_i = [];$

Repeat:

| /* Updates of states $x_i(k)$, $y_i(k)$ given by (6.3a) and (6.3b) */

| /* MXP and MNP protocol for i */

| $z_i := MXP(k); w_i := MNP(k);$ (Using (6.8a) and (6.8b))

| $StorageBuffer_i \leftarrow N_i^-;$

| /* Store last D values of all in-Neighbors. */

| **if** $k = lD$ **then**

| | **for** $j \in N_i^-$ **do**

| | | **if** $\left\{ \left(\frac{x_{ij}[1 : (k-1)D]}{y_{ij}[1 : (k-1)D]} - MXP(k) \right) \geq 0 \text{ or} \right.$

| | | $\left. \left(\frac{x_{ij}[1 : (k-1)D]}{y_{ij}[1 : (k-1)D]} - MNP(k) \right) \leq 0 \right\}$ **then**

| | | $AttackFlag_i = True;$

| | | **emit:** $AttackFlag_i \rightarrow N_i^+;$

| | | /* Send AttackFlag for termination. */

| | **else**

| | | $StorageBuffer_i \leftarrow [];$

| | **end**

| | **end**

| | $l = l + 1;$

| **end**

| $k = k + 1;$

Output: $AttackFlag_i$ // Detection of intruder by node i

hand are difficult to detect. These attacks are responsible for diverting the power network from the actual operating point. Although, these attacks are not as severe as the vandalism attacks in their impact, the difficulty lies in detection of such attacks. A manipulation attack running for a long time may lead to large revenue losses.

Intruder model: We consider a class of intruders whose objective is to alter the consensus value by θ (called attack strength), viz. the consensus algorithm converges to $\alpha - \theta$ or $\alpha + \theta$ where $\alpha = \frac{\sum_{i=1}^N x_i(0)}{N}$. Without loss of generality, we will analyze the case of intruder trying to steer the consensus to $(\alpha - \theta)$ where $\theta > 0$.

6.5.1 Assumptions on intruder attack model

We provide the assumptions imposed on our attack model:

- A1.** All the attacks are intended to manipulate the smart micro-grid performance (by manipulating the consensus value). The intruder is motivated to increase its profit by increasing its share of the total power supplied to the loads (similar to competitive Transactive Grids). Furthermore, we assume that attacks do not occur at $k = 0$.
- A2.** The intruder has as much global information available to itself as every other agent in the network.

Based on the above assumptions we now provide various attack strategies:

- 1. Constant (False) data injection attacks:** Here the motive of the attacker is to modify the consensus value by injecting a constant false value into the network. In such attacks the ratio of the state updates of the intruder node remains constant after some iteration k' i.e.

$$\frac{x_q(k)}{y_q(k)} = \frac{x^*}{y^*}, \quad (6.13)$$

for $k \geq k'$ where, q is the intruder node, and x^* and y^* are the intruder injected constant values. Note, that these kind of intrusion attacks also encompass stuck nodes. We extend the work of [132] requiring doubly-stochastic weight matrix and strong assumptions on the induced sub-graph after removing the intruder in the network be strongly connected. We present a theorem

below which shows for the ratio consensus protocol all nodes in the network will converge to the value injected by the intruder.

Theorem 12 *With the attack strategy defined in (6.13), the ratio of numerator and denominator states $\frac{x_i(k)}{y_i(k)}$ for all nodes i converges to a common limit (consensus value). Further, the limit is equal to $\frac{x^*}{y^*}$.*

Proof: See Appendix (Section 6.7). □

2. Data Manipulation attacks: Let T denote the set of discrete-time attack instants when an intruder injects malicious values in the network. Let the l^{th} attack by the intruder i occurs at instant t , then,

$$x_i(t+1) = \sum_{j \in N_i^- \cup i} p_{ij} x_j(t) - \delta_{k_l}, \quad (6.14)$$

where, $t \geq l$ and $\sum_{l \in T} \delta_{k_l} = N\theta$. State y_i is updated according to (6.3b). One such attack strategy is to choose δ_0 and a to achieve a desired attack, θ , within m number of attacks is,

$$\delta_0 = \left(\frac{1-a}{1-a^m} \right) N\theta, \quad a \in [0, 1], \quad (6.15)$$

with, $\delta_l = a^{l-1} \delta_0$ for some $\delta_0 > 0$.

The next theorem shows that the intruder has an attack strategy that can steer the ratio consensus protocol to achieve desired state in the network.

Theorem 13 *The intruder attack strategy given in (6.14) steers the consensus value to $(\alpha - \theta)$, where $\alpha = \frac{1}{N} \sum_{j \in V} x_j(0)$.*

Proof: (By Induction) Let α be the consensus value without attack. $\sum_{j \in V} x_j(0) = N\alpha$ (as $\sum_{j \in V} y_j(0) = N$). It is sufficient to prove that under the attack strategy (6.14) $\sum_{j \in V} x_j(k) = N\alpha - N\theta$. Consider i to be the intruder node and let $k_1, k_2, \dots \in T$ be the attack instants where

k_l denotes the l^{th} attack instant. From Lemma 6.2.1 with $l = 1$,

$$\begin{aligned} \sum_{j \in V} x_j(k_1) &= \sum_{\substack{j \in V \\ j \neq i}} \sum_{l \in N_j^- \cup j} p_{lj} x_j(k_1 - 1) \\ &\quad + \sum_{l \in N_i^- \cup i} p_{li} x_i(k_1 - 1) - \delta_{k_1} \\ &= \sum_{j \in V} \sum_{l \in N_j^- \cup j} p_{lj} x_j(k_1 - 1) - \delta_{k_1} \end{aligned}$$

(Induction Hypothesis) Let, for $l = m'$, $\sum_{j \in V} x_j(k_{m'}) = N\alpha - \sum_{l=1}^{m'} \delta_{k_l}$. For $l = m' + 1$ and since no attack occurs between the instants $k_{m'}$ and $k_{m'+1}$, proceeding as above,

$$\begin{aligned} \sum_{j \in V} x_j(k_{m'+1}) &= \sum_{j \in V} \sum_{l \in N_j^- \cup j} p_{lj} x_j(k_{m'+1} - 1) - \delta_{k_{m'+1}} \\ &\text{(Using Lemma 6.2.1 and no attack between } k_{m'} \text{ and } k_{m'+1}), \\ &= \sum_{j \in V} \sum_{l \in N_j^- \cup j} p_{lj} x_j(k_{m'}) - \delta_{k_{m'+1}} \\ &= \sum_{j \in V} \sum_{l \in N_j^- \cup j} p_{lj} x_j(k_{m'} - 1) - \sum_{l=1}^{m'} \delta_{k_l} - \delta_{k_{m'+1}} \\ &= \sum_{j \in V} \sum_{l \in N_j^- \cup j} p_{lj} x_j(k_{m'} - 1) - \sum_{l=1}^{m'+1} \delta_{k_l}. \end{aligned}$$

Therefore, induction holds. Hence, $\sum_{j \in V} x_j(k_m) = N\alpha - \sum_{\ell=1}^m \delta_{k_\ell} = N\alpha - N\theta$. Thus, from [84] the ratio converges to $\lim_{k \rightarrow \infty} \frac{x_j(k)}{y_j(k)} = \frac{N\alpha - N\theta}{\sum_{j \in V} y_j(0)} = \alpha - \theta$. $\square$

Remark 13.1 Here, the attack strategy depends upon the knowledge of N (more generally $\sum_{j \in V} y_j(0)$) by the attacker node. During imperfect knowledge of the network, attacker can steer the consensus only to $\alpha - \frac{N'}{N}\theta$, where N' is an estimate of N by attacker.

6.6 Results

We now provide the validation of our detection scheme for a smart micro-grid involving distributed power apportioning as the cyber-physical system.

6.6.1 Simulation Results

In this section, we present simulation results to demonstrate distributed detection of an intruder attack in a power network. Intruder models developed in Section 6.5 will be used to validate the detection strategy. Fig. 6.1(b) shows the state trajectories of the DGs and the $M(k)$ and $m(k)$ values following the distributed apportioning algorithm for communication network of Fig. 6.1 without any intruder in the network ($\alpha = 0.6$). The nodes have minimum and maximum power capacities as: $\pi^{min}(W) = [1000 \ 1000 \ 1000 \ 1000 \ 1000 \ 1000]^T$, $\pi^{max}(W) = [1000 \ 2000 \ 5000 \ 6000 \ 4000 \ 3000]^T$ with a total aggregator demand of 15 kW ($\leq \sum_{j \in V} \pi_j^{max}$). Fig. 6.2 presents simulation results for constant data injection attacks. Here, the malicious node 5 attacks the network by constantly injecting a ratio of 0.45 after iteration 30 ($\theta = 25\%$), emulating an actual aggregator demand of 12.75 kW as seen by the remaining (honest) nodes. Thus, the total power delivered by the rest of the power network becomes 10.4 kW, and node 5 can inject its maximum power (4 kW), making the total power delivered by the network as 14.4 kW. It is important to note that aggregator cannot distinguish between the two scenarios where the power network's total capacity is just 14.4 kW (deficit) or the case where attack is causing the total power delivered to be 14.4 kW. It can be observed that the intruder node is able to steer the consensus to its desired ratio (0.45 in this case) in the absence of any detection strategy. However, in the presence of the detection algorithm, intrusion is detected when node 5 violates Lemma 6.4.1 in the next $2D$ iterations (iteration number 36) resulting in detection ($AttackFlag = 1$) (Fig. 6.2). The following results are obtained by running simulations for a set of 10,000 graphs with 1000 nodes with a maximum possible diameter of 40. At each run we pick a graph with arbitrary initial conditions for all nodes. The attack strength (θ) is varied from 1% to 90% of the consensus value (α). We created 30 million possible scenarios by varying the graph topology, attack strength ($\theta\%$ of the consensus value α), intruder node, time instant of attack, number of attacks. The detection accuracy of the algorithm for all cases is shown in

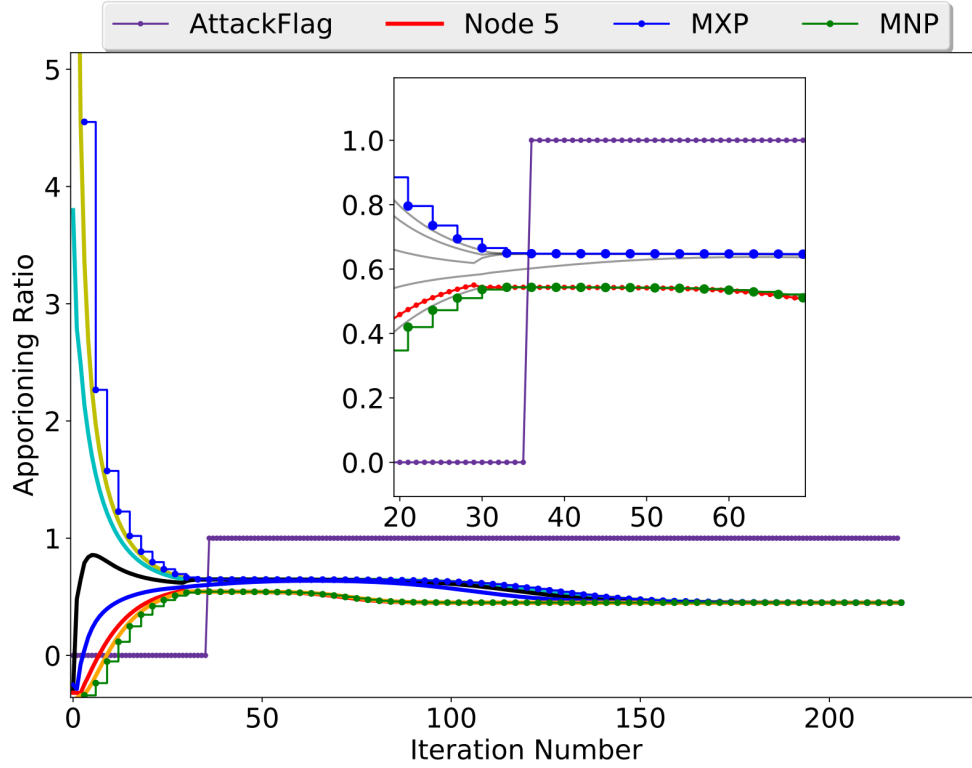


Figure 6.2: A 6-node power network with the malicious node 5 launching a constant injection attack resulting in detection.

Fig. 6.3.

Remark 13.2 *Our algorithm detects smaller attacks ($\theta < 5\%$) with an average accuracy of 89% which are harder to detect. In such cases the intruder cannot steer the final consensus to achieve larger deviations. Medium attacks ($5 \leq \theta \leq 10\%$) are detected with an accuracy $> 99.5\%$ where as stronger attacks ($\theta > 10\%$) are always detected.*

Remark 13.3 *The outliers (red crosses in Fig. 6.3) in the simulations that avoided detection for smaller and medium attack strengths, were mostly observed to be cases where the intruder could inject a large value when the range of minimum and maximum initial values across all the nodes in the network were large. However, such a strategy doesn't guarantee success in attack as intruder has no knowledge of this range in this framework.*

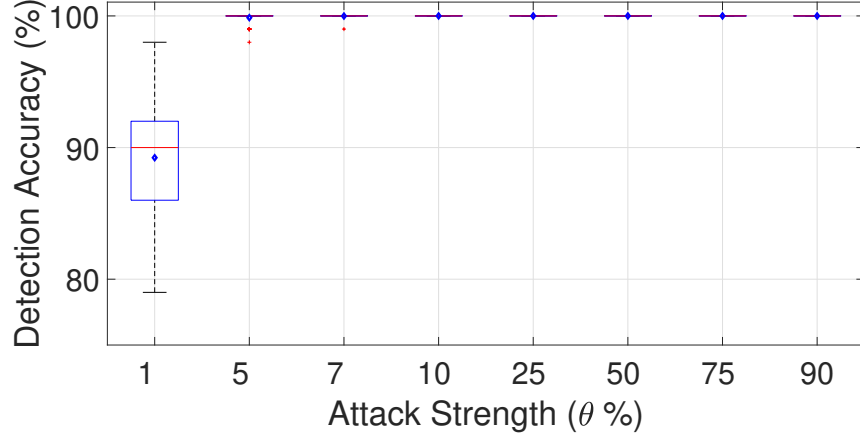


Figure 6.3: Instances of random graphs generated for 1000 node network for attack statistics.

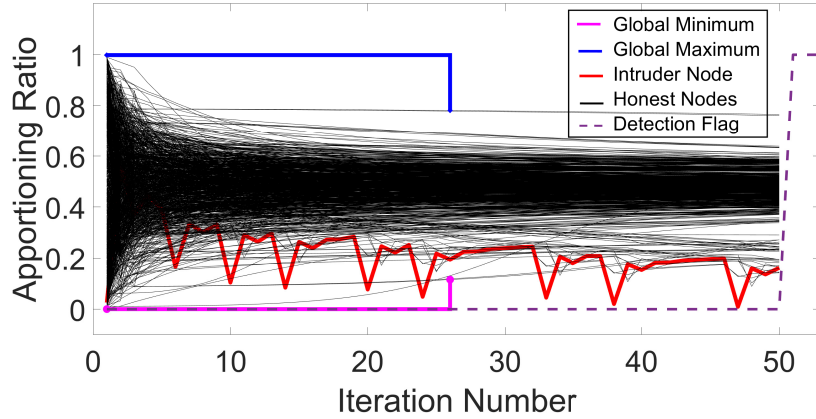


Figure 6.4: Malicious node launching data manipulation attack in a 1000-node power network. Detection at iteration 50 ($2D$).

An example case of data manipulation attack is shown in Fig. 6.4. The global maximum and global minimum are shifted by D iterations to the left for clarity but are obtained only after D iterations in the protocol. Here the intruder node with an attack strength of 3% following (6.14) and (6.15) with $a = 0.95$, $m = 95$ and $\delta_0 = 7.3 \times 10^{-4}$. However, it violates the MNP criteria in (6.11) leading to detection and sending $AttackFlag = 1$ at iteration number 50 ($2D$).

6.7 Appendix

proof:[Proof of Theorem 12] Without loss of generality we can rearrange the update law such that first m nodes are intruders as $X(k+1) = \tilde{P}X(k)$, where $\tilde{P} = [\mathbf{I}_m \ \mathbf{0}; \ \mathbf{B} \ \mathbf{A}]$. $X(k) \in \mathbb{R}^N$ with $X(k) = [x_1(k) \ x_2(k) \ \dots \ x_N(k)]^T$ is a column vector, $\mathbf{I}_m$ is the identity matrix of size m , $\mathbf{0}$ is a matrix of size $m \times (N-m)$ with all entries equal to zero, $\mathbf{B} \in \mathbb{R}^{(N-m) \times m}$, $\mathbf{A} \in \mathbb{R}^{(N-m) \times (N-m)}$ and $k > k'$. The first m rows corresponds to updates of intruder nodes $i \in Q$ according to (6.13). Note that $\mathbf{B}$ has at least one non-zero element, since the communication graph with the attacker is strongly connected. Further, $\mathbf{A}$ is a column sub-stochastic matrix as it is a sub-matrix of a column-stochastic matrix. Now, $[\mathbf{X}_m(k+1) \ \mathbf{X}_r(k+1)]^T = \tilde{P} [\mathbf{X}_m(k) \ \mathbf{X}_r(k)]^T$ where, $\mathbf{X}_m(k) = [x_1(k) \ x_2(k) \ \dots \ x_m(k)]^T$ denotes the column vector corresponding to malicious nodes, $\mathbf{X}_r(k) = [x_{m+1}(k) \ x_{m+2}(k) \ \dots \ x_N(k)]^T$ is the column vector corresponding to remaining (honest) nodes. Now, the remaining graph after removing the malicious node can either be a strongly connected graph or a set of strongly connected sub-graphs.

Case 1. Remaining graph is strongly connected

This implies that $\mathbf{A}$ is irreducible [133]. As $\mathbf{A}$ is column sub-stochastic, the magnitude of all the eigenvalues of $\mathbf{A}$ are less than 1 i.e. $|\lambda_i(\mathbf{A})| < 1$ ([134]). Then as $t \rightarrow \infty$

$$\begin{aligned} \lim_{t \rightarrow \infty} \tilde{P}^t &= \lim_{t \rightarrow \infty} \begin{bmatrix} \mathbf{I}_m & \mathbf{0} \\ (\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \dots + \mathbf{A}^t)\mathbf{B} & \mathbf{A}^t \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{I}_m & \mathbf{0} \\ \underbrace{(\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}}_{\mathbf{B}} & \mathbf{0}_{\mathbf{A}} \end{bmatrix} \end{aligned}$$

where $\mathbf{0}_{\mathbf{A}}$ is the matrix with same size as $\mathbf{A}$, whose entries are all equal to 0. Note, we used the

fact that, $\lim_{t \rightarrow \infty} (\mathbf{I} - \mathbf{A})(\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \dots + \mathbf{A}^t) = \mathbf{I}$ and $\lim_{t \rightarrow \infty} \mathbf{A}^t = \mathbf{0}_\mathbf{A}$. Thus, as $t \rightarrow \infty$,

$$\begin{aligned} \lim_{t \rightarrow \infty} \begin{bmatrix} \mathbf{X}_m(t) \\ \mathbf{X}_r(t) \end{bmatrix} &= \begin{bmatrix} \mathbf{I}_m & \mathbf{0} \\ \tilde{\mathbf{B}} & \mathbf{0}_\mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{X}_m(k') \\ \mathbf{X}_r(k') \end{bmatrix} \\ \implies \lim_{t \rightarrow \infty} \mathbf{X}_r(t) &= \tilde{\mathbf{B}} \mathbf{X}_m(k') \end{aligned}$$

Case 2. Remaining graph is a set of l strongly connected sub-graphs

In this case, $\mathbf{A}$ can be rearranged as a block diagonal matrix given as, $\mathbf{A} = \text{diag}(\mathbf{A}_{1r}, \mathbf{A}_{2r}, \dots, \mathbf{A}_{lr})$, where $r \in \{1, 2, \dots, l\}$ and all $\mathbf{A}_{ir}$ are irreducible [133]. Then, $\mathbf{X}_r(k)$ can be written as,

$$\begin{bmatrix} \mathbf{X}_{1r}(k+1) \\ \mathbf{X}_{2r}(k+1) \\ \vdots \\ \mathbf{X}_{lr}(k+1) \end{bmatrix} = \mathbf{M} \begin{bmatrix} \mathbf{X}_m(k) \\ \mathbf{X}_{1r}(k) \\ \mathbf{X}_{2r}(k) \\ \vdots \\ \mathbf{X}_{lr}(k) \end{bmatrix}$$

where $\mathbf{X}_{ir}$ is the column vector corresponding to nodes of the i_{th} strongly connected sub-graph and

$$\mathbf{M} \triangleq \begin{bmatrix} \mathbf{B}_{1r} & \mathbf{A}_{1r} & 0 & \dots & 0 \\ \mathbf{B}_{2r} & 0 & \mathbf{A}_{2r} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{B}_{lr} & 0 & 0 & \dots & \mathbf{A}_{lr} \end{bmatrix}.$$

This implies,

$$\mathbf{X}_{ir}(k+1) = \mathbf{B}_{ir} \mathbf{X}_m(k) + \mathbf{A}_{ir} \mathbf{X}_{ir}(k)$$

Now, proceeding as in case 1, for $t \rightarrow \infty$, $\mathbf{X}_{ir}(t) \rightarrow \tilde{\mathbf{B}}_{ir} \mathbf{X}_m(k')$ with $\tilde{\mathbf{B}}_{ir} = (\mathbf{I} - \mathbf{A}_{ir})^{-1} \mathbf{B}_{ir}$ for $i \in \{1, 2, \dots, l\}$. Also, $\tilde{\mathbf{B}} = [\tilde{\mathbf{B}}_{1r} \ \tilde{\mathbf{B}}_{2r} \ \dots \ \tilde{\mathbf{B}}_{lr}]^T$. Then for both the cases, $\lim_{t \rightarrow \infty} x_j(t) = (\tilde{\mathbf{B}})_j x^*$ and $\lim_{t \rightarrow \infty} y_j(t) = (\tilde{\mathbf{B}})_j y^*$ for all $j \in V \setminus Q$, where $(\tilde{\mathbf{B}})_j$ is j_{th} element of $\tilde{\mathbf{B}}$. Hence, $\lim_{t \rightarrow \infty} \frac{x_j(t)}{y_j(t)} = \frac{x^*}{y^*}$, for all $j \in V$.

Chapter 7

Thesis Conclusion

With the increase in renewable energy resources and distributed energy resources in the distribution power system, there is a rising interest to investigate their impact on power system and its control especially in islanded (grid-forming) microgrids. Advancements in power electronics, information and communication technologies (ICT) render avenues that are enablers in designing the grid of the future. This dissertation focused on challenging the conventional methodology used in multi-inverter microgrids and showed the viability and analysis of implementing ICTs to regulate system voltage and frequency while meeting standardization metrics on power sharing both in fundamental and harmonic frequency domain. Furthermore, a completely distributed framework for aggregation of smaller DERs to form a virtual power plant to meet the demands of the rest of the grid for increased grid resiliency was also presented. Experimental controller hardware-in-the-loop and power-hardware-in-the-loop validations were performed for these proposed methodologies to demonstrate their viability and applicability to real-world implementations.

In chapter 2, we proposed and demonstrated a voltage-active power half-droop-based controller to improve active power sharing accuracy in a resistive LV microgrid. A loop shaping-based control law was designed for a single inverter unit to enable close regulation of the reference sinusoidal voltage signal to be tracked. A novel isochronous architecture was also proposed to maintain the system frequency and enable active power sharing in the multi-inverter system.

The main advantage of the proposed architecture is to keep the reactive power flows small among the inverters without implementing $Q - \omega$ droop law, thereby, reducing the overall complexity of the inverter controllers. Moreover, analytical results were derived to provide design guidelines for selecting droop gains when there are active power mismatches and uncertainties in the load present in the LV microgrid. Experimental results were provided for various scenarios that validate the performance and capabilities of the proposed controller architecture.

In Chapter 3, we developed and implemented the design of a virtual impedance shaping method for low-voltage AC microgrids. An SRF-based extracting filter is proposed for extracting fundamental and selective harmonic components of inverter line currents and virtual impedances, implemented as weighing filters, are designed based on one or more performance parameters of the microgrid system such as symmetric or asymmetric harmonic current sharing. An isochronous voltage-active power droop law is proposed to maintain active and reactive power sharing by providing VSIs access to a time clock/interval signal. Analysis of the design of virtual impedances are provided for LV microgrids. Finally, validations are obtained through SIMULINK/MATLAB simulations as well as CHIL-based real time simulation on a two inverter set-up to demonstrate the performance of the proposed methodology when implemented on low cost micro-controller devices.

Chapter 4 and Chapter 5 delve into developing distributed algorithms amenable towards power apportioning for providing ancillary services in highly renewable grids.

In Chapter 4, we presented the analytical results for distributed finite time termination of ratio consensus protocol. The highlight of this work is the extension of the framework of Max-Min consensus for finite time termination of ratio consensus based on a user specified threshold. The monotonic property of the maximum and minimum of the ratio of the nodal states after every diameter number of iterations is proven and utilized to design the distributed finite time termination algorithm. The effectiveness of the algorithm is demonstrated through simulations as well as on a network of Raspberry Pis.

Chapter 5 developed a scalable distributed framework for coordinating and aggregating large numbers of DERs to provide ancillary service support in the form of SFR to a bulk power system. The results of finite-time termination of ratio consensus were extended to propose a distributed

power apportioning protocol to meet the time-varying aggregator command within a specified tolerance by participating DERs in the presence of bounded communication delays while prioritizing RES in the network. DER responses faster than state-of-the-art distributed approaches required for SFR services were achieved by implementing an early dispatch mechanism along with a brown-start approach to further improve DERs' ramping performance. Analytical results were derived to show that aggregator command is met within a desired tolerance distributively by all the participating DERs while respecting local capacity constraints in finite-time making this protocol viable for real-world implementation. A novel PHIL test-bed was developed to validate the efficacy of the distributed algorithm at a scale that has not been achieved earlier and validating the proposed protocol containing 40+ physical hardware devices (generation), 48 Raspberry Pi agents, 202 simulated distributed controller nodes, varied communication protocols and an underlying real-world power system model. Experimental results, validated on a unique PHIL test-bed at scale, show that the required performance metrics were met (initial and ramp responses were achieved to be less than 5 sec. and < 1 minute for the setup, respectively). The proposed framework and demonstrations validate that coordination of DERs in distribution networks can be achieved on a sufficiently large scale to provide essential secondary frequency response services. Such a validation for distributed DER coordination and control at scale, to the best of our knowledge, has not been achieved earlier.

Finally, in Chapter 6, we considered the problem of detecting malicious attacks in distributed cyber-physical systems implementing finite-time termination distributed ratio consensus protocols developed in Chapter 4 and Chapter 5. Attack models for intruders were proposed and analyzed that have not been previously considered in the literature. An efficient distributed intruder detection algorithm was presented, analyzed and validated using simulations. The proposed detection methodology makes use of only local information of the nodes under limited storage and computation requirements as opposed to existing detection methods. Analyzing the efficacy of the proposed scheme under time varying topologies and web-socket based experiments on a communication network are future directions for this work.

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