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A communications system perspective for dynamic mode atomic force microscopy, with applications to high-density storage and nanoimaging

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**A communications system perspective for dynamic mode atomic force
microscopy, with applications to high-density storage and nanoimaging**

by

Naveen Kumar

A dissertation submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

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Ames, Iowa

2010

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DEDICATION

I would like to dedicate this dissertation to my parents.

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ABSTRACT

In recent times, the atomic force microscope (AFM) has been used in various fields like biology, chemistry, physics and medicine for obtaining atomic level images. The AFM is a high-resolution microscope which can provide the resolution on the order of fractions of a nanometer. It has applications in the field of material characterization, probe based data storage, nano-imaging etc. The prevalent mode of using the AFM is the static mode where the cantilever is in continuous contact with the sample. This is harsh on the probe and the sample. The problem of probe and sample wear can be partly addressed by using the dynamic mode operation with the high quality factor cantilevers. In the dynamic mode operation, the cantilever is forced sinusoidally using a dither piezo. The oscillating cantilever gently taps the sample which reduces the probe-sample wear. In this dissertation, we demonstrate that viewing the dynamic mode operation from a communication systems perspective can yield huge gains in nano-interrogation speed and fidelity.

In the first part of the dissertation, we have considered a data storage system that operates by encoding information as topographic profiles on a polymer medium. A cantilever probe with a sharp tip (few nm radius) is used to create and sense the presence of topographic profiles, resulting in a density of few Tb per in.². The usage of the static mode is harsh on the probe and the media. In this work, the high quality factor dynamic mode operation, which alleviates the probe-media wear, is analyzed. The read operation is modeled as a communication channel which incorporates system memory due to inter-symbol interference and the cantilever state. We demonstrate an appropriate level of abstraction of this complex nanoscale system that obviates the need for an involved physical model. Next, a solution to the maximum likelihood sequence detection problem based on the Viterbi algorithm is devised. Experimental

and simulation results demonstrate that the performance of this detector is several orders of magnitude better than the performance of other existing schemes.

In the second part of the dissertation, we have considered another interesting application of the dynamic mode AFM in the field of nano-imaging. Nano-imaging has played a vital role in biology, chemistry and physics as it enables interrogation of material with sub-nanometer resolution. However, current nano-imaging techniques are too slow to be useful in the high speed applications of interest such as studying the evolution of certain biological processes over time that involve very small time scales. In this work, we present a high speed one-bit imaging technique using the dynamic mode AFM with a high quality factor cantilever. We propose a communication channel model for the cantilever based nano-imaging system. Next, we devise an imaging algorithm that incorporates a learned prior from the previous scan line while detecting the features on the current scan line. Experimental results demonstrate that our proposed algorithm provides significantly better image resolution compared to current nano-imaging techniques at high scanning speed.

While modeling the probe-based data storage system and the cantilever based nano-imaging system, it has been observed that the channel models exhibit the behavior similar to inter-symbol-interference (ISI) channel with data dependent time-correlated noise. The Viterbi algorithm can be adapted for performing maximum likelihood sequence detection in such channels. However, the problem of finding an analytical upper bound on the bit error rate of the Viterbi detector in this case has not been fully investigated. In the third part of the dissertation, we have considered a subset of the class of ISI channels with data dependent Gauss-Markov noise. We derive an upper bound on the pairwise error probability (PEP) between the transmitted bit sequence and the decoded bit sequence that can be expressed as a product of functions depending on current and previous states in the (incorrect) decoded sequence and the (correct) transmitted sequence. In general, the PEP is asymmetric. The average BER over all possible bit sequences is then determined using a pairwise state diagram. Simulations results demonstrate that analytic bound on BER is tight in high SNR regime.

CHAPTER 1. INTRODUCTION

The invention of atomic force microscope (AFM), by Binnig et. al. in 1986 [1], has resulted in major breakthroughs in various fields like biology, chemistry, physics and medicine over the years. The ability to see things at the level of a few nanometers is critical in many of these domains. The AFM can provide the resolution on the order of fractions of a nanometer. Thus, it can be used for developing various applications in those domains, for example imaging the movement of kinesins along microtubules, imaging the samples for fault detection in semiconductor fabrication, developing storage device which can provide areal densities close to 4 Tb/in² [41] etc. Another interesting application of the AFM is material characterization in which mechanical [2], electrical [3], thermal [4] and optical [5] properties of the material can be interrogated. Many probe based sensors are available these days which can sense topography and material properties of the sample [6].

AFM setup consists of a cantilever beam that is supported at one end with a sharp tip at another end as a means to determine the topography of the sample. A representative cantilever has dimensions of 100 μm length, 20 μm breadth and 5 μm thickness. The tip at the end of the cantilever has a tapered end that can be atomically sharp. The forces between the tip and the sample, to be interrogated, are primarily due to the interatomic forces that are effective in the 4-5 nm range. A typical qualitative force profile with respect to the separation between the tip and the sample is shown in Figure 1.1 that is characterized by long range attractive forces and short range repulsive forces. When there is a hill on the sample, the tip-sample separation is reduced, which results in an increased force on the cantilever leading to a different effect on the cantilever deflection compared to when there is a valley in the sample topography. There are various means of measuring the cantilever deflection. In the standard atomic force microscope

setup, which has formed the basis of this dissertation, the cantilever deflection is measured by a beam-bounce method where a laser is incident on the back of the cantilever surface and the laser is reflected from the cantilever surface into a split photodiode. The photodiode collects the incident laser energy and provides a measure of the cantilever deflection. The advantage of the beam-bounce method is the high resolution (low measurement noise) and high bandwidth (in the 2-3 MHz) range. The disadvantage is that it is more cumbersome for integrating this method into a parallel operation where multiple cantilevers operate in parallel. There are attractive measurement mechanisms that integrate the cantilever motion sensing onto the cantilever itself. These include piezo-resistive sensing [13] and thermal sensing [16] that sense the motion of the cantilever by monitoring the changes in resistance of the piezo material due to a strain and the changes in the resistance of the cantilever material due to temperature changes respectively. The static and dynamic mode operation are two most prevalent modes of operation to actuate the cantilever. In the static mode, the cantilever is analogous to a gramophone needle (cantilever tip) of the gramophone player that moves due to the topography of the record (sample) i.e. the cantilever is in continuous contact with the sample. The information content is present in low frequency in this case. However, it can be shown experimentally that the system gain at low frequency is very small. Therefore, in order to overcome measurement noise at output, the interaction force between tip and sample should be large which degrades the probe and sample over time and significantly reduces reliability. The problem of tip and sample wear can be partly addressed by using the dynamic mode operation. In the dynamic mode operation, the cantilever is forced sinusoidally using a dither piezo. The oscillating cantilever interacts with the sample intermittently as it gently taps the sample and thus the lateral forces are reduced which decreases tip-sample wear drastically [44]. For the dynamic mode operation there are also various other schemes to actuate the cantilever that include electrostatic [14], mechanical by means of a dither piezo that actuates the support of the cantilever base, magnetic [19] and piezoelectric [15].

One of other interesting applications of AFM involves force measurements for biomaterials, chemical sensing, polymers, colloidal forces, adhesion and more. The AFM instrumentation

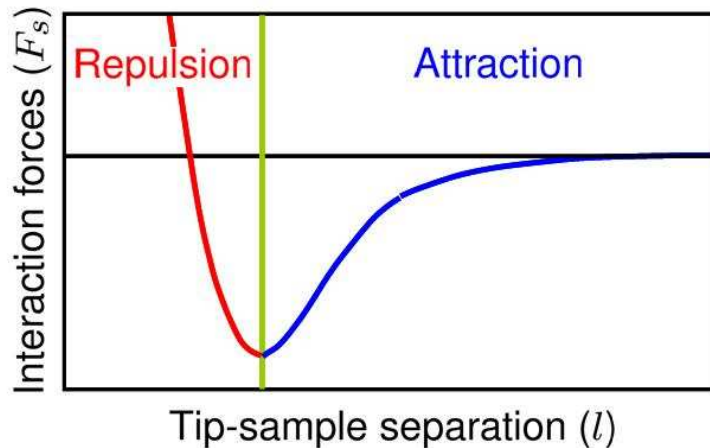


Figure 1.1 Shows the typical tip-sample interaction forces of weak long range attractive forces and strong repulsive short range forces.

is done such that it can measure the force between a tip mounted on a cantilever beam and a sample surface as a function of the tip-surface separation. This force curve can be used for various purposes like studying the bulk properties of the materials such as elasticity, plasticity and hardness etc. Similar idea like microindentation has been commonly used over the years in material science and engineering for obtaining the bulk properties of the materials. In past, the mechanical properties had been studied from the force curves obtained using AFM for various materials like rubbers (Polypropylene glycol (PPG) based), semicrystalline (iPP, HDPE, PTFE), glassy polymers (PMMA, PC), pyrolytic graphite, gold foil etc [7]. The force curves have also drawn the attention of biophysical community for assessing single-molecule inter and intramolecular interactions [8]. Thus, the applications of force curves are quite diverse and beyond the scope of this dissertation.

In this dissertation, we considered the application of the dynamic mode AFM, using a high quality factor cantilever, in the field of probe based storage. The need of probe based data storage is driven by the explosive growth of the personal computer industry and the Internet which demand for ultra-high capacity storage devices. Demands of a few Tb per in.² are predicted in the near future. Commercially used data storage techniques are primarily based on magnetic, optical and solid state technologies. However all these technologies are

reaching fundamental limits on their achievable areal densities. Magnetic storage suffers from the superparamagnetic effect that limits the minimum size of a magnetic domain. Optical devices are limited by the wavelength of the utilized laser and solid state devices are limited by the minimum size of a transistor that can be created. A promising high density storage methodology, that is presented in this dissertation utilizes a sharp tip at the end of a micro cantilever probe to create (or remove) and read indentations (see [41]). The presence/absence of an indentation represents a bit of information. This method can provide significantly higher areal densities compared to current storage technologies. Recently, experimentally achieved tip radii near 5 nm on a micro-cantilever were used to create areal densities close to 4 Tb/in². The indentations had dimensions close to the tip-radii and a height of 1 nm [42]. The areal density in this method is primarily limited by the tip geometry. The effective area of the tip that interacts with the media can be made considerably smaller with technologies such as carbon nanotube attachments (see [17, 9]) that have the promise to yield sub-nanometer small features. Thus, unlike the previous storage technologies, the fundamental limit of areal densities possible is far from being reached.

A particular realization of a probe based storage device that uses an array of cantilevers is provided in [18]. However, there are fundamental drawbacks of current probe based devices that are related to the static mode operation. In the static mode operation, the cantilever is in contact with media throughout the read operation which results in large vertical and lateral forces on the media and the tip. Thus, there is considerable wear and tear that reduces the life of the device. The problem of tip and media wear can be partly addressed by using dynamic mode operation; particularly when a cantilever with a high quality factor is employed. However the conventional dynamic methods that use high quality factors, though gentle on the medium, are too slow to be useful in data storage applications. In the dynamic mode operation given in this dissertation, the cantilever is forced sinusoidally using a dither piezo. The oscillating cantilever interacts with the medium intermittently as it gently taps the medium and thus the lateral forces are reduced which decreases the media wear [44].

At high storage densities, the readback signal suffers from increased noise and linear/nonlinear

distortions. This makes data detection more difficult, and requires powerful detection techniques. Data detection can be improved by increasing the tip-medium interaction force but this comes at the cost of increased tip and medium wear and reduces the reliability of system. Thus, what is needed are good detectors that have a low probability of error at a given tip-medium interaction.

Good detection methods in a dynamic mode operation have to circumvent the challenge of substantially increased complexity of interpreting the cantilever oscillations for information about the medium. For example, evidence of chaotic behavior [24, 36, 11] and other complicating issues like multi-valued outputs for the same input [21, 37] exist in the dynamic mode operation. Using cantilever probes that have high quality factors leads to high resolution. This is because the effect of a topographic change on the medium, on the oscillating cantilever, lasts much longer (approximately Q cantilever oscillation cycles where Q is the quality factor of the cantilever) for a high quality cantilever. Also the signal to noise ratio increases with higher quality factors (improves as $\sqrt{Q}$) [43]. However, the advantages of high quality factor become disadvantages with respect to bandwidth which is apparent as the effect of a topographic change lasts longer and therefore information on the medium has to be temporally spaced to reduce inter-symbol interference. Thus the challenges of designing good detection schemes are twofold; the first is that of modeling the characterization of the dynamics that leads to the model that predicts the essential experimental features and remains tractable for a data storage purposes and the second is to use the model to exploit the advantages of high quality factor cantilevers without sacrificing bandwidth. These challenges need to be tackled for rendering the dynamic mode operation feasible for high density data storage purposes.

Another widely prevalent use of AFM is in the field of nano-imaging and bio-manipulation at the molecular scale. For example, AFM cantilevers have been used in biological sciences for cutting DNA strands [48] and investigating the activity of RNA polymerase [47]. Various kind of biomolecules, such as phospholipids, proteins, DNA, RNA, membranes, living cells and tissues, have been imaged using AFM over the years [8]. Apart from providing the structural characterization of biomolecules, AFM can also be used to investigate mechanical, chemical and

functional properties of biomolecules [8]. These applications resulted in a major breakthroughs in various fields including biology, chemistry, physics and medicine and gave an opportunity to perceive the world at nanoscale.

Nano-imaging has been (more or less), the domain of physicists and the approaches have relied on the development of better instruments or the usage of better materials that improve the fidelity and/or the speed of imaging. However, it is evident that these approaches are limited; currently high fidelity requires low speed and vice versa (in spite of the fact that AFM is a mature technology). Current nanoimaging technology though useful, suffers from severe speed limitations. Current techniques essentially rule out imaging chemical and biological processes that evolve at time scales that are typically faster than the imaging speed, e.g. movement of kinesins along microtubules. Moreover, in many cases we are interested in fast imaging of samples that are of the order of a few square centimeters or even higher in some applications e.g., fault detection in semiconductor fabrication. There is a great need for finding an algorithm for nano-imaging which can image at a very good resolution for very high scan rates as compared to conventional imaging techniques.

In nano-imaging, the samples, to be imaged, are mostly soft which leads to the use of dynamic mode AFM. In the dynamic mode operation, the cantilever is forced sinusoidally using a dither piezo. The oscillating cantilever gently taps the sample and thus the lateral forces are reduced which decreases the sample wear [44]. The amplitude of the cantilever oscillation changes due to tip-sample interaction. The amplitude of the first harmonic of the cantilever oscillation is obtained from the cantilever deflection signal in amplitude modulation method. This amplitude signal can be used to image samples and referred as amplitude imaging [45]. In [33], the cantilever-observer architecture is introduced which removes the effect of dither and provides a better way to image samples. The innovation signal is obtained through the output of the cantilever-observer architecture. The root mean square of the innovation signal gives a fast way to image samples and referred as root mean square imaging [50]. But current imaging techniques are too slow to be useful in high fidelity imaging of chemical and biological processes that evolve at fast time scales.

For high-fidelity imaging, it is required to figure out the height and shape of the underlying sample to within 8-bit precision (for example). The imaging technique should be general enough to operate under various sample types. This makes the nano-imaging problem significantly challenging and complicated. In this dissertation, we consider a one-bit imaging, i.e., we wish to detect the presence/absence of a feature. This can be generalized to multi-bit imaging in future. In one-bit imaging technique, the raster scan is used for imaging which means that the image is subdivided into a sequence of horizontal scan lines and each scan line is imaged using the imaging scanner (AFM system scanner in our case). Each scan line consists of bit-pixels which can be either ‘0’ or ‘1’. This problem of imaging is analogous to the problem of detecting the presence/ absence of the bits in probe based data storage system. Some ideas about channel modeling for probe based data storage can be borrowed for imaging purposes. But it is important to note that the detector developed for probe based data storage cannot be used for imaging purposes. The detector for data storage device assumes equiprobable prior on the input bit sequence which is not true in the imaging scenarios as the images will have non-equiprobable priors on the input depending upon the features present in the image. This demands designing a detection strategy for nano-imaging applications which can incorporate non-equiprobable priors. Next, the imaging of chemical and biological processes, that evolve at fast time scales, drives the need for high scan rate imaging. But the feature detection at high scan rates becomes quite challenging as the tip-sample interaction duration for each feature decreases with a increase in the scan rate. Thus, the main challenge in nano-imaging is to develop an imaging technique which can incorporate the non-equiprobable priors and provide good resolution at very high scan speeds.

While modeling the probe-based data storage system, it has been observed that the channel model exhibits the behavior similar to inter-symbol-interference (ISI) channel with data dependent time-correlated noise. Many other domains involve these kind of channel models, e.g., the statistics of percolation and nonlinear effects between transitions [65, 54] in magnetic recording result in data dependent noise. The corresponding detectors have been found which significantly improve performance compared to the current state of the art [56]. It is

well known that usage of a sequence detector designed for an additive white gaussian noise (AWGN) ISI model can lead to the significant loss of performance if the data dependence and time-correlation of the noise is not taken into account.

Forney [20, 52] presented a maximum likelihood sequence detection (MLSD) solution based on the Viterbi algorithm for ISI channels with memoryless noise. The flowgraph techniques have been used to derive the upper bounds on the error probability of the detector [51, 61, 63]. The finite ISI channel with Gauss-Markov noise is considered in the work of [10, 55]. In [10], certain techniques are presented for computing an upper bound on the performance of the detector. The performance analysis of the MLSD in presence of data dependent noise have also been considered in another work of [64, 60]. However, their technique is not based on flowgraph techniques, and requires an enumeration of all error events of relevant lengths and an estimate of the corresponding pairwise error probability upper bound. It should be noted that an analytical technique for estimating detector performance is quite important since it allows us to predict the performance at high SNR's where simulation can be time-consuming.

The derivation of finding upper bound on BER in an ISI channel with additive white Gaussian noise (AWGN) is very well known over the years [63]. In this case, the upper bound on the pairwise error probability (PEP) between two state sequences can be easily factorized as a product of functions depending on current and previous states in the (incorrect) decoded sequence and the (correct) transmitted sequence. Let $\bar{S}$ and $\hat{S}$ be the transmitted and decoded state sequences respectively. Then this means that the probability that the detector prefers $\hat{S}$ to $\bar{S}$, is denoted by $P(\hat{S}|\bar{S}) \leq \prod_{k=0}^{N-1} h(\hat{S}_{k-1}^k, S_{k-1}^k)$ where h is a function of current state and previous decoded states $\hat{S}_{k-1}^k = (\hat{S}_{k-1}, \hat{S}_k)$ and actual states $S_{k-1}^k = (S_{k-1}, S_k)$. Moreover, the PEP is symmetric due to the symmetric nature of white Gaussian noise, i.e., $P(\hat{S}|\bar{S}) = P(\bar{S}|\hat{S})$. Together, these properties allow the application of the error state diagram method for finding an upper bound on the BER [63]. In contrast, neither of these properties hold for the ISI channel with data-dependent Gauss-Markov noise (considered in [10]). The signal dependent and time-correlated noise makes the PEP asymmetric. Further the PEP does not factorize in a suitable manner as required for the application of flowgraph techniques. The derivation of

upper bound on BER for such channels becomes quite challenging.

1.1 Organization of the dissertation

This dissertation emphasizes on developing a communication channel model for dynamic mode AFM. The cantilever dynamics is complex with a number of physical intricacies that can render AFM intractable for practical applications. An appropriate level of abstraction is required which will obviate the need for an involved physical model. Apart from including the physical aspects in the channel model, the channel model should also be mathematically tractable so that the communications and signal processing techniques can be applied for developing detectors for it. By using this kind of channel model in probe based data storage and nano-imaging, the good detectors can be developed which provides remarkable improvement over the current state of art. In this dissertation, these sort of issues and aspects are researched in great detail. The dissertation is organized as follows:

- Chapter 2 introduces the application of AFM in probe based data storage system. A new approach of achieving a few Tb per in.² areal densities, utilizes a cantilever probe with a sharp tip that can be used to deform and assess the topography of the material. The prevalent mode of using the cantilever probe is the static mode that is harsh on the probe and the media. In this chapter, the high quality factor dynamic mode operation, that is less harsh on the media and the probe, is analyzed. The main contributions of this chapter are summarized as,
 - The communication channel model for dynamic mode read operation using high Q cantilevers is developed.
 - A solution to the maximum likelihood sequence detection problem based on the Viterbi algorithm for the identified channel model is derived.
 - The thresholding detectors, which completely ignore inherent system memory, are also proposed.

- Chapter 3 deals with the application of AFM in the field of nano-imaging. Current nano-imaging techniques are too slow to be useful in the high speed applications of interest. A high speed one-bit imaging technique using dynamic mode AFM with a high quality factor cantilever is presented. The main contributions of this chapter are summarized as,
 - The high quality factor cantilever system using a Markovian model which incorporates the inherent system memory due to the inter-symbol interference and the cantilever state is proposed for the cantilever based nano-imaging system.
 - The imaging problem as one of finding the maximum a posteriori (MAP) symbol detector for the model is posed which is solved by adapting the BCJR algorithm for the channel model.
 - An improved MAP symbol detector that incorporates a learned prior from the previous scan line while detecting the features on the current scan line is proposed.
- Chapter 4 presents inter-symbol interference (ISI) channels with data dependent Gauss Markov noise to model read channels in magnetic recording and probe based data storage systems. The Viterbi algorithm is used for performing maximum likelihood sequence detection in such channels. However, the problem of finding an analytical upper bound on the bit error rate of the Viterbi detector has not been fully researched. Current techniques rely on an exhaustive enumeration of short error events and determine the BER using a union bound. The main contributions of this chapter are summarized as,
 - A subset of the class of ISI channels with data dependent Gauss-Markov noise is considered. An upper bound on the pairwise error probability (PEP) between the transmitted bit sequence and the decoded bit sequence is derived.
 - The average BER over all possible bit sequences is determined using a pairwise state diagram.
 - An analytic bound on BER is derived for the considered channel model.
- Chapter 5 summarizes all the findings and contributions of the dissertation and provides the future work. A discussion about all the experimental and simulation results is pro-

vided to highlight the significant improvement obtained using the proposed techniques presented in this dissertation.

CHAPTER 2. High-density Data Storage Based on Dynamic Mode Atomic Force Microscopy: A Communications Perspective

2.1 Introduction

Present day high density storage devices are primarily based on magnetic, optical and solid state technologies. Advanced signal processing and detection techniques have played an important role in the design of all data storage systems [29, 27, 28, 30, 31, 10, 20]. Indeed techniques such as partial-response max-likelihood [28, 32, 29] were responsible for significantly improving magnetic disk technology.

In this chapter, we consider a promising high density storage methodology which utilizes a sharp tip at the end of a micro cantilever probe to create, remove and read indentations (see [41]). The presence/absence of an indentation represents a bit of information. The main advantage of this method is the significantly higher areal densities compared to conventional technologies that are possible. Recently, experimentally achieved tip radii near 5 nm on a micro-cantilever were used to create areal densities close to 1 Tb/in.² [41].

A particular realization of a probe based storage device that uses an array of cantilevers, along with the static mode operation is provided in [18]. However, there are fundamental drawbacks of this technique. In the static mode operation, the cantilever is in contact with media throughout the read operation which results in large vertical and lateral forces on the media and the tip. Moreover, significant information content is present in the low frequency region of the cantilever deflection and it can be shown experimentally that the system gain at low frequency is very small. Therefore, in order to overcome the measurement noise at the output, the interaction force between the tip and the medium has to be large. This degrades the medium and the probe over time, resulting in reduced device lifetime.

The problem of tip and media wear can be partly addressed by using the dynamic mode operation; particularly when a cantilever with a high quality factor is employed. In the dynamic mode operation, the cantilever is forced sinusoidally using a dither piezo. The oscillating cantilever gently taps the medium and thus the lateral forces are reduced which decreases the media wear [44]. Using cantilever probes that have high quality factors leads to high resolution, since the effect of a topographic change on the medium on the oscillating cantilever lasts much longer (approximately Q cantilever oscillation cycles, where each cycle is $1/f_0$ seconds long and Q and f_0 is the quality factor and the resonant frequency of the cantilever respectively). Moreover, the SNR improves as $\sqrt{Q}$ [43]. However, this also results in severe inter-symbol-interference, unless the topographic changes are spaced far apart. Spacing the changes far apart is undesirable from the storage viewpoint as it implies lower areal density. Another issue is that the cantilever exhibits complicated nonlinear dynamics. For example, if there is a sequence of hard hits on the media, then the next hit results in a milder response, i.e., the cantilever itself has inherent memory, that cannot be modeled as ISI. Conventional dynamic mode methods described in [33], that utilize high-Q cantilevers are not suitable for data storage applications. This is primarily because they are unable to deal with ISI and the nonlinear channel characteristics. The current techniques can be considered analogous to peak detection techniques in magnetic storage [30].

In this work we demonstrate that these issues can be addressed by modeling the dynamic mode operation as a communication system and developing high performance detectors for it. Note that corresponding activities have been undertaken in the past for technologies such as magnetic and optical storage [27], e.g., in magnetic storage, PRML techniques, resulted in tremendous improvements. In our work, the main issues are, (a) developing a model for the cantilever dynamics that predicts essential experimental features and remains tractable for data storage purposes, and (b) designing high-performance detectors for this model, that allow the usage of high quality cantilevers, without sacrificing areal density. As discussed in the sequel, several concepts such as Markovian modeling of the cantilever dynamics and Viterbi detection in the presence of noise with memory [10], play a key role in our approach.

In this chapter, a dynamic mode read operation is researched where the probe is oscillated and the media information is modulated on the cantilever probe's oscillations. It is demonstrated that an appropriate level of abstraction is possible that obviates the need for an involved physical model. The read operation is modeled as a communication channel which incorporates the system memory due to inter-symbol interference and the cantilever state that can be identified using training data. Using the identified model, a solution to the maximum likelihood sequence detection problem based on the Viterbi algorithm is devised. Experimental and simulation results which corroborate the analysis of the detector, demonstrate that the performance of this detector is several orders of magnitude better than the performance of other existing schemes and confirm performance gains that can render the dynamic mode operation feasible for high density data storage purposes.

Our work will motivate research for fabrication of prototypes that are massively parallel and employ high quality cantilevers (such as those used with the static mode [41] and intermittent contact dynamic mode but with low-Q [14]). In current prototypes, the cantilever detection is integrated into the cantilever structure and the cantilevers are actuated electrostatically. Even though the experimental setup reported in this chapter uses a particular scheme for measuring the cantilever detection and for actuating the cantilever, the paradigm developed for data detection is largely applicable in principle to other modes of detection and actuation of the cantilever. The analysis criteria primarily assume that high quality factor cantilevers are employed and that a dynamic mode operation is pursued.

2.2 Background and related work.

Probe based high density data storage devices employ a cantilever beam that is supported at one end and has a sharp tip at another end as a means to determine the topography of the media on which information is stored. The information on the media is encoded in terms of topographic profiles. A raised topographic profile is considered a high bit and a lowered topographic profile is considered a low bit. There are various means of measuring the cantilever deflection. In the standard atomic force microscope setup, which has formed the basis of probe

based data storage, the cantilever deflection is measured by a beam-bounce method where a laser is incident on the back of the cantilever surface and the laser is reflected from the cantilever surface into a split photodiode. The photodiode collects the incident laser energy and provides a measure of the cantilever deflection (see Figure 2.1(a)). The advantage of the beam-bounce method is the high resolution (low measurement noise) and high bandwidth (in the 2-3 MHz) range. The disadvantage is that it cannot be easily integrated into an operation where multiple cantilevers operate in parallel. There are attractive measurement mechanisms that integrate the cantilever motion sensing onto the cantilever itself. These include piezo-resistive sensing [13] and thermal sensing [16]. For the dynamic mode operation there are various schemes to actuate the cantilever that include electrostatic [14], mechanical by means of a dither piezo that actuates the support of the cantilever base, magnetic [19] and piezoelectric [15]. In this chapter, it is assumed that the cantilever is actuated by a dither piezo and the sensing mechanism employed is the beam bounce method (see Figure 2.1(a)).

2.2.1 Models of cantilever probe, the measurement process and the tip-media interaction

A first mode approximation of the cantilever is given by the spring mass damper dynamics described by

$$\ddot{p} + \frac{\omega_0}{Q}\dot{p} + \omega_0^2 p = f(t), \quad y = p + v, \quad (2.1)$$

where $\ddot{p} = \frac{d^2 p}{dt^2}$, p , f , y and v denote the deflection of the tip, the force on the cantilever, the measured deflection and the measurement noise respectively whereas the parameters ω_0 and Q are the first modal frequency (resonant frequency) and the quality factor of the cantilever respectively. The input-output transfer function with input f and output p is given as $G = \frac{1}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$. The cantilever model described above can be identified precisely (see [34]).

The interaction force, h , between the tip and the media depends on the deflection p of the cantilever tip. Such a dependence is well characterized by the Lennard-Jones like force that is typically characterized by weak long-range attractive forces and strong short range repulsive forces (see Figure 2.1(c)). Thus, the probe based data storage system can be viewed as an

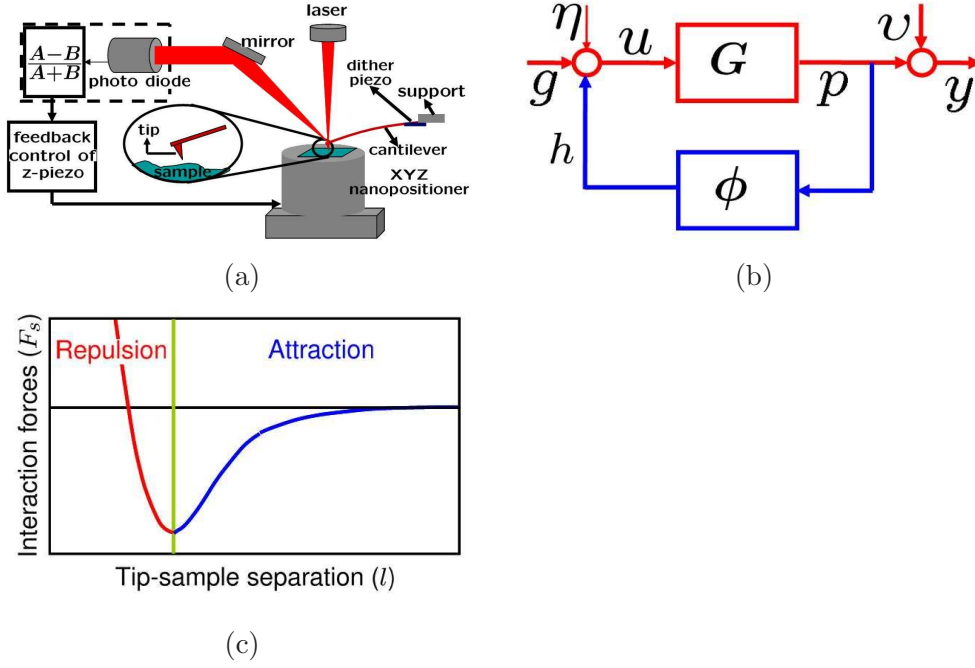


Figure 2.1 (a) Shows the main components of a probe based storage device. The main probe is a cantilever with a tip at one end that interacts with the media. The support end can be forced using a dither piezo. The deflection of the tip-end is measured by a laser-mirror-photodiode arrangement. The controller employs the deflection measurement to keep the probe engaged with the media. (b) Shows a block diagram representation of the cantilever system G being forced by white noise (η), tip-media force h and the dither forcing g . The output of the block G , the deflection p is corrupted by measurement noise v that results in the measurement y . Tip media force $h = \phi(p)$. (c) Shows the typical tip-media interaction forces of weak long range attractive forces and strong repulsive short range forces.

interconnection of a linear cantilever system G with the nonlinear tip-media interaction forces in feedback (see Figure 2.1(b) and note that $p = G(h + \eta + g)$ with $h = \phi(p)$ [38]).

2.2.2 Cantilever-Observer Model

A state space representation of the filter G can be obtained as $\dot{\bar{x}} = A\bar{x} + B\mathbf{f}$, $y = C\bar{x} + v$ where $\bar{x} = [p \ \dot{p}]^T$ and $\mathbf{f} = \eta + g$ (assuming no media forces h) and A , B and C are given by,

$$A = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & -\omega_0/Q \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Based on the model of the cantilever, an observer to monitor the state of the cantilever can be implemented [25] (see Figure 2.2). The observer dynamics and the associated state estimation error dynamics is given by,

$$\begin{array}{l}
 \overbrace{\begin{array}{l} \dot{\hat{x}} = A\hat{x} + Bg + L(y - \hat{y}); \hat{x}(0) = \hat{x}_0, \\ \hat{y} = C\hat{x}, \end{array}}^{\text{Observer}} \\
 \underbrace{\begin{array}{l} \dot{\tilde{x}} = A\bar{x} + B(g + \eta) - A\hat{x} - Bg - L(y - \hat{y}), \\ \quad = (A - LC)\tilde{x} + B\eta - Lv, \\ \tilde{x}(0) = \bar{x}(0) - \hat{x}(0), \end{array}}^{\text{State Estimation Error Dynamics}}
 \end{array}$$

where L is the gain of the observer, $\hat{x}$ is the estimate of the state $\bar{x}$ and g is the external known dither forcing applied to the cantilever. The error in the estimate is given by $\tilde{x} = \bar{x} - \hat{x}$, whereas the error in the estimate of the output y is given by, $e = y - \hat{y} = C\tilde{x} + v$. The error between the observed state and the actual state of the cantilever, when no noise terms or media forces are present ($\eta = v = h = 0$) is only due to the mismatch in the initial conditions of the observer and the cantilever-tip. Note that the cantilever tip interacts with the media only for a small portion of an oscillation. It is shown in [33] that such a tip-media interaction can be modeled well as an impact force (in other words as an impulsive force) on the cantilever that translates into an initial condition reset of the cantilever state. The error process is white if the Kalman gain is used for L [25]. For cantilever deflection sensors with low enough and realizable levels of measurement noise, the effective length of the impulse response of the system with media force as input and the error signal e as the output can be made as short as four periods of the cantilevers first resonant frequency.

As described in [33], the discretized model of the cantilever dynamics is given by

$$\begin{aligned}
 x_{k+1} &= Fx_k + G(g_k + \eta_k) + \delta_{\theta, k+1}\nu, \\
 y_k &= Hx_k + v_k, k \geq 0,
 \end{aligned} \tag{2.2}$$

where the matrices F , G , and H are obtained from matrices A , B and C using the zero order hold discretization at a desired sampling frequency and $\delta_{i,j}$ denotes the dirac delta function.

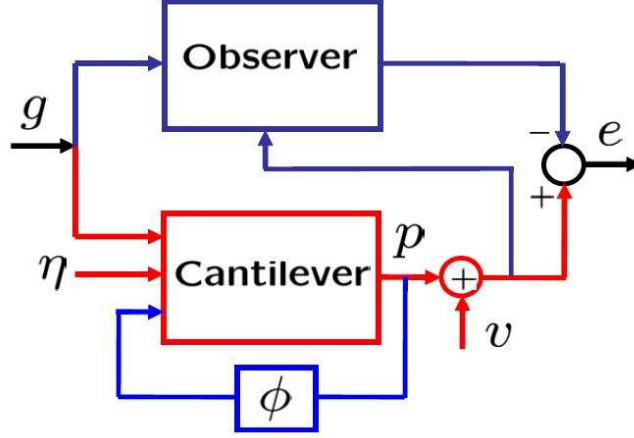


Figure 2.2 An observer architecture for the system in Figure 2.1(b)

θ denotes the time instant when the impact between the cantilever tip and the media occurs and ν signifies the value of the impact. The impact results in an instantaneous change or jump in the state by ν at time instant θ . When a Kalman observer is used, the profile in the error signal due to the media can be pre-calculated as,

$$e_k = y_k - \hat{y}_k = \Gamma_{k;\theta} \nu + n_k, \quad (2.3)$$

where $\{\Gamma_{k;\theta} \nu\}$ is a known dynamic state profile with an unknown arrival time θ defined by $\Gamma_{k;\theta} = H(F - L_K H)^{k-\theta}$, for $k \geq \theta$. L_K is the Kalman observer gain, n_k is a zero mean white noise sequence which is the measurement residual had the impact not occurred and θ is assumed to be equal to 0 for simplicity. The statistics of n are given by, $E\{n_j n_k^T\} = V \delta_{jk}$ where $V = H P_{\bar{x}} H^T + R$ and $P_{\bar{x}}$ is the steady state error covariance obtained from the Kalman filter that depends on P and R which are the variances of the thermal noise and measurement noise respectively.

2.3 Channel model and detectors

2.3.1 Reformulation of state space representation

It is to be noted that although we have modeled the cantilever system as a spring-mass-damper model (second order system with no zeros and two stable poles)(see (2.1)), the exper-

imentally identified channel transfer function that is more accurate in practice has right half plane zeros that are attributed to delays present in the electronics. Given this scenario, the state space representation used in [33] leads to a discrete channel with two inputs as seen in (2.3) because the structure of B is no longer in the form of $[0 \ 1]^T$. However, source information enters the channel as a single input as the tip-medium interaction force. The problem can be reformulated as one of a channel being driven by a single input by choosing an appropriate state space representation. For the state space model of the cantilever, it is known that the pair (A, B) is controllable which implies there exists a transformation which will convert the state space into a controllable canonical form such that $B = [0 \ 1]^T$. This kind of structure of B will force the discretized model (2.2) to be such that one component of ν is equal to 0. With B chosen as above, the entire system can be visualized as a channel that has a single source. In this chapter, the single source model is used as it simplifies the detector structure and analysis substantially.

2.3.2 Channel Model

The cantilever based data storage system can be modeled as a communication channel as shown in Figure 2.3. The components of this model are explained below in detail.

Shaping Filter $b(t)$: The model takes as input the bit sequence $\bar{a} = (a_0, a_1 \dots a_{N-1})$ where $a_k, k = 1, \dots, N-1$ is equally likely to be 0 or 1. In the probe storage context, ‘0’ refers to the topographic profile being *low* and ‘1’ refers to the topographic profile being *high*. Each bit has a duration of T seconds. This duration can be found based on the length of the topographic profile specifying a single bit and the speed of the scanner. The height of the high bit is denoted by A . The cantilever interacts with the media by gently tapping it when it is high. When the media is low, typically no interaction takes place. We model the effect of the medium height using a filter with impulse response $b(t)$ (shown in Figure 2.3) that takes as input, the input bit impulse train $a(t) = \sum_{k=0}^{N-1} a_k \delta(t - kT)$. The output of the filter is given by $\check{a}(t) = \sum_{k=0}^{N-1} a_k b(t - kT)$.

Nonlinearity Block (ϕ): The cantilever oscillates at frequency f_0 which means that in each cantilever cycle of duration $T_c = 1/f_0$, the cantilever hits the media at most once if the media is high during a time T_c . Due to the dynamics of the system it may not hit the media, even if it is high. The magnitude of impact on the media is not constant and changes according to the state of the cantilever prior to the interaction with the media. We note that a very accurate modeling of the cantilever trajectory will require the solution of complex nonlinear equations corresponding to the cantilever dynamics and knowledge of the bit profile so that each interaction is known. In this work we model the impact values of the tip-media interaction by means of a probabilistic Markov model that depends on the previous bits. This obviates the need for a detailed model. We assume that in each high bit duration T , the cantilever hits the media q times (i.e. $T = qT_c$) with varying magnitudes. Therefore, for N bits, the output of the nonlinearity block is given by,

$$\tilde{a}(t) = \sum_{k=0}^{Nq-1} \nu_k(\bar{a})\delta(t - kT_c),$$

where ν_k denotes the magnitude of the k^{th} impact of the cantilever on the medium. Here, we approximate the nonlinearity block output as a sequence of impulsive force inputs to the cantilever. The strength of the impulsive hit at any instant is dependent on previous impulsive hits; precisely because the previous interactions affect the amplitude of the oscillations that in turn affect how hard the hit is at a particular instance. The exact dependence is very hard to model deterministically and therefore we chose a Markov model, as given below for the sequence of impact magnitudes for a single bit duration,

$$\bar{\nu}_i = \bar{\mathfrak{G}}(a_i, a_{i-1}, \dots, a_{i-m}) + \bar{\mathfrak{b}}_i \quad (2.4)$$

where $\bar{\mathfrak{G}}(a_i, a_{i-1}, \dots, a_{i-m})$ is a function of $\bar{\nu}_i = [\nu_{iq} \ \nu_{iq+1} \dots \nu_{(i+1)q-1}]^T$ and the current and the last m bits. Here m denotes the system memory and $\bar{\mathfrak{b}}_i$ is a zero mean i.i.d. Gaussian vector of length q . The appropriateness of the model will be demonstrated by our experimental results.

Channel Response ($\Gamma(t)$): The Markovian modeling of the output of the nonlinearity block as discussed above allows us to break the feedback loop in Figure 2.2 (see also [33]). The rest

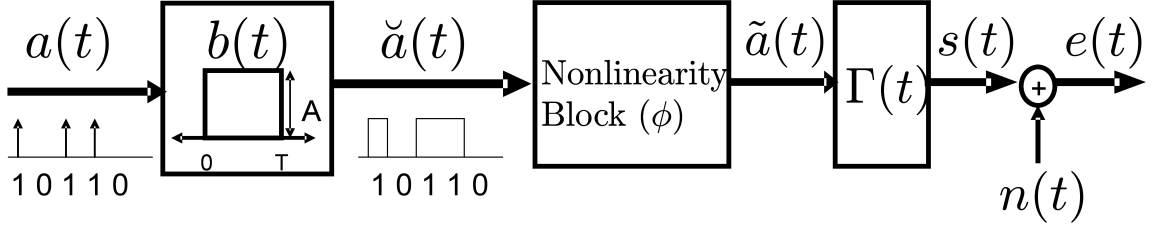


Figure 2.3 Continuous time channel model of probe based data storage system

of the system can then be modeled by treating it as a linear system with impulse response $\Gamma(t)$. $\Gamma(t)$ is the error between the cantilever tip deflection and the tip deflection as estimated by the observer when the cantilever tip is subjected to an impulsive force. It can be found in closed form for a given set of parameters of cantilever-observer system (see (2.3)).

Channel Noise ($n(t)$): The measurement noise (from the imprecision in measuring the cantilever position) and thermal noise (from modeling mismatches) can be modeled by a single zero mean white Gaussian noise process ($n(t)$) with power spectral density equal to V .

The continuous time innovation output $e(t)$ becomes,

$$e(t) = s(t, \bar{\nu}(\bar{a})) + n(t),$$

where $s(t, \bar{\nu}(\bar{a})) = \sum_{k=0}^{Nq-1} \nu_k(\bar{a}) \Gamma(t - kT_c)$ and $\bar{\nu}(\bar{a}) = (\nu_0(\bar{a}), \nu_1(\bar{a}) \dots \nu_{Nq-1}(\bar{a}))$. The sequence of impact values $\bar{\nu}_i$ is assumed to follow a Markovian model as explained above, $\Gamma(t)$ is the channel impulse response and $n(t)$ is a zero mean white Gaussian noise process.

2.3.3 Sufficient Statistics for Channel model

Before providing sufficient statistics we consolidate the notation used. The source stream is N elements long ($\bar{a}$ denotes the sequence of source bits), with the topographic profile and the scan speed is chosen such that the cantilever impacts any topographic profile q times. Thus there are Nq possible hits with $\bar{\nu}(\bar{a})$ denoting the sequence of strength of the Nq impulsive hits on the cantilever. Furthermore, the set of strengths of impulsive force inputs, which is q elements long, during the i^{th} topographic profile encoding the i^{th} source symbol is denoted

by $\bar{\nu}_i$. Given the probabilistic model on $\bar{\nu}$ and finite bit sequence $(\bar{a})$, an information lossless decomposition of $e(t)$ by expansion over an orthonormal finite-dimensional basis with dimension $\tilde{N}$ can be achieved where $\tilde{N}$ orthonormal basis functions span the signal space formed by $s(t, \bar{\nu}(\bar{a}))$. The components of $e(t)$ over $\tilde{N}$ orthonormal basis functions are given by,

$$\bar{\mathbf{e}} = \bar{s}(\bar{\nu}(\bar{a})) + \bar{n},$$

where $\bar{\mathbf{e}} = (\mathbf{e}_0, \mathbf{e}_1 \dots \mathbf{e}_{\tilde{N}})$, $\bar{s}(\bar{\nu}(\bar{a})) = (s_0, s_1 \dots s_{\tilde{N}})$, $\bar{n} = (n_0, n_1 \dots n_{\tilde{N}})$ and $\bar{n} \sim N(0, V I_{\tilde{N} \times \tilde{N}})$ where $I_{\tilde{N} \times \tilde{N}}$ stands for $\tilde{N} \times \tilde{N}$ identity matrix [20]. The maximum likelihood estimate of the bit sequence can be found as,

$$\hat{\bar{a}} = \arg \max_{\bar{a} \in \{0,1\}^N} f(\bar{\mathbf{e}}|\bar{a})$$

where $\hat{\bar{a}} = (\hat{a}_0, \hat{a}_1 \dots \hat{a}_{N-1})$ is the estimated bit sequence and f denotes a pdf. The term $f(\bar{\mathbf{e}}|\bar{a})$ can be further simplified as,

$$\begin{aligned} f(\bar{\mathbf{e}}|\bar{a}) &= \int_{\bar{\nu}} f(\bar{\mathbf{e}}|\bar{a}, \bar{\nu}) f(\bar{\nu}|\bar{a}) d\bar{\nu} \\ &= \int_{\bar{\nu}} \frac{1}{(2\pi V)^{\frac{\tilde{N}}{2}}} \times \exp \frac{-\|\bar{\mathbf{e}} - \bar{s}(\bar{\nu}(\bar{a}))\|^2}{2V} f(\bar{\nu}|\bar{a}) d\bar{\nu} \\ &= \frac{1}{(2\pi V)^{\frac{\tilde{N}}{2}}} \exp \frac{-\|\bar{\mathbf{e}}\|^2}{2V} \times \int_{\bar{\nu}} \exp \frac{-(\|\bar{s}(\bar{\nu}(\bar{a}))\|^2 - 2\bar{\mathbf{e}}^T \bar{s}(\bar{\nu}(\bar{a})))}{2V} f(\bar{\nu}|\bar{a}) d\bar{\nu} \end{aligned}$$

where $\|\cdot\|^2$ denotes Euclidean norm, $f(\bar{\mathbf{e}}|\bar{a}, \bar{\nu})$ and $f(\bar{\nu}|\bar{a})$ denote the respective conditional pdf's and $\bar{\nu} = (\nu_0, \nu_1 \dots \nu_{Nq-1})$. The correlation between $\bar{\mathbf{e}}$ and $\bar{s}(\bar{\nu}(\bar{a}))$ can be equivalently expressed as an integral over time because of the orthogonal decomposition procedure i.e.,

$$\begin{aligned} \bar{\mathbf{e}}^T \bar{s}(\bar{\nu}(\bar{a})) &= \int_{-\infty}^{\infty} e(t) s(t, \bar{\nu}(\bar{a})) dt \\ &= \sum_{k=0}^{Nq-1} \nu_k \int_{-\infty}^{\infty} e(t) \Gamma(t - kT_c) dt \\ &= \sum_{k=0}^{Nq-1} \nu_k z'_k = \bar{\nu}^T \bar{z}'. \end{aligned}$$

where $\bar{\nu} = (\nu_0, \nu_1 \dots \nu_{Nq-1})$, $\bar{z}' = (z'_0, z'_1 \dots z'_{Nq-1})$ and $z'_k = \int_{-\infty}^{\infty} e(t) \Gamma(t - kT_c) dt$ for $0 \leq k \leq Nq - 1$ is the output of a matched filter $\Gamma(-t)$ with input $e(t)$ sampled at $t = kT_c$.

The term $f(\bar{\mathbf{e}}|\bar{a})$ can now be written as,

$$\begin{aligned} f(\bar{\mathbf{e}}|\bar{a}) &= \frac{1}{(2\pi V)^{\frac{\tilde{N}}{2}}} \exp \frac{-\|\bar{\mathbf{e}}\|^2}{2V} \times \int_{\bar{\nu}} \exp \frac{-(\|\bar{s}(\bar{\nu}(\bar{a}))\|^2 - 2\bar{\mathbf{e}}^T \bar{s}(\bar{\nu}(\bar{a})))}{2V} f(\bar{\nu}|\bar{a}) d\bar{\nu} \\ &= \underbrace{\frac{1}{(2\pi V)^{\frac{\tilde{N}}{2}}} \exp \frac{-\|\bar{\mathbf{e}}\|^2}{2V}}_{\mathfrak{h}(\bar{\mathbf{e}})} \times \underbrace{\int_{\bar{\nu}} \exp \frac{-\|\bar{s}(\bar{\nu}(\bar{a}))\|^2}{2V} \exp \frac{\bar{\nu}^T \bar{z}'}{V} f(\bar{\nu}|\bar{a}) d\bar{\nu}}_{\mathfrak{F}(\bar{z}'|\bar{a})} \end{aligned}$$

So $f(\bar{\mathbf{e}}|\bar{a})$ can be factorized into $\mathfrak{h}(\bar{\mathbf{e}})$ (dependent only on $\bar{\mathbf{e}}$) and $\mathfrak{F}(\bar{z}'|\bar{a})$ (for a given $\bar{a}$ dependent only on $\bar{z}'$). Using the Fisher-Neyman factorization theorem [12], we can claim that $\bar{z}'$ is a vector of sufficient statistics for the detection process i.e.

$$\frac{f(\bar{\mathbf{e}}|\bar{a})}{f(\bar{z}'|\bar{a})} = \mathcal{C},$$

where $\mathcal{C}$ is a constant independent of $\bar{a}$. So we can reformulate the detection problem as,

$$\hat{\bar{a}} = \arg \max_{\bar{a} \in \{0,1\}^N} f(\bar{z}'|\bar{a})$$

which means that bit detection problem depends only on the matched filter outputs ($\bar{z}'$). These matched filter outputs for $0 \leq k \leq Nq - 1$ can be further simplified as,

$$z'_k = \sum_{k_1=0}^{Nq-1} \nu_{k_1}(\bar{a}) h'_{k-k_1} + n'_k,$$

where $h'_{k-k_1} = \int_{-\infty}^{\infty} \Gamma(t - kT_c) \Gamma(t - k_1 T_c) dt$ and $n'_k = \int_{-\infty}^{\infty} n(t) \Gamma(t - kT_c) dt$. $E(n'_k n'_{k'})$ is,

$$\begin{aligned} E(n'_k n'_{k'}) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(n(t) n(\tau)) \Gamma(t - kT_c) \Gamma(\tau - k'T_c) dt d\tau \\ &= V R_{k-k'}, \end{aligned}$$

where $R_{k-k'} = \int_{-\infty}^{\infty} \Gamma(t - kT_c) \Gamma(t - k'T_c) dt$. A whitening matched filter can be determined to whiten output noise n'_k [20]. We shall denote the discretized output of whitened matched filter shown in Figure 3.1 as z_k ,

$$z_k = \sum_{k_1=0}^I \nu_{k-k_1}(\bar{a}) h_{k_1} + n_k,$$

where the filter $\{h_k\}_{k=0,1,\dots,I}$ denotes the effect of the whitened matched filter and the sequence $\{n_k\}$ represents the Gaussian noise with variance V .

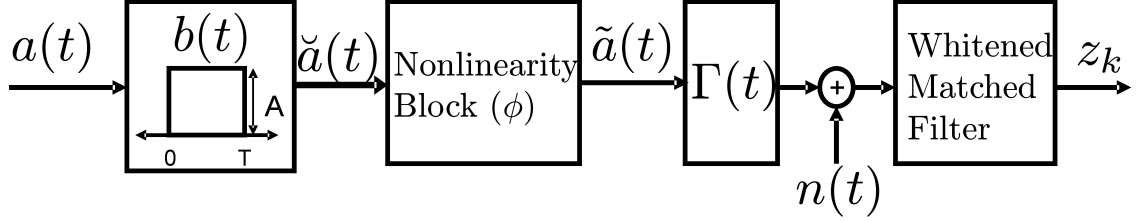


Figure 2.4 Discretized channel model with whitened matched filter

2.3.4 Viterbi Detector Design

Note that the outputs of the whitened matched filter $\bar{z}$, continue to remain sufficient statistics for the detection problem. Therefore, we can reformulate the detection strategy as,

$$\begin{aligned}\hat{\bar{a}} &= \arg \max_{\bar{a} \in \{0,1\}^N} f(\bar{z}|\bar{a}) \\ &= \arg \max_{\bar{a} \in \{0,1\}^N} \prod_{i=0}^{N-1} f(\bar{z}_i|\bar{a}, \bar{z}_0^{i-1})\end{aligned}\quad (2.5)$$

where $\bar{z} = [z_0 \ z_1 \ \dots \ z_{Nq-1}]^T$, $\bar{z}_i$ is the received output vector corresponding to the i^{th} input bit, i.e., $\bar{z}_i = [z_{iq} \ z_{iq+1} \ \dots \ z_{(i+1)q-1}]^T$ and $\bar{z}_0^{i-1} = [\bar{z}_0^T \ \bar{z}_1^T \ \dots \ \bar{z}_{i-1}^T]^T$. In our model, the channel is characterized by finite impulse response of length I i.e. $h_i = 0$ for $i < 0$ and $i > I$ and we assume that $I \leq m_I q$ i.e. the inter-symbol-interference (ISI) length in terms of q hits is equal to m_I . Let m be the system memory (see (3.1)). The length of channel response is known which means that m_I is known but the value of m cannot be found because it depends on the experimental parameters of the system. In the experimental results section, we describe how we find the value of m from experimental data. The received output vector $\bar{z}_i$ can now be written as,

$$\begin{aligned}\bar{z}_i &= \begin{pmatrix} h_I & . & . & h_0 & 0 & . & . & 0 \\ 0 & h_I & . & . & h_0 & 0 & . & 0 \\ . & . & . & . & . & . & . & . \\ 0 & . & . & 0 & h_I & . & . & h_0 \end{pmatrix} \begin{pmatrix} \nu_{iq-I} \\ \nu_{1+iq-I} \\ \vdots \\ \nu_{(i+1)q-1} \end{pmatrix} + \bar{n}_i \\ &= H\bar{\nu}_{i-m_I}^i + \bar{n}_i,\end{aligned}$$

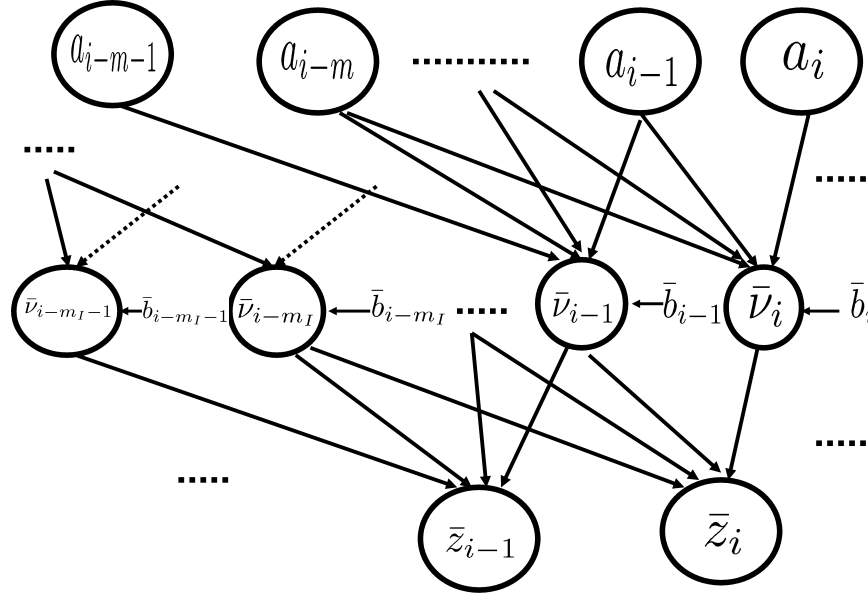


Figure 2.5 Dependency graph for the model with $I = m_I q$ and m is the system memory of the system

where $\bar{\nu}_i = [\nu_{iq} \nu_{iq+1} \dots \nu_{(i+1)q-1}]^T$, $\bar{\nu}_{i-m_I}^i = [\bar{\nu}_{i-m_I}^T \dots \bar{\nu}_i^T]^T$ and $\bar{n}_i = [n_{iq} n_{1+iq} \dots n_{(i+1)q-1}]^T$.

Our next task is to simplify the factorization in (2.5) so that decoding can be made tractable. We construct the dependency graph of the concerned quantities which is shown in Figure 2.5. Using the Bayes ball algorithm [39], we conclude that

$$f(\bar{z}_i | \bar{\nu}_{i-m_I}^i, \bar{a}, \bar{z}_0^{i-1}) = f(\bar{z}_i | \bar{\nu}_{i-m_I}^i), \quad (2.6)$$

$$f(\bar{\nu}_{i-m_I} | \bar{a}, \bar{z}_0^{i-1}) = f(\bar{\nu}_{i-m_I} | a_0^{i-1}, \bar{z}_0^{i-1}), \quad (2.7)$$

$$f(\bar{\nu}_{i-k} | \bar{\nu}_{i-m_I}^{i-k-1}, \bar{a}, \bar{z}_0^{i-1}) = f(\bar{\nu}_{i-k} | \bar{\nu}_{i-m_I}^{i-k-1}, a_0^{i-m_I-1}, a_{i-m-k}^{i-1}, \bar{z}_0^{i-1}), \forall 1 \leq k \leq m_I - 1 \quad (2.8)$$

$$f(\bar{\nu}_i | \bar{\nu}_{i-m_I}^{i-1}, \bar{a}, \bar{z}_0^{i-1}) = f(\bar{\nu}_i | a_{i-m}^i) \quad (2.9)$$

where $a_0^{i-1} = [a_0 \ a_1 \ \dots \ a_{i-1}]$. Although the conditional pdf $f(\bar{\nu}_{i-k} | \bar{\nu}_{i-m_I}^{i-k-1}, \bar{a}, \bar{z}_0^{i-1})$ and $f(\bar{\nu}_{i-m_I} | \bar{a}, \bar{z}_0^{i-1})$ depend on the entire past, we assume that these dependencies are rapidly decreasing with increase in past time. This is observed in simulation and experimental data as well. For making the detection process more tractable, we make the following assumptions

on this dependence,

$$f(\bar{\nu}_{i-m_I}|a_0^{i-1}, \bar{z}_0^{i-1}) \approx f(\bar{\nu}_{i-m_I}|a_{i-m-m_I}^{i-1}, \bar{z}_{i-m_I}^{i-1}) \quad (2.10)$$

$$\begin{aligned} f(\bar{\nu}_{i-k}|\bar{\nu}_{i-m_I}^{i-k-1}, a_0^{i-m_I-1}, a_{i-m-k}^{i-1}, \bar{z}_0^{i-1}) \\ \approx f(\bar{\nu}_{i-k}|\bar{\nu}_{i-m_I}^{i-k-1}, a_{i-k-m}^{i-1}, \bar{z}_{i-k}^{i-1}), \forall 1 \leq k \leq m_I - 1 \end{aligned} \quad (2.11)$$

i.e. the dependence is restricted to only the immediate neighbors in the dependency graph. Using the above assumptions and dependency graph results, $f(\bar{z}_i|\bar{a}, \bar{z}_0^{i-1})$ can be further simplified as [57, 56],

$$\begin{aligned} f(\bar{z}_i|\bar{a}, \bar{z}_0^{i-1}) &= \int f(\bar{z}_i|\bar{\nu}_{i-m_I}^i, \bar{a}, \bar{z}_0^{i-1}) f(\bar{\nu}_{i-m_I}^i|\bar{a}, \bar{z}_0^{i-1}) d\bar{\nu}_{i-m_I}^i \\ &= \int f(\bar{z}_i|\bar{\nu}_{i-m_I}^i, \bar{a}, \bar{z}_0^{i-1}) f(\bar{\nu}_{i-m_I}|\bar{a}, \bar{z}_0^{i-1}) \\ &\quad \cdot \Pi_{k=1}^{m_I-1} f(\bar{\nu}_{i-k}|\bar{\nu}_{i-m_I}^{i-k-1}, \bar{a}, \bar{z}_0^{i-1}) f(\bar{\nu}_i|\bar{\nu}_{i-m_I}^{i-1}, \bar{a}, \bar{z}_0^{i-1}) d\bar{\nu}_{i-m_I}^i \\ &= \int f(\bar{z}_i|\bar{\nu}_{i-m_I}^i) f(\bar{\nu}_{i-m_I}|a_0^{i-1}, \bar{z}_0^{i-1}) \\ &\quad \cdot \Pi_{k=1}^{m_I-1} f(\bar{\nu}_{i-k}|\bar{\nu}_{i-m_I}^{i-k-1}, a_0^{i-m_I-1}, a_{i-m-k}^{i-1}, \bar{z}_0^{i-1}) \\ &\quad \cdot f(\bar{\nu}_i|a_{i-m}^i) d\bar{\nu}_{i-m_I}^i \text{ (Using (2.6),(2.7),(2.8),(2.9))} \\ &= \int f(\bar{z}_i|\bar{\nu}_{i-m_I}^i) f(\bar{\nu}_{i-m_I}|a_{i-m-m_I}^{i-1}, \bar{z}_{i-m_I}^{i-1}) \\ &\quad \cdot \Pi_{k=1}^{m_I-1} f(\bar{\nu}_{i-k}|\bar{\nu}_{i-m_I}^{i-k-1}, a_{i-k-m}^{i-1}, \bar{z}_{i-k}^{i-1}) \\ &\quad \cdot f(\bar{\nu}_i|a_{i-m}^i) d\bar{\nu}_{i-m_I}^i \text{ (Using (2.10),(2.11))} \\ &= \int f(\bar{z}_i|\bar{\nu}_{i-m_I}^i, a_{i-m-m_I}^i, \bar{z}_{i-m_I}^{i-1}) f(\bar{\nu}_{i-m_I}^i|a_{i-m-m_I}^i, \bar{z}_{i-m_I}^{i-1}) d\bar{\nu}_{i-m_I}^i \\ &= f(\bar{z}_i|a_{i-m-m_I}^i, \bar{z}_{i-m_I}^{i-1}). \end{aligned}$$

By defining a state $S_i = a_{i-m-m_I+1}^i$, this can be further expressed as $f(\bar{z}_i|S_i, S_{i-1}, \bar{z}_{i-m_I}^{i-1})$.

Again using Bayes ball algorithm, we conclude that

$$f(\bar{z}_{i-m_I}^i|\bar{\nu}_{i-2m_I}^i, a_{i-m-m_I}^i) = f(\bar{z}_{i-m_I}^i|\bar{\nu}_{i-2m_I}^i), \quad (2.12)$$

$$\begin{aligned} \Pi_{k=1}^{2m_I-1} f(\bar{\nu}_{i-2m_I+k}|\bar{\nu}_{i-2m_I}^{i-2m_I+k-1}, a_{i-m-m_I}^i) &= \Pi_{k=1}^{m_I-1} f(\bar{\nu}_{i-2m_I+k}|\bar{\nu}_{i-2m_I}^{i-2m_I+k-1}, a_{i-m-m_I}^i) \\ &\quad \cdot \Pi_{k=m_I}^{2m_I-1} f(\bar{\nu}_{i-2m_I+k}|\bar{\nu}_{i-2m_I}^{i-2m_I+k-1}, a_{i-2m_I-m+k}^i), \end{aligned} \quad (2.13)$$

$$f(\bar{\nu}_i|\bar{\nu}_{i-2m_I}^{i-1}, a_{i-m-m_I}^i) = f(\bar{\nu}_i|a_{i-m}^i). \quad (2.14)$$

The pdf of $\bar{z}_{i-m_I}^i = [\bar{z}_{i-m_I}^T \dots \bar{z}_i^T]^T$ given current state S_i and previous state S_{i-1} is given by,

$$\begin{aligned}
f(\bar{z}_{i-m_I}^i | S_i, S_{i-1}) &= f(\bar{z}_{i-m_I}^i | a_{i-m-m_I}^i) \\
&= \int f(\bar{z}_{i-m_I}^i | \bar{\nu}_{i-2m_I}^i, a_{i-m-m_I}^i) f(\bar{\nu}_{i-2m_I}^i | a_{i-m-m_I}^i) d\bar{\nu}_{i-2m_I}^i \\
&= \int f(\bar{z}_{i-m_I}^i | \bar{\nu}_{i-2m_I}^i, a_{i-m-m_I}^i) f(\bar{\nu}_{i-2m_I}^i | a_{i-m-m_I}^i) \\
&\quad \cdot \prod_{k=1}^{2m_I-1} f(\bar{\nu}_{i-2m_I+k}^i | \bar{\nu}_{i-2m_I}^{i-2m_I+k-1}, a_{i-m-m_I}^i) \\
&\quad \cdot f(\bar{\nu}_i | \bar{\nu}_{i-2m_I}^{i-1}, a_{i-m-m_I}^i) d\bar{\nu}_{i-2m_I}^i \\
&= \int f(\bar{z}_{i-m_I}^i | \bar{\nu}_{i-2m_I}^i) f(\bar{\nu}_{i-2m_I}^i | a_{i-m-m_I}^i) \\
&\quad \cdot \prod_{k=1}^{m_I-1} f(\bar{\nu}_{i-2m_I+k}^i | \bar{\nu}_{i-2m_I}^{i-2m_I+k-1}, a_{i-m-m_I}^i) \\
&\quad \cdot \prod_{k=m_I}^{2m_I-1} f(\bar{\nu}_{i-2m_I+k}^i | a_{i-2m_I-m+k}^i) \\
&\quad \cdot f(\bar{\nu}_i | a_{i-m}^i) d\bar{\nu}_{i-2m_I}^i \text{ (Using (2.12), (2.13), (2.14))}
\end{aligned}$$

where the last step is obtained using results from dependency graph and all the terms in the last step except $f(\bar{\nu}_{i-2m_I}^i | a_{i-m-m_I}^i)$ and $\prod_{k=1}^{m_I-1} f(\bar{\nu}_{i-2m_I+k}^i | \bar{\nu}_{i-2m_I}^{i-2m_I+k-1}, a_{i-m-m_I}^i)$ are Gaussian distributed. This implies that the pdf of $\bar{z}_{i-m_I}^i$ given (S_i, S_{i-1}) is not exactly Gaussian distributed. If the number of states in the detector is increased it can be modeled as a Gaussian which means that the term like $f(\bar{\nu}_{i-2m_I}^i | a_{i-m-m_I}^i)$ can be made Gaussian distributed by increasing the number of states, but this increases the complexity. In order to keep the decoding tractable we make the assumption that $f(\bar{z}_{i-m_I}^i | S_i, S_{i-1})$ is Gaussian i.e. $f(\bar{z}_{i-m_I}^i | S_i, S_{i-1}) \sim N(\bar{\mathcal{Y}}(S_i, S_{i-1}), \mathcal{C}(S_i, S_{i-1}))$, where $\bar{\mathcal{Y}}(S_i, S_{i-1})$ is the mean and $\mathcal{C}(S_i, S_{i-1})$ is the covariance. With our state definition, we can reformulate the detection problem as a maximum likelihood state sequence detection problem [10],

$$\begin{aligned}
\hat{\bar{S}} &= \arg \max_{all \bar{S}} f(\bar{z} | \bar{S}) \\
&= \arg \max_{all \bar{S}} \prod_{i=0}^{N-1} f(\bar{z}_i | \bar{S}, \bar{z}_0 \dots \bar{z}_{i-1})
\end{aligned}$$

$$\begin{aligned}
&= \arg \max_{\text{all } \hat{S}} \prod_{i=0}^{N-1} f(\bar{z}_i | S_i, S_{i-1}, \bar{z}_{i-m_I}^{i-1}) \\
&= \arg \max_{\text{all } \hat{S}} \prod_{i=0}^{N-1} \frac{f(\bar{z}_{i-m_I}^i | S_i, S_{i-1})}{f(\bar{z}_{i-m_I}^{i-1} | S_i, S_{i-1})} \\
&= \arg \min_{\text{all } \hat{S}} \sum_{i=0}^{N-1} \log \frac{|\mathcal{C}(S_i, S_{i-1})|}{|c(S_i, S_{i-1})|} + (\bar{z}_{i-m_I}^i - \bar{\mathcal{Y}}(S_i, S_{i-1}))^T \\
&\quad \cdot \mathcal{C}(S_i, S_{i-1})^{-1} (\bar{z}_{i-m_I}^i - \bar{\mathcal{Y}}(S_i, S_{i-1})) - (\bar{z}_{i-m_I}^{i-1} \\
&\quad - \bar{\mathcal{Y}}(S_i, S_{i-1}))^T c(S_i, S_{i-1})^{-1} (\bar{z}_{i-m_I}^{i-1} - \bar{\mathcal{Y}}(S_i, S_{i-1}))
\end{aligned}$$

where $\hat{S}$ is estimated state sequence, $c(S_i, S_{i-1})$ is the upper $m_I q \times m_I q$ principal minor of $\mathcal{C}(S_i, S_{i-1})$ and $\bar{\mathcal{Y}}(S_i, S_{i-1})$ collects the first $m_I q$ elements of $\bar{\mathcal{Y}}(S_i, S_{i-1})$. It is assumed that the first state is known. With metric given above, Viterbi decoding can be applied to get the maximum likelihood state sequence and the corresponding bit sequence.

2.3.5 LMP, GLRT and Bayes Detector

In [33], the hit detection algorithm is proposed which ignores the modeling of channel memory and works well only when the hits are sufficiently apart. In [49], various detectors for hit detection like locally most powerful (LMP), generalized likelihood ratio test (GLRT) and Bayes detector are presented. These detectors also ignore the system memory and perform detection of single hits. Subsequently a majority type rule is used for bit detection. The continuous time innovation ($e(t)$) is sampled at very high sampling rate $1/T_s$ such that $T_s \ll T_c$. As the channel response ($\Gamma(t)$) is finite length, the sampled channel response is assumed to have the finite length equal to M . The sampled channel response is given by,

$$\Gamma_0 = [\Gamma(t)|_{t=0} \ \Gamma(t)|_{t=T_s} \ \dots \ \Gamma(t)|_{t=(M-1)T_s}]^T$$

Determining when the cantilever is “hitting” the media and when it is not, is formulated as a binary hypothesis testing problem with the following hypotheses,

$$H_0 : \bar{e} = \bar{n},$$

$$H_1 : \bar{e} = \Gamma_0 \nu + \bar{n}$$

where the sampled innovation vector $\bar{e} = [e_1 \ e_2 \dots e_M]^T$, $\bar{n} = [n_1 \ n_2 \dots n_M]^T$, Γ_0 is the sampled channel response, ν signifies the value of the impact on media and $V I_{M \times M}$ denotes the covariance matrix of $\bar{n}$ where $I_{M \times M}$ stands for $M \times M$ identity matrix. In case of locally most powerful (LMP) test given in [26], the likelihood ratio is given by [49],

$$l_{lmp}(M) = \frac{\partial}{\partial \nu} (\log \frac{f(\bar{e}|H_1)}{f(\bar{e}|H_0)})|_{\nu=0} = \bar{e}^T V^{-1} \Gamma_0.$$

where l_{lmp} denotes likelihood ratio for LMP. In our model, there are q number of hits in one bit duration. Let $l_{k,lmp}$ be the likelihood ratio corresponding to k^{th} hit. The decision rule for the detection of one bit in this case is defined as,

$$Max \left(l_{1,lmp}(M), l_{2,lmp}(M) \dots l_{q,lmp}(M) \right) \leq_1^0 \tau_1 \quad (2.15)$$

where τ_1 is LMP threshold. The likelihood ratio in the case of GLRT is [49],

$$l_{glrt}(M) = \log \frac{f(\bar{e}|H_1, \nu = \tilde{\nu})}{f(\bar{e}|H_0)} = l_{lmp}^2,$$

where $\tilde{\nu}$ is maximum likelihood (ML) estimate of ν i.e. $\tilde{\nu} = \arg \max_{\nu} f(\bar{e}|H_1)$, l_{lmp} and l_{glrt} are likelihood ratios for LMP and GLRT case respectively. The decision rule for the bit detection in this case is defined in a similar manner given in (2.15).

Simulations from a Simulink model of the system can be run for a large number of hits in order to gather statistics on the discretized output of nonlinearity block which models the tip-media force. We modeled the statistics for ν by a Gaussian pdf with the appropriate mean and variance. With known mean and variance of ν the likelihood ratio for Bayes test is [49],

$$l_{bayes}(M) = \log \frac{f(\bar{e}|H_1)}{f(\bar{e}|H_0)} = \bar{e}^T V^{-1} \mu' + \frac{1}{2} \bar{e}^T V' \bar{e} - \bar{e}^T V' \mu',$$

where $\mu' = \Gamma_0 \alpha$ and $V' = \frac{\Gamma_0 \Gamma_0^T}{(\frac{V^2}{\lambda^2} + V \Gamma_0^T \Gamma_0)}$ and $\nu \sim N(\alpha, \lambda^2)$. The decision rule in this case is also defined in a similar manner given in (2.15). Note that ν is a measure of the tip-medium interaction force and as such it is difficult to experimentally verify the value of this force accurately which means the Bayes test cannot be applied for the bit detection on actual experimental data.

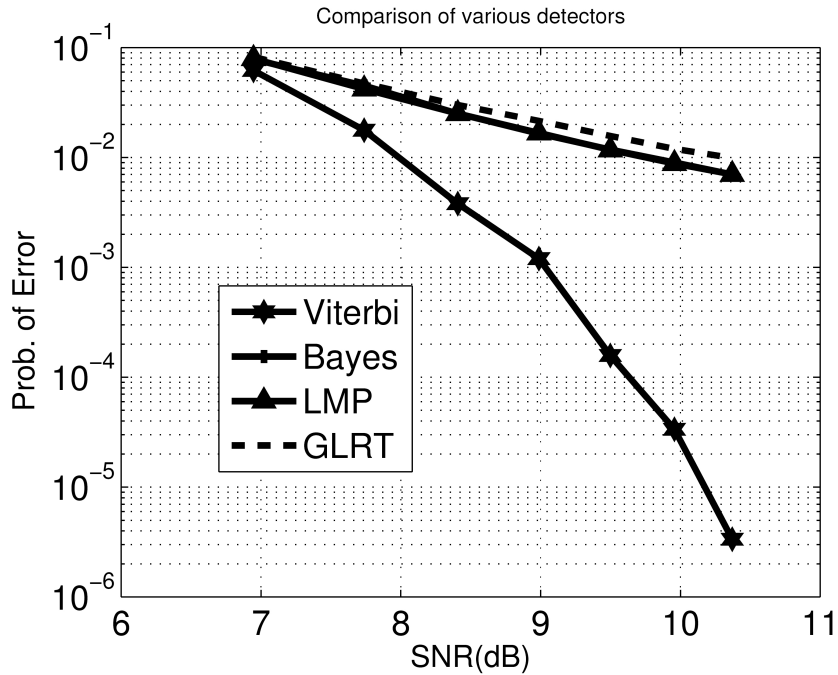


Figure 2.6 Comparison of various detectors for simulation data. The Bayes curve is not visible in the graph as it coincides with the LMP curve.

2.4 Simulation Results

We performed simulations with the following parameters. The first resonant frequency of the cantilever $f_0 = 63.15$ KHz, quality factor $Q = 206$, the value of forcing amplitude equal to 24 nm, tip-media separation is 28 nm, the number of hits in high bit duration is equal to 13 i.e. $q = 13$, discretized thermal and measurement noise variance are 0.1 and 0.001 respectively. A Kalman observer was designed and the length of the channel impulse response (I) was approximately 24 which means that m_I is equal to 2. We set the value of the system memory, $m = 1$. Using a higher value of m results in a more complex detector. We used a topographic profile where high and low regions denote bits '1' and '0' respectively and the bit sequence is generated randomly. The simulation was performed with the above parameters using the Simulink model that mimics the experimental station that provides a qualitative as well as a quantitative match to the experimental data. Tip-media interaction was varied by changing the height of media corresponding to bit '1'. We define the system SNR as the

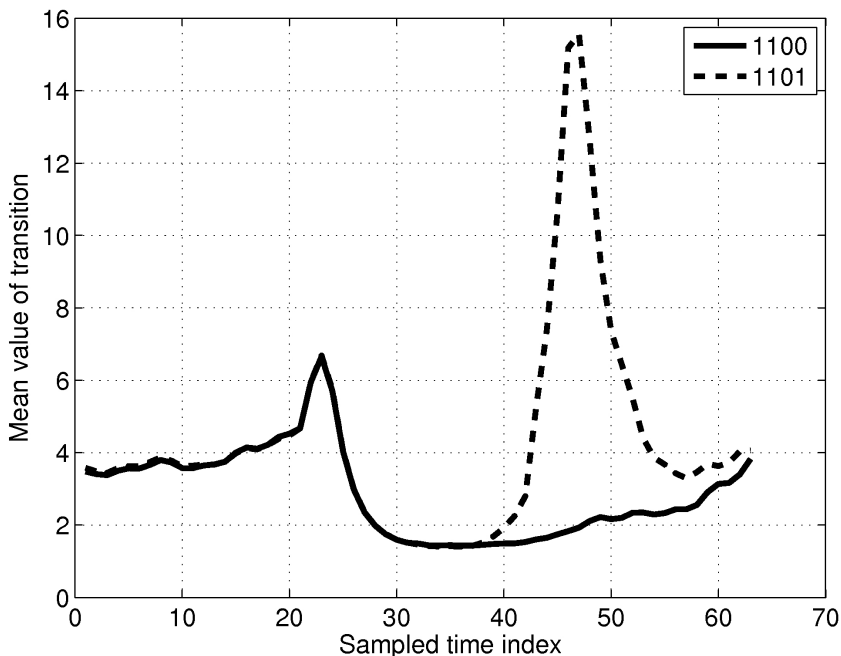


Figure 2.7 Mean vector for 2 state transitions for $300 \mu s$ bit width from experimental data where ‘1100’ and ‘1101’ represents transition from state ‘110’ to state ‘100’ and ‘101’ respectively

nominal tip-media interaction (nm) divided by total noise variance.

In Figure 2.6, we compare the results of four different detectors. The LMP, GLRT and Bayes detector perform hit detection, as against bit detection. In these detectors, the system memory is not taken into account. It is clear that the minimum probability of error for all detectors decreases as the tip-media interaction increases which makes SNR higher. The intuition behind this result is that hits become harder on media if tip-media interaction is increased which makes detection easier. The Viterbi detector gives best performance among all detectors because it incorporates the Markovian property of ν in the metric used for detection. At an SNR of 10.4 dB the Viterbi detector has a BER of 3×10^{-6} as against the LMP detector that has 7×10^{-3} .

2.5 Experimental Results

In experiments, a cantilever with resonant frequency $f_0 = 71.78$ KHz and quality factor $Q = 67.55$ is oscillated near its resonant frequency. A freshly cleaved mica sheet is placed on

top of a high bandwidth piezo. This piezo can position the media (mica sheet) in z-direction with respect to cantilever tip. A random sequence of bits is generated through an FPGA board and applied to the z-piezo. High level is equivalent to 1 V and represents bit ‘1’ and low level is 0 V and represents bit ‘0’ thus creating a pseudo media profile of 6 nm height. The bit width can be changed using FPGA controller from 60 – 350 μs . The tip is engaged with the media at a single point and its instantaneous amplitude in response to its interaction with z piezo is monitored. The controller gain is kept sufficiently low such that the operation is effectively in open loop. The gain is sufficient to cancel piezo drift and maintain a certain level of tip-media interaction. An observer is implemented in another FPGA board which is based on the cantilever’s free air model and takes dither and deflection signals as its input and provides innovation signal at the output. The innovation signal is used to detect bits by comparing various bit detection algorithms. The experiments were performed on Multimode AFM, from Veeco Instruments. Considering a bit width of 40 nm and scan time of 60 μs gives a tip velocity equal to $2/3 \times 10^{-3}$ m/sec. The total scan size of the media is 100 micron which means the cantilever will take 0.15 seconds to complete one full scan. Read scan speed for this operation is 6.66 Hz. The read scan speed for different bit widths can be found in a similar manner.

The cantilever model is identified using the frequency sweep method wherein excitation frequency ω of $g(t) = A_0 \sin \omega t$ of dither piezo is varied from 0 – 100 KHz and $p(t)$ is recorded. Magnitude and phase information about $G(i\omega)$ is obtained by evaluating the ratios between steady state amplitude and phase of output vs input excitation respectively. A second order transfer function is obtained that best fits the experimentally identified magnitude and phase responses of the cantilever. A , B and C matrices are obtained from the state space realization of the identified second order transfer function. F , G and H can be further found using the zero order hold discretization at a desired sampling frequency. The discretized state space of the cantilever model is used to find the discretized channel impulse response $\Gamma_{k;\theta}$ (see (2.3)).

For 300 μs bit width, there are around 21 hits in high bit duration and Viterbi decoding is applied on the innovation signal obtained from experiment. For experimental model, I is

approximately 24 which means m_I is equal to 2. It is hard to estimate the system memory (m) from experimental parameters. Fortunately, there is a way around for this. As shown in the derivation of the detector, by making appropriate approximations, the final detector only requires the mean and the covariance of each branch in the trellis. These can be found by using training data and assuming various values of m . We have varied m from 0 to 2 and found the corresponding BER using these values of m . The total number of states in the Viterbi detector is 2^{m+m_I} . We have observed that for $m > 1$, the improvement in BER is quite marginal as compared to the increased complexity of Viterbi decoding. Accordingly we are using $m = 1$ for which the BER from Viterbi decoding is equal to 1×10^{-5} whereas the BER from LMP test is 0.26. The BER in the case of Viterbi decoding is significantly smaller when compared to the BER for usual thresholding detectors. If the bit width is decreased to $60 \mu s$ which means there are around 4 hits in the high bit duration, the BER for Viterbi decoding is 7.56×10^{-2} whereas the BER for LMP is 0.49 which means that LMP is doing almost no bit detection. As the bit width is decreased, there is more ISI between adjacent bits which increases the BER. The BER for different bit widths from all the detectors is shown in Figure 2.8. It can be clearly seen that Viterbi decoding gives remarkable results on experimental data as compared to the LMP detector. The Viterbi detector exploits the cantilever dynamics by modeling the mean and covariance matrix for different state transitions. We have plotted the mean vectors for 2 state transitions with $300 \mu s$ bit width in Figure 2.7. There are around 21 hits in one bit duration. The Viterbi decoding contains 8 states and 16 possible state transitions. In Figure 2.7, there is a clear distinction in mean vectors for different transitions which makes the Viterbi detector quite robust. Thresholding detectors like LMP and GLRT perform very badly on experimental data. For a bit sequence like '000011111', the cantilever gets enough time to go into steady state in the beginning and hits quite hard on media when bit '1' appears after a long sequence of '0' bits. The likelihood ratio for LMP and GLRT rises significantly for such high bits which can be easily detected through thresholding. However, a sequence of continuous '1' bits keeps the cantilever in steady state with the cantilever hitting the media mildly which means the likelihood ratio remains small for these bits. Thus it is very likely that long sequence of '1'

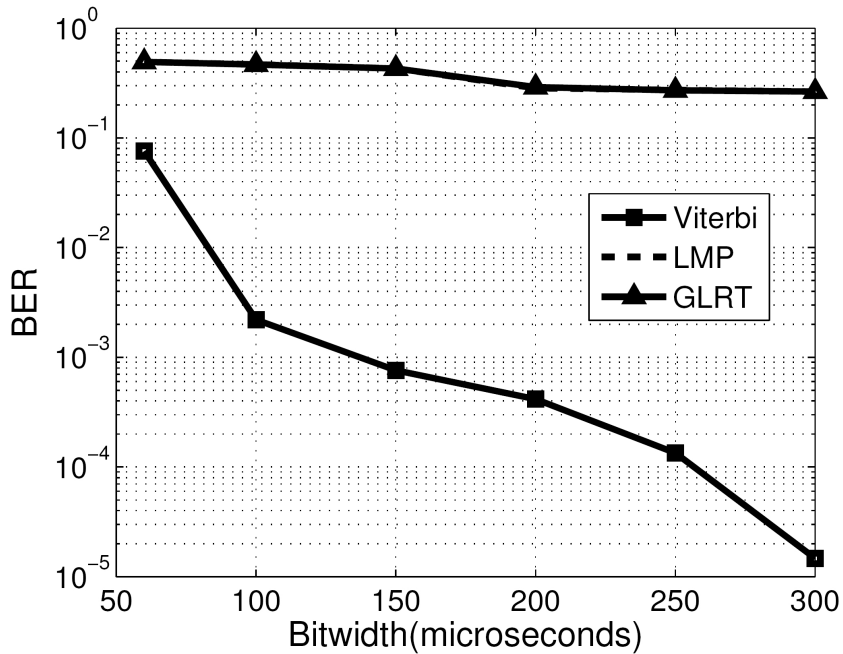


Figure 2.8 BER for Viterbi, LMP and GLRT for different bit widths varying from 60 μs to 300 μs for experimental data. There is a very marginal difference between LMP and GLRT curve which is not visible in the graph but LMP does perform better than GLRT.

bits will not get detected by threshold detectors.

2.6 Conclusions

We presented the dynamic mode operation of a cantilever probe with a high quality factor and demonstrated its applicability to a high-density probe storage system. The system is modeled as a communication system by modeling the cantilever interaction with media. The bit detection problem is solved by posing it as a maximum likelihood sequence detection followed by Viterbi decoding. The main requirements for the proposed algorithm are (a) the availability of training sequences which can provide the statistics for different state transitions, (b) differences between the tip-media interaction magnitude between ‘0’ and ‘1’ bit and (c) an accurate characterization of the linear model of the cantilever in free air. Simulation and experimental results show that the Viterbi detector outperforms LMP, GLRT and Bayes detector and gives remarkably low BER. The work reported in this chapter demonstrates that competitive metrics

can be achieved and enables probe based high density data storage, where high quality factor probes can be used in the dynamic mode operation. Thus, it alleviates the issues of media and tip wear in probe based high density data storage.

CHAPTER 3. High-speed nano-imaging using dynamic mode AFM: A MAP detection approach

3.1 Introduction

The technologies of nano-interrogation and nano-imaging have resulted in major breakthroughs in various fields, including biology, chemistry, physics and medicine. The ability to see things at the level of a few nanometers is critical in many of these domains. This has been made possible through instruments such as atomic force microscopes (AFM), scanning tunneling microscopes (STM) etc. Current nanoimaging technology though useful, suffers from severe speed limitations. Current techniques essentially rule out imaging chemical and biological processes that evolve at time scales that are typically faster than the imaging speed, e.g. movement of kinesins along microtubules. Moreover, in many cases we are interested in fast imaging of samples that are of the order of a few square centimeters or even higher in some applications e.g., fault detection in semiconductor fabrication.

AFM plays a vital role to control, manipulate and interrogate matter at the atomic scale. For example, many applications of AFM cantilevers in biological sciences involve cutting DNA strands [48] and investigating the activity of RNA polymerase [47]. In nano-imaging, the samples, to be imaged, are mostly soft which leads to the use of dynamic mode AFM. In the dynamic mode operation, the cantilever is forced sinusoidally using a dither piezo. The oscillating cantilever gently taps the sample and thus the lateral forces are reduced which decreases the sample wear [44]. The amplitude of the cantilever oscillation changes due to tip-sample interaction. The amplitude of the first harmonic of the cantilever oscillation is obtained from the cantilever deflection signal in amplitude modulation method. This amplitude signal can be used to image samples and referred as amplitude imaging [45]. In [33], the cantilever-

observer architecture is introduced which removes the effect of dither and provides a better way to image samples. The innovation signal is obtained through the output of the cantilever-observer architecture. The root mean square of the innovation signal gives a fast way to image samples and referred as root mean square imaging [50]. But current imaging techniques are too slow to be useful in high fidelity imaging of chemical and biological processes that evolve at fast time scales.

In [56], we have considered the development of a high-density data storage device using dynamic mode AFM. Here, the information is encoded using nanoscale topographic profiles, e.g., ‘1’ - indentation and ‘0’ - no indentation. In order to enable high access speeds, one needs to be able to infer the underlying bit pattern at a fast rate. We have shown that under practically validated modeling assumptions, the entire system can be viewed as a communication system. Moreover, one can map the overall problem of detecting the presence/ absence of the bits as one of Bayesian inference over factor graphs. The bit detection problem requires us to decide whether or not a single topographic feature exists on the sample. Furthermore, in the data storage application one can choose the topographic features and sample material. In contrast, high-fidelity imaging requires us to figure out the height and shape of the underlying sample to within 8-bit precision (for example). Moreover, an imaging technique should be general enough to operate under various sample types. Thus, the nano-imaging problem is significantly richer and more complicated.

In this chapter, we consider one-bit imaging, i.e., we wish to detect the presence/absence of a feature. This will be generalized to multi-bit imaging in future. In one-bit imaging technique, the raster scan is used for imaging which means that the image is subdivided into a sequence of horizontal scan lines and each scan line is imaged using the imaging scanner (AFM system scanner in our case). Each scan line consists of bit-pixels which can be either ‘0’ or ‘1’. This problem of imaging is analogous to the problem of detecting the presence/absence of the bits in probe based data storage system. It allows us to use the channel model developed in [56] for imaging purposes. But it is important to note that Viterbi detector developed in [56] cannot be used for imaging purpose. Viterbi detector assumes equiprobable

prior on the input bit sequence which is not true in imaging scenarios as the images will have non-equiprobable priors on input depending upon the features present in the image. Next, the imaging of chemical and biological processes, that evolve at fast time scales, drives the need for high scan rate imaging. But the feature detection at high scan rates becomes quite challenging as the tip-sample interaction duration for each feature decreases with a increase in the scan rate. Thus, the main challenge in nano-imaging is to develop an imaging technique which can incorporate the non-equiprobable priors in the detector while detecting features on the image and provide good resolution at very high scan speeds.

In this chapter, a one-bit imaging technique using dynamic mode operation with a high quality factor cantilever is presented. The cantilever-based nano-imaging system for the one-bit imaging is modeled as an appropriate communication channel model which incorporates the inherent system memory due to the inter-symbol interference and the cantilever state. We first develop the maximum a posteriori (MAP) symbol detector by adapting the BCJR algorithm [46] for our channel model. Next, we propose an improved MAP symbol detector that incorporates a learned prior from the previous scan line while detecting the features on the current scan line. Experimental results demonstrate that our proposed algorithm provides significantly better image resolution compared to current imaging techniques at high scanning speed.

3.2 Channel model and imaging algorithm

The cantilever based nano-imaging system can be modeled as a communication channel as shown in Figure 3.1 [56]. The components of this model are explained below in detail.

Shaping Filter ($b(t)$): The model takes as input the bit-pixel sequence $(a_0, a_1 \dots a_{N-1})$ from the image sample where $a_k, k = 1, \dots, N - 1$ can be ‘0’ or ‘1’. In the nano-imaging context, ‘0’ refers to the topographic profile being *low* (absence of feature in the image) and ‘1’ refers to the topographic profile being *high* (presence of feature in the image). Each bit-pixel has a duration of T seconds. This duration can be found based on the length of the topographic feature present in image sample and the speed of the image scanner. The height of the high

bit-pixel is denoted by A . The cantilever interacts with the sample by gently tapping it when it is high. When the sample is low, typically no interaction takes place. We model the effect of the image sample height using a filter with impulse response $b(t)$ (shown in Figure 3.1) that takes as input, the input bit-pixel impulse train $a(t) = \sum_{k=0}^{N-1} a_k \delta(t - kT)$. The output of the filter is given by $\check{a}(t) = \sum_{k=0}^{N-1} a_k b(t - kT)$ which is high/low when the corresponding bit is ‘1’/‘0’.

Nonlinearity Block (ϕ): The nonlinearity block models the tip-sample interaction forces. The nonlinearity block output is modeled as a sequence of impulsive force inputs to the cantilever [56]. The cantilever oscillates at frequency f_c which means that in each cantilever cycle of duration $T_c (= 1/f_c)$, the cantilever hits the sample at most once if the sample is high during a time T_c . In each high bit duration T , the cantilever hits the sample q times (i.e. $T = qT_c$) with varying magnitudes. Therefore, for N bits, the output of the nonlinearity block is given by, $\tilde{a}(t) = \sum_{k=0}^{Nq-1} \nu_k(\bar{a}) \delta(t - kT_c)$, where ν_k denotes the magnitude of the k^{th} impact of the cantilever on the sample. The strength of the impulsive force inputs are dependent on previous impulsive hits; precisely because the previous interactions affect the amplitude of the oscillations that in turn affect how hard the hit is at a particular instant of time. A Markov model on the sequence of impact magnitudes for a single bit-pixel duration is given by [56],

$$\bar{\nu}_k = \bar{\mathfrak{G}}(a_k, a_{k-1}, \dots, a_{k-m}) + \bar{\mathfrak{b}}_k \quad (3.1)$$

where $\bar{\mathfrak{G}}(a_k, a_{k-1}, \dots, a_{k-m})$ is a function of current and last m bits, m denotes the inherent memory of the system, $\bar{\nu}_k = [\nu_{kq} \ \nu_{kq+1} \dots \nu_{(k+1)q-1}]^T$ and $\bar{\mathfrak{b}}_k$ is a zero mean i.i.d. Gaussian vector of length q .

Channel Response ($\Gamma(t)$): $\Gamma(t)$ is the impulse response of the cantilever-observer system described in [56]. The cantilever-observer system provides a better way of real-time imaging the samples with scan speed quite faster than conventional methods [50].

Channel Noise ($n(t)$): The measurement noise (from the imprecision in measuring the cantilever position) and thermal noise (from modeling mismatches) can be modeled by a single zero mean white Gaussian noise process ($n(t)$) with power spectral density equal to V .

The continuous output $e(t)$ of the channel model named as innovation signal is,

$$e(t) = \sum_{k=0}^{Nq-1} \nu_k(\bar{a}) \Gamma(t - kT_c) + n(t) = s(t, \bar{\nu}(\bar{a})) + n(t),$$

where $s(t, \bar{\nu}(\bar{a})) = \sum_{k=0}^{Nq-1} \nu_k(\bar{a}) \Gamma(t - kT_c)$ and $\bar{\nu}(\bar{a}) = (\nu_0(\bar{a}), \nu_1(\bar{a}) \dots \nu_{Nq-1}(\bar{a}))$. The sequence of impact values $\bar{\nu}_k$ is assumed to follow a Markovian model as explained above, $\Gamma(t)$ is the channel impulse response and $n(t)$ is a zero mean white Gaussian noise process.

3.2.1 Discretized Channel Model

It can be shown that the output of whitened matched filter shown in Figure 3.1 provides the sufficient statistics for channel model [56]. We shall denote the discretized output of whitened matched filter as z_k , such that $z_k = \sum_{k_1=0}^I \nu_{k-k_1}(\bar{a}) h_{k_1} + n_k$, where the filter $\{h_k\}_{k=0,1,\dots,I}$ denotes the effect of the whitened matched filter and the sequence $\{n_k\}$ represents Gaussian noise with variance V [56].

Let $\bar{z}_k$ be the received output vector corresponding to the k^{th} input bit-pixel, i.e., $\bar{z}_k = [z_{kq} \ z_{kq+1} \dots z_{(k+1)q-1}]^T$ and $\bar{z}_0^{k-1} = [\bar{z}_0^T \ \bar{z}_1^T \dots \bar{z}_{k-1}^T]^T$. In our model, the channel is characterized by a finite impulse response of length I i.e. $h_k = 0$ for $k < 0$ and $k > I$. In this work we assume that $I \leq m_I q$ i.e. the inter-symbol-interference (ISI) length in terms of q hits is equal to m_I . Let m be the inherent memory of the system (see (3.1)). The length of channel response m_I is known and the value of m can be found by the method described in [56]. The received output vector $\bar{z}_k$ can now be written as,

$$\begin{aligned} \bar{z}_k &= \begin{pmatrix} h_I & . & . & h_0 & 0 & . & . & 0 \\ 0 & h_I & . & . & h_0 & 0 & . & 0 \\ . & . & . & . & . & . & . & . \\ 0 & . & . & 0 & h_I & . & . & h_0 \end{pmatrix} \begin{pmatrix} \nu_{kq-I} \\ \nu_{kq-I+1} \\ \vdots \\ \nu_{(k+1)q-1} \end{pmatrix} + \bar{n}_k \\ &= H \bar{\nu}_{k-m_I}^k + \bar{n}_k \end{aligned}$$

where $\bar{\nu}_k = [\nu_{kq} \ \nu_{kq+1} \dots \nu_{(k+1)q-1}]^T$, $\bar{\nu}_{k-m_I}^k = [\bar{\nu}_{k-m_I}^T \dots \bar{\nu}_k^T]^T$, $\bar{n}_k = [n_{kq} \ n_{kq+1} \dots n_{(k+1)q-1}]^T$.

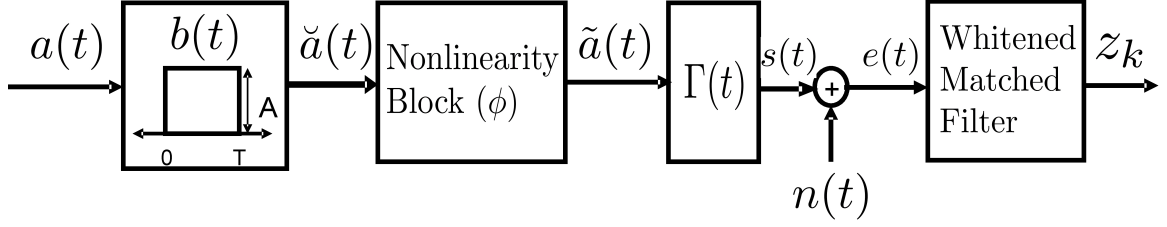


Figure 3.1 Discretized channel model with whitened matched filter

In [56], the dependency graph on the concerned quantities is constructed and $f(\bar{z}_k|\bar{a}, \bar{z}_0^{k-1})$ is simplified as,

$$f(\bar{z}_k|\bar{a}, \bar{z}_0^{k-1}) = f(\bar{z}_k|a_{k-m-m_I}^k, \bar{z}_{k-m_I}^{k-1}). \quad (3.2)$$

where $f()$ denotes the p.d.f, m is the inherent system memory and m_I is the inter-symbol-interference (ISI) length in terms of bits. By defining a state $S_k = a_{k-m-m_I+1}^k$, this can be further expressed as $f(\bar{z}_k|S_k, S_{k-1}, \bar{z}_{k-m_I}^{k-1})$.

3.2.2 BCJR Algorithm

The a posteriori probability (APP) of a symbol, a_k is defined as, $APP(a_k) = f(a_k|\bar{z})$. In MAP symbol detection, the symbol a_k is found which maximizes the $APP(a_k)$. We now derive MAP symbol detector for our channel model. The symbols ζ' , ζ and ζ'' denote the past, current and future states respectively where the state at k^{th} time instant is defined as $S_k = a_{k-m-m_I+1}^k$. In BCJR algorithm, $f(a_k, \bar{z})$ is computed instead of $f(a_k|\bar{z})$,

$$\begin{aligned} f(a_k, \bar{z}) &= \sum_{\zeta'} f(S_{k-1} = \zeta', a_k, \bar{z}) \\ &= \sum_{\zeta'} f(S_{k-1} = \zeta', S_k = \zeta, \bar{z}) \\ &= \sum_{\zeta'} f(S_{k-1} = \zeta', \bar{z}_0^{k-1}) f(S_k = \zeta, \bar{z}_k^{N-1} | S_{k-1} = \zeta', \bar{z}_0^{k-1}) \end{aligned}$$

$$\begin{aligned}
&= \sum_{\zeta'} f(S_{k-1} = \zeta', \bar{z}_0^{k-1}) f(S_k = \zeta, \bar{z}_k | S_{k-1} = \zeta', \bar{z}_0^{k-1}) \\
&\quad \cdot f(\bar{z}_{k+1}^{N-1} | S_k = \zeta, S_{k-1} = \zeta', \bar{z}_0^k) \\
&= \sum_{\zeta'} f(S_{k-1} = \zeta', \bar{z}_0^{k-1}) f(\bar{z}_k | S_k = \zeta, S_{k-1} = \zeta', \bar{z}_{k-m_I}^{k-1}) \\
&\quad \cdot f(S_k = \zeta | S_{k-1} = \zeta', \bar{z}_0^{k-1}) f(\bar{z}_{k+1}^{N-1} | S_k = \zeta, \bar{z}_{k-m_I+1}^k) \text{(Using (3.2))} \\
&= \sum_{\zeta'} f(S_{k-1} = \zeta', \bar{z}_0^{k-1}) f(\bar{z}_k | S_k = \zeta, S_{k-1} = \zeta', \bar{z}_{k-m_I}^{k-1}) \\
&\quad \cdot f(S_k = \zeta | S_{k-1} = \zeta') f(\bar{z}_{k+1}^{N-1} | S_k = \zeta, \bar{z}_{k-m_I+1}^k) \\
&= \sum_{\zeta'} f(S_{k-1} = \zeta', \bar{z}_0^{k-1}) f(\bar{z}_k | S_k = \zeta, S_{k-1} = \zeta', \bar{z}_{k-m_I}^{k-1}) \\
&\quad \cdot P(a_k) f(\bar{z}_{k+1}^{N-1} | S_k = \zeta, \bar{z}_{k-m_I+1}^k) \\
&= \sum_{\zeta'} \alpha_{k-1}(\zeta') \cdot \gamma_k(\zeta', \zeta) \cdot \beta_k(\zeta)
\end{aligned}$$

where $\alpha_{k-1}(\zeta') = f(S_{k-1} = \zeta', \bar{z}_0^{k-1})$, $\gamma_k(\zeta', \zeta) = f(\bar{z}_k | S_k = \zeta, S_{k-1} = \zeta', \bar{z}_{k-m_I}^{k-1}) P(a_k)$ and $\beta_k(\zeta) = f(\bar{z}_{k+1}^{N-1} | S_k = \zeta, \bar{z}_{k-m_I+1}^k)$.

The recursion equations on $\alpha_k(\zeta)$ and $\beta_k(\zeta)$ can be derived as follows,

$$\begin{aligned}
\alpha_k(\zeta) &= f(S_k = \zeta, \bar{z}_0^k) \\
&= \sum_{\zeta'} f(S_{k-1} = \zeta', S_k = \zeta, \bar{z}_0^k) \\
&= \sum_{\zeta'} f(S_{k-1} = \zeta', \bar{z}_0^{k-1}) f(S_k = \zeta, \bar{z}_k | S_{k-1} = \zeta', \bar{z}_0^{k-1}) \\
&= \sum_{\zeta'} f(S_{k-1} = \zeta', \bar{z}_0^{k-1}) f(S_k = \zeta, \bar{z}_k | S_{k-1} = \zeta', \bar{z}_{k-m_I}^{k-1}) \text{(Using (3.2))} \\
&= \sum_{\zeta'} \alpha_{k-1}(\zeta') \cdot \gamma_k(\zeta', \zeta)
\end{aligned}$$

$$\begin{aligned}
\beta_k(\zeta) &= f(\bar{z}_{k+1}^{N-1} | S_k = \zeta, \bar{z}_{k-m_I+1}^k) \\
&= \sum_{\zeta''} f(S_{k+1} = \zeta'', \bar{z}_{k+1}^{N-1} | S_k = \zeta, \bar{z}_{k-m_I+1}^k) \\
&= \sum_{\zeta''} f(\bar{z}_{k+2}^{N-1} | S_{k+1} = \zeta'', \bar{z}_{k-m_I+1}^{k+1}) f(S_{k+1} = \zeta'', \bar{z}_{k+1} | S_k = \zeta, \bar{z}_{k-m_I+1}^k) \\
&= \sum_{\zeta''} f(\bar{z}_{k+2}^{N-1} | S_{k+1} = \zeta'', \bar{z}_{k-m_I+2}^{k+1}) f(S_{k+1} = \zeta'', \bar{z}_{k+1} | S_k = \zeta, \bar{z}_{k-m_I+1}^k) \text{ (Using (3.2))} \\
&= \sum_{\zeta''} \beta_{k+1}(\zeta'') \cdot \gamma_{k+1}(\zeta, \zeta'')
\end{aligned}$$

It should be noted that the pair $(S_{k-1} = \zeta', a_k)$ completely determines state S_k . The variables $\alpha_k(\zeta)$ and $\beta_k(\zeta)$ has recursive structure,

$$\begin{aligned}
\alpha_k(\zeta) &= \sum_{\zeta'} \alpha_{k-1}(\zeta') \cdot \gamma_k(\zeta', \zeta) \\
\beta_k(\zeta) &= \sum_{\zeta''} \beta_{k+1}(\zeta'') \cdot \gamma_{k+1}(\zeta, \zeta'')
\end{aligned}$$

Typical Initialization for $\alpha_k(\zeta)$ and $\beta_k(\zeta)$ variables is,

$$\alpha_{-1}(\zeta) = \begin{cases} 1, & \text{if } \zeta = 0 \\ 0, & \text{if } \zeta \neq 0 \end{cases} \quad \text{and} \quad \beta_{N-1}(\zeta) = \begin{cases} 1, & \text{if } \zeta = 0 \\ 0, & \text{if } \zeta \neq 0 \end{cases}$$

In implementation, the forward and backward recursions are done in log domain. The recursion equations in log domain are as follows,

$$\begin{aligned}
\log(\alpha_k(\zeta)) &= \log \sum_{\zeta'} e^{\log(\alpha_{k-1}(\zeta')) + \log(\gamma_k(\zeta', \zeta))} \\
\log(\beta_k(\zeta)) &= \log \sum_{\zeta''} e^{\log(\beta_{k+1}(\zeta'')) + \log(\gamma_{k+1}(\zeta, \zeta''))} \\
\log(\gamma_k(\zeta', \zeta)) &= \log(f(\bar{z}_k | S_{k-1} = \zeta', S_k = \zeta, \bar{z}_{k-\zeta_I}^{k-1})) + \log(P(a_k))
\end{aligned}$$

In above equation, the term $f(\bar{z}_k | S_{k-1} = \zeta', S_k = \zeta, \bar{z}_{k-\zeta_I}^{k-1})$ can be computed by making an assumption that $f(\bar{z}_{k-m_I}^k | S_k, S_{k-1})$ is Gaussian distributed [56], i.e.,

$$f(\bar{z}_{k-m_I}^k | S_k = \zeta, S_{k-1} = \zeta') \sim N(\bar{\mathcal{Y}}(\zeta, \zeta'), \mathcal{C}(\zeta, \zeta')),$$

where $\bar{\mathcal{Y}}(\zeta, \zeta')$ is the mean vector and $\mathcal{C}(\zeta, \zeta')$ is the covariance matrix. Now the term $\log(f(\bar{z}_k | S_{k-1} = \zeta', S_k = \zeta, \bar{z}_{k-m_I}^{k-1}))$ can be computed as [59],

$$\begin{aligned} \log(f(\bar{z}_k | S_k = \zeta, S_{k-1} = \zeta', \bar{z}_{k-m_I}^{k-1})) &= \log\left(\frac{f(\bar{z}_{k-m_I}^k | S_k = \zeta, S_{k-1} = \zeta')}{f(\bar{z}_{k-m_I}^{k-1} | S_k = \zeta, S_{k-1} = \zeta')}\right) \\ &= -\frac{1}{2} \log \frac{|\mathcal{C}(\zeta, \zeta')|}{|c(\zeta, \zeta')|} - \frac{1}{2} (\bar{z}_{k-m_I}^k - \bar{\mathcal{Y}}(\zeta, \zeta'))^T \\ &\quad \cdot \mathcal{C}(\zeta, \zeta')^{-1} (\bar{z}_{k-m_I}^k - \bar{\mathcal{Y}}(\zeta, \zeta')) + \frac{1}{2} (\bar{z}_{k-m_I}^{k-1} \\ &\quad - \bar{\mathcal{Y}}(\zeta, \zeta'))^T c(\zeta, \zeta')^{-1} (\bar{z}_{k-m_I}^{k-1} - \bar{\mathcal{Y}}(\zeta, \zeta')) \end{aligned}$$

where $c(\zeta, \zeta')$ is the upper $m_I q \times m_I q$ principal minor of $\mathcal{C}(\zeta, \zeta')$ and $\bar{\mathcal{Y}}(\zeta, \zeta')$ collects the first $m_I q$ elements of $\bar{\mathcal{Y}}(\zeta, \zeta')$. Using above results, $\log(f(a_k, \bar{z}))$ can be computed as,

$$\log(f(a_k, \bar{z})) = \log \sum_{\zeta'} e^{\log(\alpha_{k-1}(\zeta')) + \log(\gamma_k(\zeta', \zeta)) + \log(\beta_k(\zeta))}$$

The log-likelihood ratio can be computed using the above results,

$$\begin{aligned} L(a_k) &= \log \frac{f(a_k = 0 | \bar{z})}{f(a_k = 1 | \bar{z})} \\ &= \log(f(a_k = 0, \bar{z})) - \log(f(a_k = 1, \bar{z})) \end{aligned}$$

The forward and backward recursions given above can be used to find $L(a_k)$. The decision rule for finding the bit is given by, $L(a_k) \stackrel{1}{\leq} 0$. It is important to note here that BCJR algorithm developed for our channel model is different from conventional BCJR algorithm. Unlike the conventional BCJR algorithm, the recursive variables $\beta_k(\zeta)$ and $\gamma_k(\zeta', \zeta)$ contain the pdf terms which are dependent on $\bar{z}_0^k$ and $\bar{z}_0^{k-1}$ respectively.

3.2.3 Imaging Algorithm

The raster scan is used for imaging which means that the image is subdivided into a sequence of horizontal scan lines. The features in the images are assumed to be spatially continuous which means that the image does not change much between two consecutive scan lines. This information can be incorporated in the BCJR algorithm to provide better imaging. The advantages of using BCJR algorithm are twofold; the first is that the log-likelihood ratios obtained from decoding one scan line can directly give probabilistic prior on each symbol in

Algorithm 1 : BCJR Imaging Algorithm.

1. Decode the first scan line with the equiprobable prior on the input and obtain the probabilistic prior on input symbol for the first scan line.
 2. Decode the second scan line using the probabilistic prior on input symbol from first scan line and again obtain the probabilistic prior on input symbol for the second scan line.
 3. Decode the third scan line using the prior from second line and keep decoding the subsequent scan lines in a similar manner until the whole binary image is reconstructed.
-

the scan line and the second is that this probabilistic prior on each symbol can be used in computing the term $\gamma_k(\zeta', \zeta)$ while decoding the next scan line. It should be noted that the current scan line can be decoded using the probabilistic prior obtained from previous scan line. But in the start of imaging, no information about the first scan line is available which forces us to use the equiprobable prior on input in decoding the first scan line. Using the above ideas, the BCJR imaging algorithm is summarized in Algorithm 1.

The current state of art techniques include a) Amplitude imaging - this imaging is done through envelope of deflection signal [45] b) Root mean square imaging - the root mean square of the innovation signal is used for this imaging [50] and (c) LMP imaging - locally most powerful test is used on the innovation signal in this case to reconstruct the binary image [49].

3.3 Experimental Results

We performed experiments for one-bit imaging with a cantilever with resonant frequency $f_0 = 74.73$ KHz and quality factor $Q = 140.68$. A freshly cleaved mica sheet is used as an imaging sample. The experiments were performed on Multimode AFM, from Veeco Instruments. We have performed the imaging at different scan rate. One pixel in the image corresponds to high and low topographic profile. We considered the test patten shown in Figure 3.2 (a), that is of dimension $2.7 \mu\text{m} \times 10 \mu\text{m}$. In first set of experiments, we took the scan rate such that each pixel remains high or low for $120 \mu\text{sec}$ which means cantilever will hit around 8 times if the topographic profile is high. Each line scan of size $20 \mu\text{m}$ will get done in $\frac{1}{74.73 \times 10^3} \times 8 \times 512 \times 2$ seconds. The scan rate in this case will be $20/8/512/2 \times 74.73 \times 10^3 = 182.44 \mu\text{m}/\text{sec}$. In Fig-

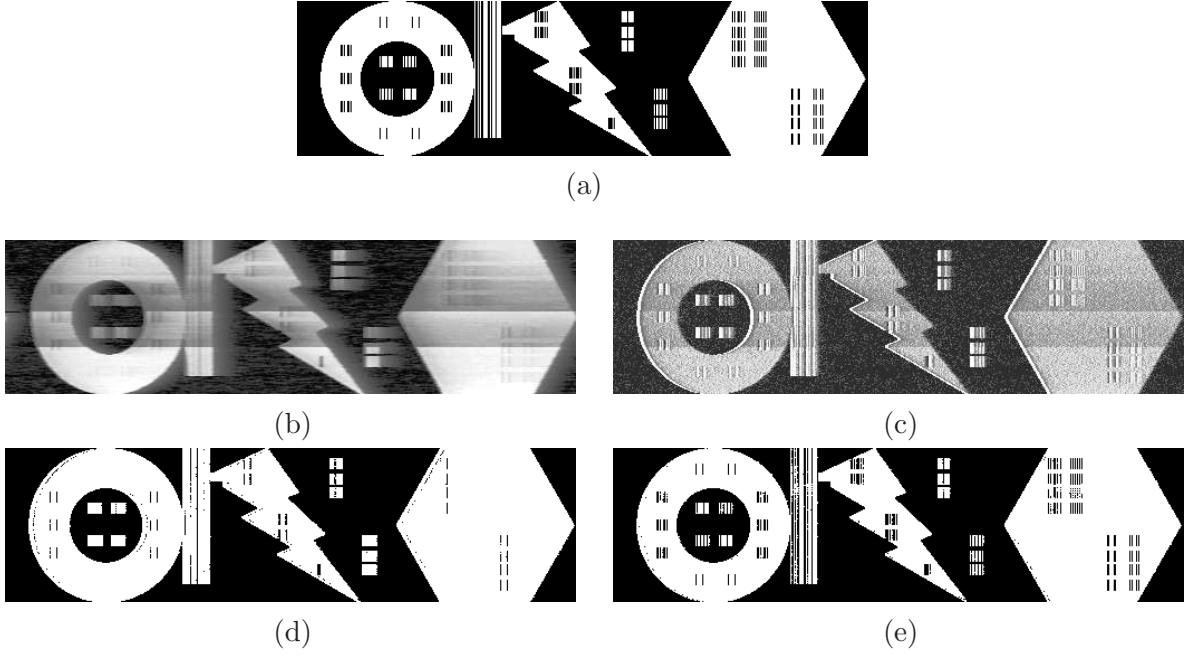


Figure 3.2 Comparison of imaging techniques at a scan rate of $182.44 \mu\text{m/sec}$. (a) Reference test pattern, (b) Amplitude imaging (c) Root mean square imaging (d) LMP imaging (e) BCJR Imaging

Figure 3.2, the performance of current state of the art techniques is compared with our proposed technique at $182.44 \mu\text{m/sec}$ imaging speed. In order to see the improvement provided by our method, we have zoomed in the images and shown the spatial features in Figure 3.3. In Figure 3.3, it is clearly seen that our algorithm cleanly resolves all the spatial features present in the image whereas other imaging techniques cannot. It is evident that the proposed technique has much better fidelity providing better resolution even at high scan speeds.

In another set of experiments, we took the scan rate such that each pixel remains high or low for $80 \mu\text{sec}$ which means cantilever will hit around 5 times if the topographic profile is high. In this case, the scan rate is given by $20/5/512/2 \times 74.73 \times 10^3 = 291.91 \mu\text{m/sec}$. The images at $291.91 \mu\text{m/sec}$ scan rate are shown in Figure 3.4. The zoomed spatial features at this scan rate are shown in Figure 3.5. Even in this case, our proposed techniques cleanly resolves all the features whereas other imaging techniques cannot. We have even done the experiments at very high scan rate $729.78 \mu\text{m/sec}$. The images at $729.78 \mu\text{m/sec}$ scan rate are shown in

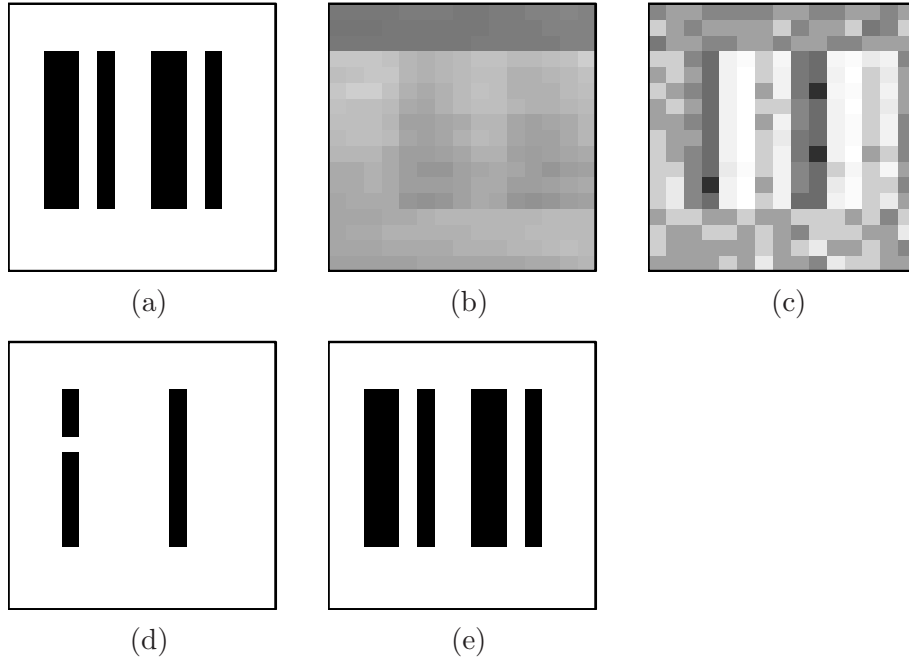


Figure 3.3 Comparison of the feature resolution provided by different techniques at scan rate of $182.44 \mu\text{m}/\text{sec}$. A zoomed image is provided for facilitating visual comparison. (a) Reference test pattern, (b) Amplitude imaging (c) Root mean square imaging (d) LMP imaging (e) BCJR Imaging

Figure 3.6. The zoomed spatial features at this scan rate are shown in Figure 3.7. It can be observed that our proposed techniques is not able to resolve the high spatial frequency features in the image but it does resolve low spatial frequency features whereas other imaging methods again completely fail. In nutshell, our proposed technique outperforms all the current state of art imaging techniques.

Another interesting aspect in the imaging which we have researched is that how the presence of noise in output affects different imaging techniques. It should be noted that the AFM system used here is based on beam-bounce method where a laser is incident on the back of the cantilever surface and the laser is reflected from the cantilever surface into a split photodiode. The advantage of the beam-bounce method is that it gives low measurement noise. If another kind of actuation and sensing mechanisms are used, it will lead to increase in thermal and measurement noise. We have simulated this case by adding white noise to experimental data and observed that current state of art imaging techniques are not even able to image low

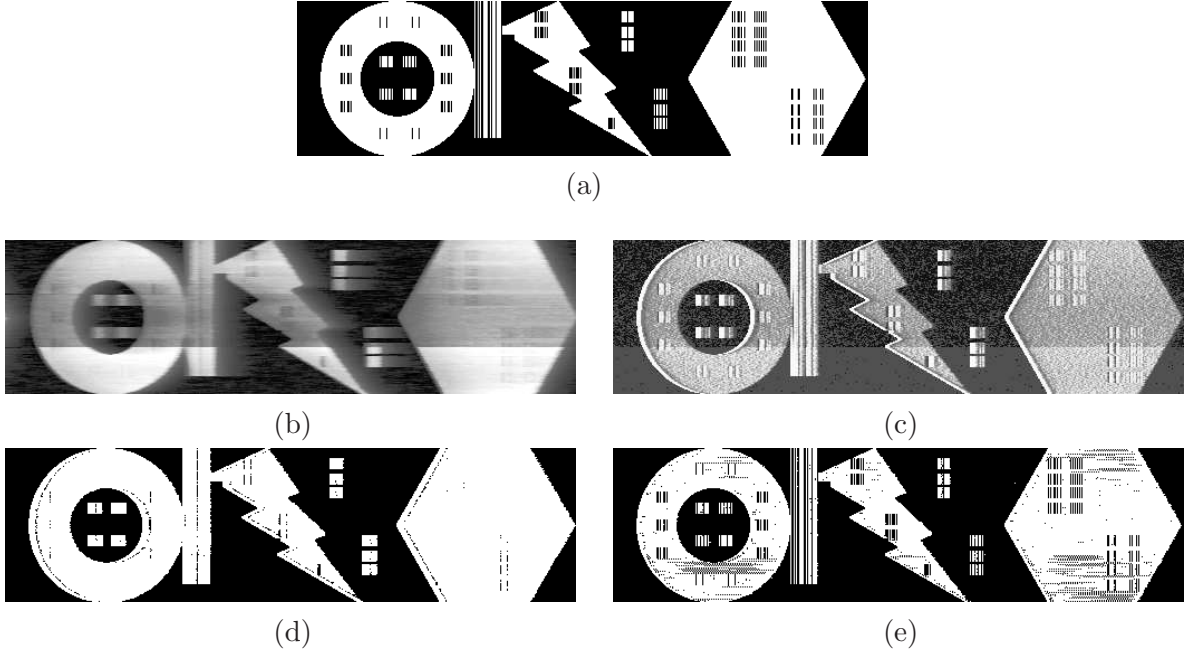


Figure 3.4 Comparison of imaging techniques at a scan rate of $291.91 \mu\text{m/sec}$. (a) Reference test pattern, (b) Amplitude imaging (c) Root mean square imaging (d) LMP imaging (e) BCJR Imaging

frequency spatial features like sequence of ones or zeros whereas our proposed algorithm still gives good imaging performance. The intuition behind this result is that amplitude and root mean square imaging methods are unable to filter out the noise and gives bad performance. The LMP imaging filters out the noise but fails due to thresholding whereas our proposed method works as it is based on the difference between mean vectors and covariance matrix of the state transitions obtained from training [56].

Video Rate Imaging

We have done an experiment in which a phase change is observed on the topographic sample over time. Consider a case when all the pixels in the image are high or low in equiprobable manner. Over the time, the pixels in the image changes from low to high and high to low. The probability of a pixel going from high to low is very less as compared to the probability of a pixel going from low to high. If we observe this phenomenon over the time, image will be

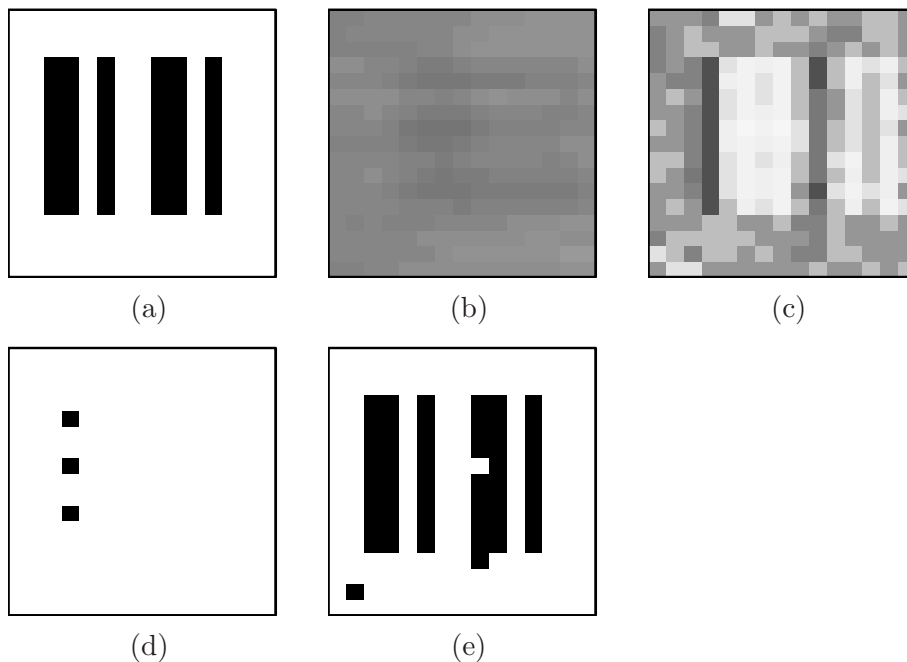


Figure 3.5 Comparison of the feature resolution provided by different techniques at scan rate of $291.91 \mu\text{m}/\text{sec}$. A zoomed image is provided for facilitating visual comparison. (a) Reference test pattern, (b) Amplitude imaging (c) Root mean square imaging (d) LMP imaging (e) BCJR Imaging

having more number of pixels turning into ‘1’ as time goes on. This kind of phase change is studied in the experiment. We performed the experiment in which the input image changes in this manner over time. We used BCJR imaging algorithm to reconstruct the frames over the time at an imaging speed of $182.44 \mu\text{m}/\text{sec}$. In Figure 3.8, it can be easily seen that our proposed techniques clearly shows how the phase is changed over the time in different frames. Current imaging techniques again completely fail in this case. Such video rate imaging is very useful in studying the evolution of a biological process over time.

3.4 Conclusions

We have presented the channel model for the cantilever based nano-imaging system. We have developed the maximum a posteriori (MAP) symbol detector which does take into account the image prior while detecting the features on the image. The MAP symbol detection problem is solved using the BCJR algorithm. We proposed the BCJR imaging algorithm for the raster

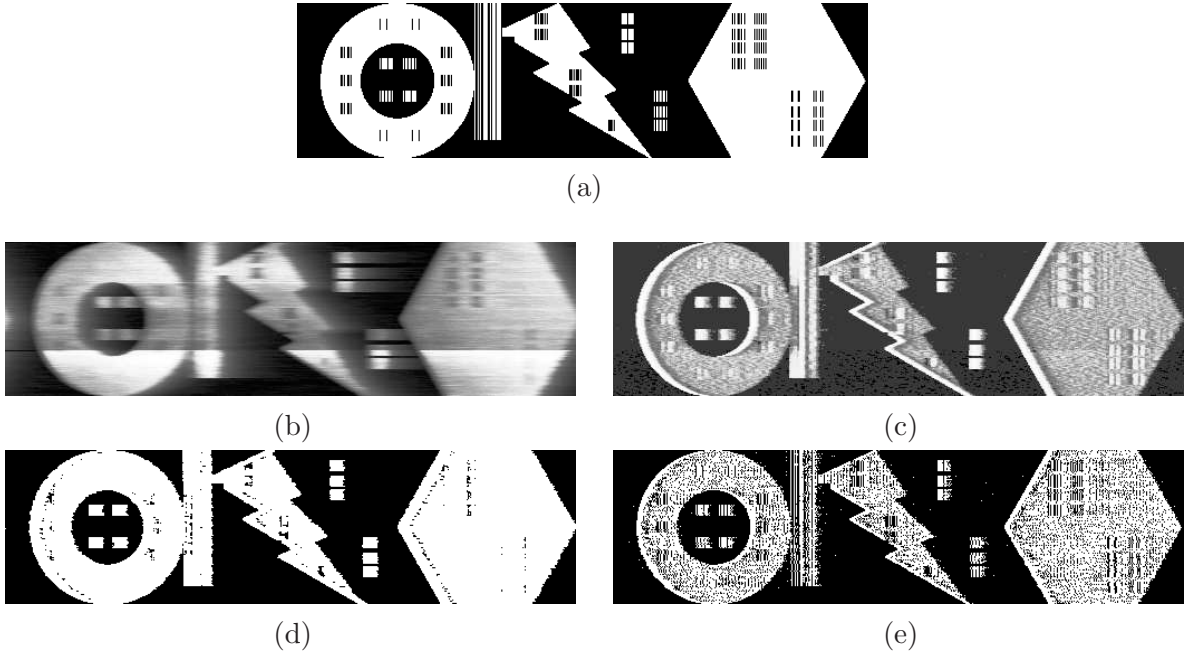


Figure 3.6 Comparison of imaging techniques at a scan rate of $729.78 \mu\text{m}/\text{sec}$. (a) Reference test pattern, (b) Amplitude imaging (c) Root mean square imaging (d) LMP imaging (e) BCJR Imaging

scan imaging which incorporates the input prior from previous line scan while detecting the features on the current line scan. Experimental results corroborate the analysis of the detector, demonstrate that our proposed algorithm does provide better image resolution compared to current imaging techniques at high scanning speed. In future, these techniques will enable video rate imaging of molecular scale phenomenon. This will address the issue of being able to visualize and understand dynamics at the nano-scale.

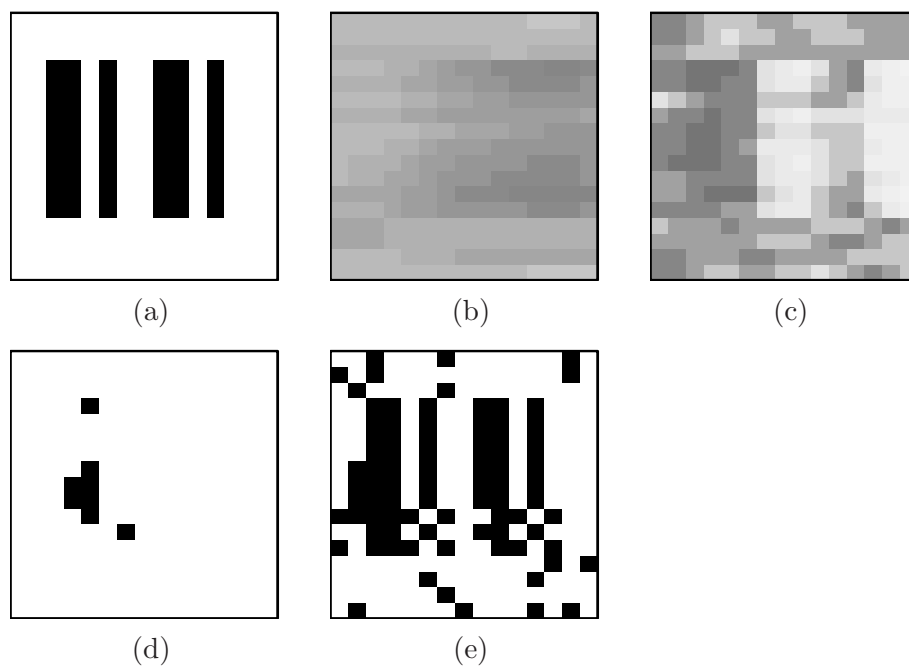


Figure 3.7 Comparison of the feature resolution provided by different techniques at scan rate of $729.78 \mu\text{m}/\text{sec}$. A zoomed image is provided for facilitating visual comparison. (a) Reference test pattern, (b) Amplitude imaging (c) Root mean square imaging (d) LMP imaging (e) BCJR Imaging

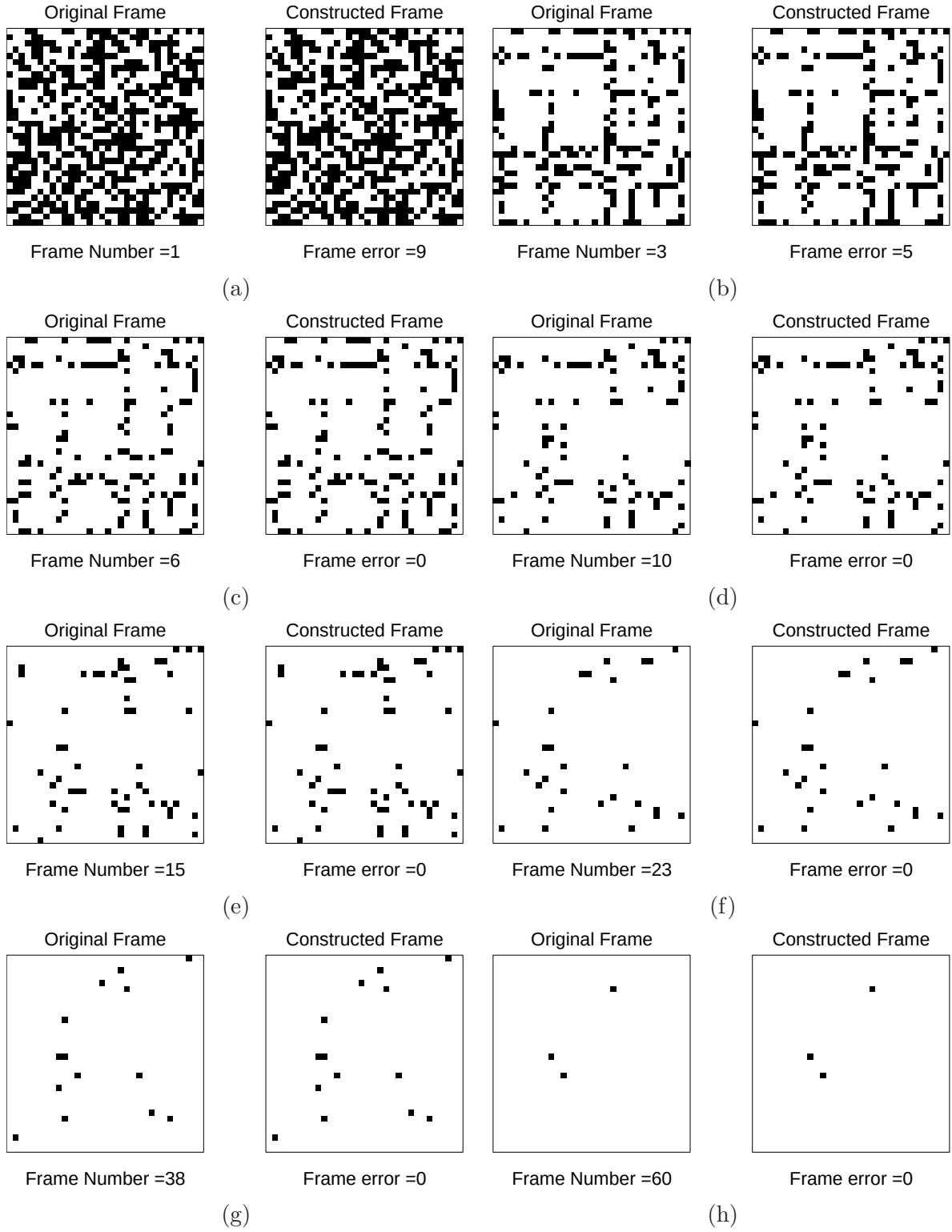


Figure 3.8 Figures (a) - (h) show the original image and the reconstructed image using the BCJR approach for various frames at an imaging speed of $182.44 \mu\text{m}/\text{sec}$.

CHAPTER 4. Performance evaluation for ML sequence detection in ISI channels with Gauss Markov Noise

4.1 Introduction

Maximum likelihood sequence detection (MLSD) in channels with inter-symbol-interference and data dependent time-correlated noise is an important problem in many domains. For example, in magnetic recording, the statistics of percolation and nonlinear effects between transitions [65, 54] result in noise that exhibits data-dependent time-correlation. Recently, similar noise models for nanotechnology based probe storage have also been developed and the corresponding detectors have been found to have significantly improved performance compared to the current state of the art [56]. It is well-recognized that a sequence detector designed for an AWGN ISI model can have a significant loss of performance if the data dependence and time-correlation of the noise is not taken into account.

In the case of finite ISI channels with memoryless noise, Forney [20, 52] presented an MLSD solution based on the Viterbi algorithm. Upper bounds on the error probability of the detector can be derived based on flowgraph techniques [51, 61, 63]. The work of [10, 55], considered channels with finite ISI and noise modeled by a finite memory Gauss-Markov process. In [10], certain approaches (see section V in [10]) are presented for computing an upper bound on the performance of the detector. Another work of [64, 60], also presents the performance analysis of the MLSD in presence of data dependent noise. However, their technique is not based on flowgraph techniques, and requires an enumeration of all error events of relevant lengths and an estimate of the corresponding pairwise error probability upper bound. We emphasize that an analytical technique for estimating detector performance is of great value since it allows us to predict the performance at high SNR's where simulation can be time-consuming.

In an ISI channel with additive white Gaussian noise (AWGN), the upper bound on the pairwise error probability (PEP) between two state sequences can be easily factorized as a product of functions depending on current and previous states in the (incorrect) decoded sequence and the (correct) transmitted sequence. Let $\bar{S}$ and $\hat{S}$ be the transmitted and decoded state sequences respectively. Then this means that the probability that the detector prefers $\hat{S}$ to $\bar{S}$, is denoted by $P(\hat{S}|\bar{S}) \leq \prod_{k=0}^{N-1} h(\hat{S}_{k-1}^k, S_{k-1}^k)$ where h is a function of current state and previous decoded states $\hat{S}_{k-1}^k = (\hat{S}_{k-1}, \hat{S}_k)$ and actual states $S_{k-1}^k = (S_{k-1}, S_k)$. Moreover, the PEP is symmetric due to the symmetric nature of white Gaussian noise, i.e., $P(\hat{S}|\bar{S}) = P(\bar{S}|\hat{S})$. Together, these properties allow the application of the error state diagram method for finding an upper bound on the BER [63].

In contrast, for the ISI channel with data-dependent Gauss-Markov noise (considered in [10]), neither of these properties hold. The signal dependent and time-correlated noise makes the PEP asymmetric. Further the PEP does not factorize in a suitable manner as required for the application of flowgraph techniques. This makes the estimation of BER for such channels, quite challenging.

In this chapter, we consider a subset of the class of ISI channels with Gauss-Markov noise. For these channels, we arrive at an upper bound to the PEP that can be expressed as a product of functions depending on current and previous states in the (incorrect) decoded sequence and the (correct) transmitted sequence. The asymmetric character of the PEP, i.e., the fact that $P(\hat{S}|\bar{S}) \neq P(\bar{S}|\hat{S})$ necessitates an average over all correct and erroneous state sequences. We show that this can be achieved using the concept of the “pairwise state diagram” [51]. Based on this, we present an analytical technique for determining an upper bound on the BER. Simulations results show that our proposed bound is tight in the high SNR regime.

4.2 Channel model and Viterbi detector

We introduce the channel model and the corresponding detector in this section. A word about notation. In what follows, if z_k is a discrete-time indexed sequence at k^{th} time instant, the column vector of sequence samples from time instant k_1 to k_2 is denoted by $z_{k_1}^{k_2} = [z_{k_1} \dots z_{k_2}]^T$

where $k_1 \leq k_2$. We will use the notation $f(\cdot|\cdot)$ to denote a conditional pdf. The precise pdf under consideration will be evident from the context of the discussion.

4.2.1 Channel Model

Let a_k denote the k^{th} source bit that is equally likely to be 0 or 1. The channel output shown in Figure 4.1 with intersymbol interference (ISI) of length I is given by,

$$z_k = y(a_{k-I}^k) + n_k,$$

where $y(a_{k-I}^k)$ is the noiseless channel output dependent only on the $I + 1$ past transmitted bits. The noise n_k is modeled as a signal dependent Gauss-Markov noise process with memory length L as explained below.

$$n_k = \bar{b}^T n_{k-L}^{k-1} + \sigma(a_{k-I}^k) w_k,$$

where the vector $\bar{b}$ represents L coefficients of an autoregressive filter, $\sigma(a_{k-I}^k)$ is signal dependent parameters and w_k is a zero mean unit variance i.i.d Gaussian random variable. Note that in the most general model (considered in [10]), even the autoregressive filter $\bar{b}$ would depend on the data sequence $\bar{a}$. However, in this work, we only work with models where $\bar{b}$ is fixed. We revisit this point in Section 4.3. The noise n_k can be rewritten as,

$$n_k = \bar{b}^T \begin{pmatrix} z_{k-L} - y(a_{k-L-I}^{k-L}) \\ \cdot \\ \cdot \\ \cdot \\ z_{k-1} - y(a_{k-1-I}^{k-1}) \end{pmatrix} + \sigma(a_{k-I}^k) w_k$$

This implies that

$$z_k = y(a_{k-I}^k) + \bar{b}^T \begin{pmatrix} z_{k-L} - y(a_{k-L-I}^{k-L}) \\ \cdot \\ \cdot \\ \cdot \\ z_{k-1} - y(a_{k-1-I}^{k-1}) \end{pmatrix} + \sigma(a_{k-I}^k) w_k$$

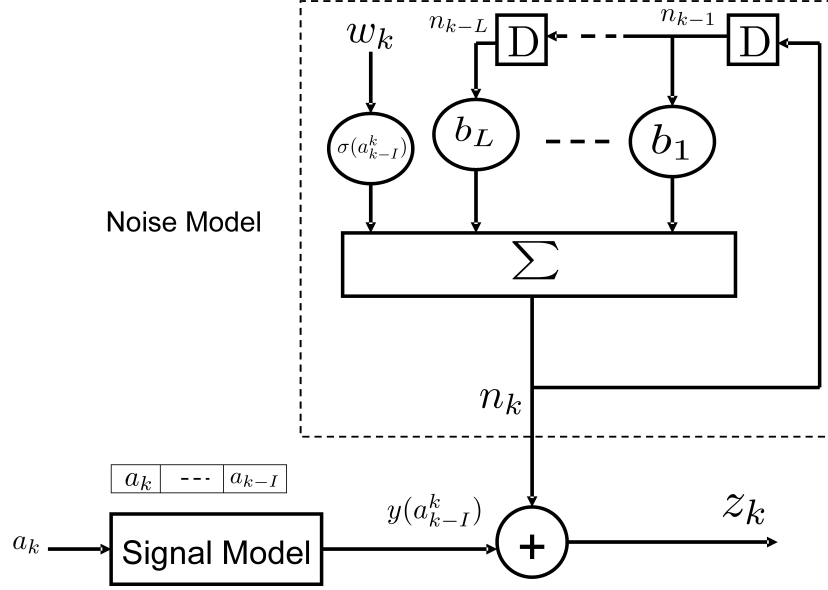


Figure 4.1 Channel model with Gauss-Markov noise.

From above analysis, we can conclude that

$$f(z_k | z_0^{k-1}, \bar{a}) = f(z_k | z_{k-L}^{k-1}, a_{k-L-I}^k), \quad (4.1)$$

where we recall that $f(\cdot|\cdot)$ represents the conditional pdf.

4.2.2 Viterbi Detector

The maximum likelihood estimate of the bit sequence denoted $\hat{\bar{a}}$ is given by

$$\begin{aligned} \hat{\bar{a}} &= \arg \max_{\bar{a} \in \{0,1\}^N} f(\bar{z} | \bar{a}) \\ &= \arg \max_{\bar{a} \in \{0,1\}^N} \prod_{k=0}^{N-1} f(z_k | z_0^{k-1}, \bar{a}) \\ &= \arg \max_{\bar{a} \in \{0,1\}^N} \prod_{k=0}^{N-1} f(z_k | z_{k-L}^{k-1}, a_{k-L-I}^k) \quad (\text{Using (4.1)}) \\ &= \arg \max_{\bar{a} \in \{0,1\}^N} \prod_{k=0}^{N-1} \frac{f(z_{k-L}^k | a_{k-L-I}^k)}{f(z_{k-L}^{k-1} | a_{k-L-I}^k)} \end{aligned}$$

We define a state $S_k = a_{k-L-I+1}^k$ (there will be a total of 2^{L+I} states). With this definition, $f(z_{k-L}^k | a_{k-L-I}^k) = f(z_{k-L}^k | S_{k-1}^k)$. Moreover it is Gaussian distributed,

$$f(z_{k-L}^k | S_{k-1}^k) \sim N(\bar{\mu}(S_{k-1}^k), C(S_{k-1}^k))$$

where $\bar{\mu}(S_{k-1}^k)$ is the mean vector and $C(S_{k-1}^k)$ is the covariance matrix.

With our state definition, we can reformulate the detection problem as the following MLSD problem.

$$\begin{aligned}
\hat{\hat{S}} &= \arg \max_{\hat{S}} \prod_{k=0}^{N-1} \frac{f(z_{k-L}^k | a_{k-L-I}^k)}{f(z_{k-L}^{k-1} | a_{k-L-I}^k)} \\
&= \arg \max_{\hat{S}} \prod_{k=0}^{N-1} \frac{f(z_{k-L}^k | S_{k-1}^k)}{f(z_{k-L}^{k-1} | S_{k-1}^k)} \\
&= \arg \min_{\hat{S}} \sum_{k=0}^{N-1} \log \frac{|C(S_{k-1}^k)|}{|\mathbf{c}(S_{k-1}^k)|} \\
&+ (z_{k-L}^k - \bar{\mu}(S_{k-1}^k))^T C(S_{k-1}^k)^{-1} (z_{k-L}^k - \bar{\mu}(S_{k-1}^k)) \\
&- (z_{k-L}^{k-1} - \bar{\mu}'(S_{k-1}^k))^T \mathbf{c}(S_{k-1}^k)^{-1} (z_{k-L}^{k-1} - \bar{\mu}'(S_{k-1}^k))
\end{aligned}$$

where $\hat{\hat{S}}$ is the estimated state sequence, $\mathbf{c}(S_{k-1}^k)$ is the upper $L \times L$ principal minor of $C(S_{k-1}^k)$ and $\bar{\mu}'(S_{k-1}^k)$ collects the first L elements of $\bar{\mu}(S_{k-1}^k)$. It is assumed that the first state is known. With the metric given above, Viterbi decoding can be applied to get the ML state sequence [62] and the corresponding bit sequence.

The matrix $C(S_{k-1}^k)$ is of dimension $(L+1) \times (L+1)$. For higher values of L , the complexity of detector increases as the decoding metric involves the inversion of the matrix $C(S_{k-1}^k)$. However, the matrix inversion lemma can be used here to obtain

$$\begin{aligned}
C(S_{k-1}^k)^{-1} &= \begin{bmatrix} \mathbf{c}(S_{k-1}^k) & \bar{c} \\ \bar{c}^T & c \end{bmatrix}^{-1} \\
&= \begin{bmatrix} \mathbf{c}(S_{k-1}^k)^{-1} & 0 \\ 0 & 0 \end{bmatrix} + \frac{\bar{w}(S_{k-1}^k) \bar{w}(S_{k-1}^k)^T}{\gamma(S_{k-1}^k)},
\end{aligned} \tag{4.2}$$

where

$$\bar{w}(S_{k-1}^k) = \begin{bmatrix} -\mathbf{c}(S_{k-1}^k)^{-1} \bar{c} \\ 1 \end{bmatrix} = \begin{bmatrix} -\bar{b} \\ 1 \end{bmatrix}, \text{ and}$$

$$\gamma(S_{k-1}^k) = (c - \bar{c}^T \mathbf{c}(S_{k-1}^k)^{-1} \bar{c}) = \sigma^2(a_{k-I}^k).$$

Using (4.2), we can simplify the detector as follows.

$$\hat{S} = \arg \min_{\bar{S}} \sum_{k=0}^{N-1} \log \sigma^2(a_{k-I}^k) + \frac{([- \bar{b}^T \ 1](z_{k-L}^k - \bar{\mu}(S_{k-1}^k)))^2}{\sigma^2(a_{k-I}^k)}.$$

It should be noted that the above expression does not involve any matrix inversion. This reduces the complexity of the detector substantially. Another observation is that the Viterbi decoding metric involves passing z_{k-L}^k through a filter $[-\bar{b}^T \ 1]$ which is the inverse of the autoregressive filter of noise process n_k shown in Figure 4.1. The metric first uncorrelates the noise with an FIR filter and then applies the Euclidean metric to the output of the filter.

4.3 Upper Bound on BER

As discussed previously, the channel model under consideration (cf. Section 4.2), is such that the corresponding PEP is asymmetric, and moreover does not factorize as a product of appropriate functions as required by flowgraph techniques. We now show that we can address this issue by using the Gallager upper bounding technique [53], coupled with a suitable change of variables.

Denote an error event of length N as $\epsilon_N = (\bar{S}, \hat{S})$ such that $\bar{S}$ and $\hat{S}$ are valid state sequences and $S_k = \hat{S}_k$, $S_{k+N} = \hat{S}_{k+N}$, $S_{k+j} \neq \hat{S}_{k+j}$ for $1 \leq j \leq N-1$ and $S_{k+j} = \hat{S}_{k+j}$ for other values of j where $\hat{S}_k$ and S_k are the estimated and correct state respectively. Using this, an upper bound on the BER can be found as follows [51],

$$P_b(e) \leq \sum_{N=1}^{\infty} \sum_{\bar{S}} P(\bar{S}) \sum_{\hat{S}: (\bar{S}, \hat{S}) \in E_N} \nu(\bar{S}, \hat{S}) P(\hat{S}|\bar{S}),$$

where $\nu(\bar{S}, \hat{S})$ is the number of erroneous bits along the sequences $\bar{S}$ and $\hat{S}$ and E_N is the set of all error events ϵ_N of length N . The number of erroneous bits is given by

$$\nu(\bar{S}, \hat{S}) = \frac{d}{dZ} [\Pi_{k=0}^{N-1} Z^{\delta(a_k, \hat{a}_k)}] \Big|_{Z=1}$$

where $\delta(a_k, \hat{a}_k) = 1$, if $a_k \neq \hat{a}_k$ and Z is a dummy variable. Using this the upper bound above can be expressed as

$$P_b(e) \leq \sum_{N=1}^{\infty} \sum_{\bar{S}} P(\bar{S}) \sum_{\hat{S}: (\bar{S}, \hat{S}) \in E_N} \frac{d}{dZ} [\Pi_{k=0}^{N-1} Z^{\delta(a_k, \hat{a}_k)}] \Big|_{Z=1} P(\hat{S}|\bar{S})$$

where $P(\bar{S}) = P(S_0)P(S_1|S_0) \dots P(S_N|S_{N-1}) = \frac{1}{M} \cdot \frac{1}{2^N}$ if $\bar{S}$ is valid state sequence, (M is the number of states). The upper bound on the PEP can be simplified using Gallager's technique [53] as shown below. Let $A(\bar{S}, \hat{S}) = \{\bar{z} : f(\bar{z}|\hat{S}) \geq f(\bar{z}|\bar{S})\}$. Note that using previous arguments, we also have that $A(\bar{S}, \hat{S}) = \left\{ \bar{z} : \prod_{k=0}^{N-1} \frac{f(z_k|\hat{S}_{k-1}^k, z_{k-L}^{k-1})}{f(z_k|S_{k-1}^k, z_{k-L}^{k-1})} \geq 1 \right\}$. Now $P(\hat{S}|\bar{S})$ is [58],

$$\begin{aligned}
P(\hat{S}|\bar{S}) &= P(\hat{S}_0 \dots \hat{S}_{N-1} | S_0 \dots S_{N-1}) \\
&= \int_{A(\bar{S}, \hat{S})} \prod_{k=0}^{N-1} f(z_k | S_{k-1}^k, z_{k-L}^{k-1}) d\bar{z} \\
&\leq \min_{\forall \rho_k} \int \prod_{k=0}^{N-1} f(z_k | S_{k-1}^k, z_{k-L}^{k-1}) \\
&\quad \cdot \prod_{k=0}^{N-1} \left(\frac{f(z_k | \hat{S}_{k-1}^k, z_{k-L}^{k-1})}{f(z_k | S_{k-1}^k, z_{k-L}^{k-1})} \right)^{\rho_k} d\bar{z} \\
&= \min_{\forall \rho_k} \int \prod_{k=0}^{N-1} (f(z_k | S_{k-1}^k, z_{k-L}^{k-1}))^{1-\rho_k} \\
&\quad \cdot (f(z_k | \hat{S}_{k-1}^k, z_{k-L}^{k-1}))^{\rho_k} d\bar{z}
\end{aligned}$$

where $0 \leq \rho_k \leq 1$ for $k = 0, \dots, N-1$.

The above integral can be simplified as follows.

$$\begin{aligned}
&\int \prod_{k=0}^{N-1} (f(z_k | S_{k-1}^k, z_{k-L}^{k-1}))^{1-\rho_k} (f(z_k | \hat{S}_{k-1}^k, z_{k-L}^{k-1}))^{\rho_k} d\bar{z} \\
&= \int \prod_{k=0}^{N-1} \left(\frac{f(z_{k-L}^k | S_{k-1}^k)}{f(z_{k-L}^{k-1} | S_{k-1}^k)} \right)^{1-\rho_k} \left(\frac{f(z_{k-L}^k | \hat{S}_{k-1}^k)}{f(z_{k-L}^{k-1} | \hat{S}_{k-1}^k)} \right)^{\rho_k} d\bar{z} \\
&= \int \prod_{k=0}^{N-1} \frac{1}{\sqrt{2\pi} \sigma^{1-\rho_k} (a_{k-I}^k)^{\hat{\sigma} \rho_k} (\hat{a}_{k-I}^k)} \\
&\quad \cdot \exp \left(-\frac{(1-\rho_k)([-\bar{b}^T \ 1](z_{k-L}^k - \bar{\mu}(S_{k-1}^k)))^2}{2\sigma^2(a_{k-I}^k)} - \frac{\rho_k([- \bar{b}^T \ 1](z_{k-L}^k - \hat{\mu}(\hat{S}_{k-1}^k)))^2}{2\hat{\sigma}^2(\hat{a}_{k-I}^k)} \right) d\bar{z} \\
&= \prod_{k=0}^{N-1} \int \frac{1}{\sqrt{2\pi} \sigma^{1-\rho_k} (a_{k-I}^k)^{\hat{\sigma} \rho_k} (\hat{a}_{k-I}^k)} \\
&\quad \cdot \exp \left(-\frac{(1-\rho_k)(u_k - \mathfrak{M}(S_{k-1}^k))^2}{2\sigma^2(a_{k-I}^k)} - \frac{\rho_k(u_k - \hat{\mathfrak{M}}(\hat{S}_{k-1}^k))^2}{2\hat{\sigma}^2(\hat{a}_{k-I}^k)} \right) du_k
\end{aligned}$$

where $u_k = [-\bar{b}^T \ 1] \cdot [z_{k-L} \dots z_k]^T$, $\mathfrak{M}(S_{k-1}^k) = [-\bar{b}^T \ 1] \cdot \bar{\mu}(S_{k-1}^k)$, $\hat{\mathfrak{M}}(\hat{S}_{k-1}^k) = [-\bar{b}^T \ 1] \cdot \hat{\mu}(\hat{S}_{k-1}^k)$. The Jacobian matrix for the change of variables has determinant equal to 1, since the corresponding matrix of partial derivatives has ones on the diagonal and is lower triangular. Note that the change of variables decouples the original expression, so that it can be expressed

as the product of N independent integrals. Now we can simplify the PEP as follows.

$$\begin{aligned}
P(\hat{a}|\bar{a}) &\leq \min_{\forall \rho_k} \Pi_{k=0}^{N-1} \int \frac{1}{\sqrt{2\pi}\sigma^{1-\rho_k}(a_{k-I}^k)\hat{\sigma}^{\rho_k}(\hat{a}_{k-I}^k)} \\
&\quad \cdot \exp\left(-\frac{(1-\rho_k)(u_k - \mathfrak{M}(S_{k-1}^k))^2}{2\sigma^2(a_{k-I}^k)} - \frac{\rho_k(u_k - \hat{\mathfrak{M}}(\hat{S}_{k-1}^k))^2}{2\hat{\sigma}^2(\hat{a}_{k-I}^k)}\right) du_k \\
&= \Pi_{k=0}^{N-1} \min_{\rho_k} \int \frac{1}{\sqrt{2\pi}\sigma^{1-\rho_k}(a_{k-I}^k)\hat{\sigma}^{\rho_k}(\hat{a}_{k-I}^k)} \\
&\quad \cdot \exp\left(-\frac{(1-\rho_k)(u_k - \mathfrak{M}(S_{k-1}^k))^2}{2\sigma^2(a_{k-I}^k)} - \frac{\rho_k(u_k - \hat{\mathfrak{M}}(\hat{S}_{k-1}^k))^2}{2\hat{\sigma}^2(\hat{a}_{k-I}^k)}\right) du_k \tag{4.3} \\
&= \Pi_{k=0}^{N-1} \min_{\rho_k} \frac{\sigma^{\rho_k}(a_{k-I}^k)\hat{\sigma}^{1-\rho_k}(\hat{a}_{k-I}^k)}{\sqrt{(1-\rho_k)\hat{\sigma}^2(\hat{a}_{k-I}^k) + \rho_k\sigma^2(a_{k-I}^k)}} \\
&\quad \cdot \exp\left(-\frac{(1-\rho_k)\hat{\sigma}^2(\hat{a}_{k-I}^k)\mathfrak{M}^2(S_{k-1}^k) + \rho_k\sigma^2(a_{k-I}^k)\hat{\mathfrak{M}}^2(\hat{S}_{k-1}^k)}{2\sigma^2(a_{k-I}^k)\hat{\sigma}^2(\hat{a}_{k-I}^k)}\right) \\
&\quad + \frac{((1-\rho_k)\hat{\sigma}^2(\hat{a}_{k-I}^k)\mathfrak{M}(S_{k-1}^k) + \rho_k\sigma^2(a_{k-I}^k)\hat{\mathfrak{M}}(\hat{S}_{k-1}^k))^2}{2\sigma^2(a_{k-I}^k)\hat{\sigma}^2(\hat{a}_{k-I}^k)((1-\rho_k)\hat{\sigma}^2(\hat{a}_{k-I}^k) + \rho_k\sigma^2(a_{k-I}^k))} \\
&= \Pi_{k=0}^{N-1} W(S_{k-1}^k, \hat{S}_{k-1}^k) \tag{4.4}
\end{aligned}$$

where $W(S_{k-1}^k, \hat{S}_{k-1}^k)$ is a function of $\sigma(a_{k-I}^k)$, $\hat{\sigma}(\hat{a}_{k-I}^k)$, $\mathfrak{M}(S_{k-1}^k)$ and $\hat{\mathfrak{M}}(\hat{S}_{k-1}^k)$ and the simplification of the integral in (4.3) is given in the Appendix.

It is important to note that the factorization of PEP given by (4.4) for our channel model is possible because the autoregressive filter $\bar{b}$ is not dependent on the input bit sequence. In [10], $\bar{b}$ is assumed to be data dependent given by $\bar{b}(a_{k-I}^k)$. When the autoregressive filter $\bar{b}(a_{k-I}^k)$ becomes data dependent, it is very difficult to write PEP in the form given in (4.4). In this case, the inverse of the autoregressive filter of the noise process n_k ($[-\bar{b}(a_{k-I}^k)^T \ 1]$) is state-dependent which means that the actual state transition (S_{k-1}^k) and estimated state transition ($\hat{S}_{k-1}^k$) have different filters. In this situation, the specific change of variables used above does not seem to work.

Probability of bit error can now be further simplified as [51],

$$P_b(e) \leq \sum_{N=1}^{\infty} \sum_{\bar{S}} P(\bar{S}) \sum_{\hat{S}: (\bar{S}, \hat{S}) \in E_N} \frac{d}{dZ} [\Pi_{k=0}^{N-1} Z^{\delta(a_k, \hat{a}_k)} |_{Z=1} P(\hat{S}|\bar{S})]$$

$$\begin{aligned}
&= \frac{d}{dZ} \sum_{N=1}^{\infty} \frac{1}{M} \cdot \frac{1}{2^N} \sum_{\bar{S}} \sum_{\hat{S}} \Pi_{k=0}^{N-1} Z^{\delta(a_k, \hat{a}_k)} P(\hat{S}|\bar{S})|_{Z=1} \\
&\leq \frac{d}{dZ} \sum_{N=1}^{\infty} \frac{1}{M} \cdot \frac{1}{2^N} \sum_{\bar{S}} \sum_{\hat{S}} \Pi_{k=0}^{N-1} Z^{\delta(a_k, \hat{a}_k)} \\
&\quad \cdot W(S_{k-1}^k, \hat{S}_{k-1}^k)|_{Z=1} \quad (\text{Using (4.4)}) \\
&= \frac{1}{M} \frac{d}{dZ} \sum_{N=1}^{\infty} \sum_{\bar{S}} \sum_{\hat{S}} \Pi_{k=0}^{N-1} \frac{1}{2} Z^{\delta(a_k, \hat{a}_k)} W(S_{k-1}^k, \hat{S}_{k-1}^k)|_{Z=1} \\
&= \frac{1}{M} \frac{d}{dZ} (T(Z))|_{Z=1}
\end{aligned}$$

where M is number of states. For obtaining $T(Z)$, we construct a product trellis. Consider a matrix $V(Z)$ of order $M^2 \times M^2$, where each row and column is indexed by a pair of states corresponding to the actual and the errored states. Let β_i represent a state that takes one of 2^{L+I} values. Consider the entry of $V(Z)$ indexed by $((\beta_i, \beta_j), (\beta'_i, \beta'_j))$,

$$[V(Z)]_{((\beta_i, \beta_j), (\beta'_i, \beta'_j))} = \begin{cases} \frac{1}{2} Z^{\delta(a_i, a_j)} W((\beta_i, \beta_j), (\beta'_i, \beta'_j)) \\ 0, \text{ if either } \beta_i \rightarrow \beta'_i \text{ or } \beta_j \rightarrow \beta'_j \text{ not allowed} \end{cases}$$

where $W((\beta_i, \beta_j), (\beta'_i, \beta'_j))$ can be found for state transitions $\beta_i \rightarrow \beta'_i$ and $\beta_j \rightarrow \beta'_j$ using (4.4) and a_i and a_j are latest bit in states β'_i and β'_j respectively. A product state is called good state if $\beta'_i = \beta'_j$ and bad otherwise. $V(Z)$ will have a structure which has $V_{GG}(Z)$ (good to good state transition), $V_{GB}(Z)$ (good to bad state transition), $V_{BG}(Z)$ (bad to good state transition) and $V_{BB}(Z)$ (bad to bad state transition),

$$V(Z) = \begin{bmatrix} V_{GG}(Z) & V_{GB}(Z) \\ V_{BG}(Z) & V_{BB}(Z) \end{bmatrix}$$

where the order of $V_{GG}(Z)$ matrix is $M \times M$ and the order of $V_{BB}(Z)$ is $(M^2 - M) \times (M^2 - M)$.

Now we can write $T(Z)$ as,

$$T(Z) = \mathbf{a}(Z) + \mathbf{b}(Z)(I - V_{BB}(Z))^{-1} \mathbf{c}(Z)$$

where $\mathbf{a}(Z) = \mathbf{1}^T V_{GG}(Z) \mathbf{1}$, $\mathbf{b}(Z) = \mathbf{1}^T V_{GB}(Z)$ and $\mathbf{c}(Z) = V_{BG}(Z) \mathbf{1}$. The symbol $\mathbf{1}$ denotes a vector all of whose entries are 1 and I is identity matrix of order $(M^2 - M) \times (M^2 - M)$.

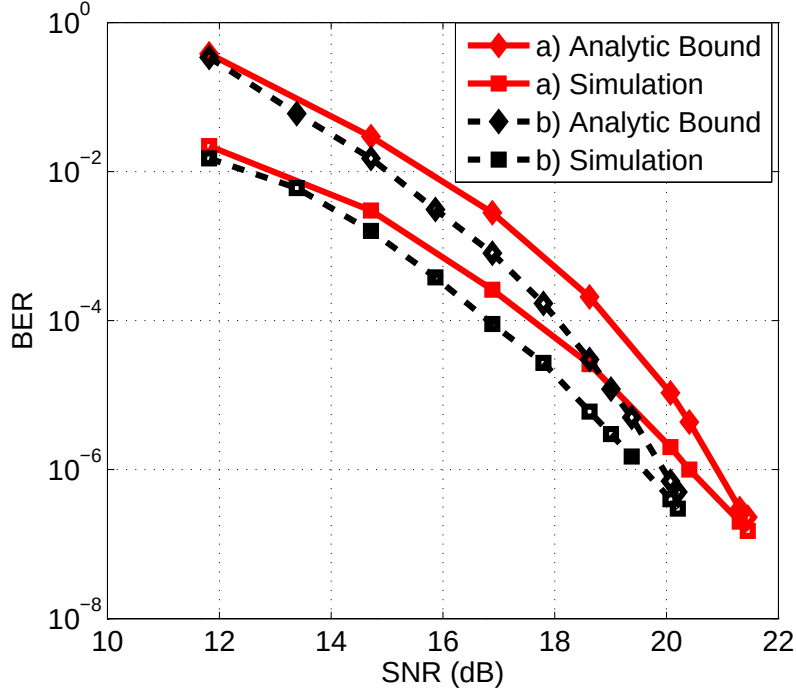


Figure 4.2 BER with different SNR for the channel model with a) 8 states in decoding and b) 16 states in decoding.

Using the above result, we can compute $P_b(e)$ as [51],

$$P_b(e) \leq \frac{1}{M} [\mathbf{a}'(1) + \mathbf{b}'^T(1)(I - V_{BB}(1))^{-1}\mathbf{c}(1) + \mathbf{b}^T(1) \cdot (I - V_{BB}(1))^{-1}\mathbf{c}'(1) + \mathbf{b}^T(1)(I - V_{BB}(1))^{-1}V'_{BB}(1)(I - V_{BB}(1))^{-1}\mathbf{c}(1)].$$

For our model, $V_{GG}(Z)$ is not a function of Z which means that $\mathbf{a}'(1) = 0$. Similarly, $\mathbf{c}(Z)$ is also not a function of Z which implies $\mathbf{c}'(1) = 0$ and it should also be noted that $\mathbf{b}'^T(1) = \mathbf{b}^T(1)$.

The new bound for our channel model is,

$$P_b(e) \leq \frac{1}{M} [\mathbf{b}^T(1)(I - V_{BB}(1))^{-1}\mathbf{c}(1) + \mathbf{b}^T(1)(I - V_{BB}(1))^{-1}V'_{BB}(1)(I - V_{BB}(1))^{-1}\mathbf{c}(1)].$$

4.4 Simulation Results

In the first set of simulations, we used the following parameters: $L = 2$ with $\bar{b} = [.1 \ .5]$ and ISI memory $I = 1$. The signal dependent noise variance for 4 states are given by $\sigma^2(00) = 1$, $\sigma^2(01) = 2$, $\sigma^2(10) = 3$ and $\sigma^2(11) = 4$. The number of states in decoding is equal to 8 in

this case. The SNR is defined as signal energy in $y(a_{k-I}^k)$ divided by total noise variance. We have used a linear signal component given as $y(a_{k-1}^k) = c(2a_k + a_{k-1})$ where the value of c can be varied to change the SNR. In Figure 4.2, the analytic bound follows the simulation BER. At an SNR of 21 dB, the analytic bound gives a BER equal to 3×10^{-7} whereas simulation BER is equal to 2×10^{-7} . The analytic bound is quite tight in high SNR regime. In another simulation, we used following parameters, $L = 3$ coefficients of an autoregressive filter is given by $\bar{b} = [.1 \ .3 \ .5]$, ISI memory (I) is equal to 1 and signal dependent noise variance for 4 states are given by $\sigma^2(00) = 1$, $\sigma^2(01) = 2$, $\sigma^2(10) = 3$ and $\sigma^2(11) = 4$. The number of states in decoding is equal to 16 in this case. In Figure 4.2, the analytic bound again follows the simulation BER for this channel model with modified channel parameters. At an SNR of 20 dB, the analytic bound gives a BER equal to 7×10^{-7} whereas simulation BER is equal to 4×10^{-7} .

4.5 Conclusions

We considered the problem of deriving an analytical upper bound for ML sequence detection in ISI channels with signal dependent Gauss-Markov noise. In these channels the pairwise error probability (PEP) is not symmetric. Moreover, it is hard to express the PEP as a product of appropriate terms that allow the application of flowgraph techniques. In this work, we considered a subset of these channels, and demonstrated an appropriate upper bound on the PEP. Using this upper bound along with pairwise state diagrams, we arrive at analytical BER bounds that are tight in the high SNR regime. These bounds have been verified by our simulation results.

CHAPTER 5. CONCLUSION AND FUTURE WORK

In this dissertation, we showed the application of the dynamic mode AFM in the field of probe based data storage and nano-imaging. In probe based data storage work, the dynamic mode operation of a cantilever probe with a high quality factor is considered. A communication channel model is proposed for the dynamic mode read operation using the Markovian modeling on tip-media interaction. The proposed channel model not only captures the physical behavior of the cantilever system, but it is also mathematically tractable for applying the bit detection techniques. Furthermore, the bit detection problem is posed as a maximum likelihood sequence detection which is solved in an efficient manner by identifying appropriate conditional independencies in the underlying model and by making physically motivated assumptions. In simulation and experimental results, the proposed Viterbi detector outperforms LMP, GLRT and Bayes detector and gives remarkably low BER. This work shows that competitive metrics can be achieved for developing probe based high density data storage, where high quality factor probes are used in the dynamic mode operation. It alleviates the issues of tip-media wear in probe based high density data storage.

An efficient error control coding system is a must for any data storage system since the sector error rate specifications are on the order of 10^{-10} for systems in daily use such as hard drives. In future work, we are expecting to achieve this BER by using appropriate coding techniques. Using run-length-limited (RLL) codes in our system is likely to improve performance. In experimental data, a small amount of jitter is inevitably present which is well handled by our algorithm. At high densities, the jitter will be significantly higher and more advanced modeling and detection techniques need to be applied in future. Our proposed algorithm for data detection works well as long as following the requirements are satisfied.

Firstly, the training sequences should be available which can provide the statistics for different state transitions. Secondly, there should be differences between the tip-media interaction magnitude between ‘0’ and ‘1’ bit. Thirdly, an accurate estimation of the channel model of the cantilever is desired. In future, we can reformulate the problem and find the feasible solution if these requirements are not fulfilled.

In nano-imaging work, we applied the ideas of channel modeling used for probe based data storage system. We again demonstrated that the channel model for the cantilever based nano-imaging system can also be modeled as a communication channel model. The assumption of equiprobable input bits is valid in the data storage system but it doesn’t hold true in nano-imaging context. It forced us to pose the imaging problem as a maximum a posteriori (MAP) symbol detection problem which does take into account the image prior while detecting the features on the image. The BCJR algorithm is used and adapted for solving the MAP symbol detection problem. Furthermore, we proposed the imaging algorithm for the raster scan imaging which incorporates the input prior from previous line scan while detecting the features on the current line scan. Experimental results showed that our proposed algorithm provides significantly better image resolution compared to current imaging techniques at high scanning speed.

Currently one-bit imaging is proposed in this dissertation. This work can be extended for multi-level imaging where not only the absence or presence of topographic profile is specified but the height of topographic profile can also be found. This kind of information is quite useful in finding the structure and movement of biological process. In future, this work will motivate the video rate imaging applications. This will radically change the visualization and the perception of the world at nano-scale.

The communication channel models developed in our work show the similar behavior as ISI channel model with Gauss-Markov noise. We considered the problem of deriving an analytical upper bound on BER which can obviate the need for time consuming simulations for finding BERs. For these channels, the flowgraph techniques cannot be applied as pairwise error probability (PEP) is not symmetric and it is hard to express the PEP as a product of appropriate

terms. We considered a subset of these channels and found an appropriate upper bound on the PEP. Using this upper bound along with pairwise state diagrams, we derived the analytical BER bounds that are tight in the high SNR regime. Our simulation results confirmed the convergence of the proposed BER bound with simulation BER in high SNR regime. It would be interesting to examine whether our current techniques can be extended to address the general channel model. Moreover, it may be possible to reduce the complexity of evaluating the bound by reducing the size of the product trellis by exploiting channel characteristics.

In nutshell, we have shown that the judicious application of communications and signal processing techniques in the field of AFM can result in major breakthroughs. Our work has paved the way for a radically different approach to probe based microscopy which was not achievable by current state-of-the-art. Even though the experimental setup presented in this dissertation uses a particular scheme for measuring the cantilever detection and for actuating the cantilever, the paradigm developed for data detection is largely applicable in principle to other modes of detection and actuation of the cantilever. These ideas of channel modeling and data detection can be used for many future applications in various research fields like nano-imaging, advanced material characterization at nanoscale etc.

APPENDIX A. ADDITIONAL MATERIAL

The integral in the equation (4.3) can be expressed in the following form,

$$\begin{aligned}
& \int \frac{1}{\sqrt{2\pi}\gamma} \exp\left(-\frac{1}{2}(\alpha(x-m)^2 + \beta(x-\hat{m})^2)\right) dx \\
&= \frac{1}{\gamma\sqrt{\alpha+\beta}} \exp\left(-\frac{\alpha m^2 + \beta \hat{m}^2}{2} + \frac{(\alpha m + \beta \hat{m})^2}{2(\alpha + \beta)}\right) \\
&\cdot \underbrace{\int \frac{\sqrt{\alpha+\beta}}{\sqrt{2\pi}} \exp\left(-\frac{\alpha+\beta}{2}\left(x - \frac{\alpha m + \beta \hat{m}}{\alpha + \beta}\right)^2\right) dx}_{=1} \\
&= \frac{1}{\gamma\sqrt{\alpha+\beta}} \exp\left(-\frac{\alpha m^2 + \beta \hat{m}^2}{2} + \frac{(\alpha m + \beta \hat{m})^2}{2(\alpha + \beta)}\right)
\end{aligned}$$

where $\alpha = \frac{(1-\rho_k)}{\sigma^2(a_{k-I}^k)}$, $\beta = \frac{\rho_k}{\hat{\sigma}^2(\hat{a}_{k-I}^k)}$, $m = \mathfrak{M}(S_{k-1}^k)$, $\gamma = \sigma^{1-\rho_k}(a_{k-I}^k)\hat{\sigma}^{\rho_k}(\hat{a}_{k-I}^k)$ and $\hat{m} = \hat{\mathfrak{M}}(\hat{S}_{k-1}^k)$. Using the above equality, we can easily simplify the RHS of equation (4.3) as,

$$\begin{aligned}
& \int \frac{1}{\sqrt{2\pi}\sigma^{1-\rho_k}(a_{k-I}^k)\hat{\sigma}^{\rho_k}(\hat{a}_{k-I}^k)} \exp\left(-\frac{(1-\rho_k)(u_k - \mathfrak{M}(S_{k-1}^k))^2}{2\sigma^2(a_{k-I}^k)} - \frac{\rho_k(u_k - \hat{\mathfrak{M}}(\hat{S}_{k-1}^k))^2}{2\hat{\sigma}^2(\hat{a}_{k-I}^k)}\right) du_k \\
&= \frac{1}{\sigma^{1-\rho_k}(a_{k-I}^k)\hat{\sigma}^{\rho_k}(\hat{a}_{k-I}^k)} \cdot \frac{\sigma(a_{k-I}^k)\hat{\sigma}(\hat{a}_{k-I}^k)}{\sqrt{(1-\rho_k)\hat{\sigma}^2(\hat{a}_{k-I}^k) + \rho_k\sigma^2(a_{k-I}^k)}} \\
&\cdot \exp\left(-\frac{(1-\rho_k)\hat{\sigma}^2(\hat{a}_{k-I}^k)\mathfrak{M}^2(S_{k-1}^k) + \rho_k\sigma^2(a_{k-I}^k)\hat{\mathfrak{M}}^2(\hat{S}_{k-1}^k)}{2\sigma^2(a_{k-I}^k)\hat{\sigma}^2(\hat{a}_{k-I}^k)}\right) \\
&+ \frac{((1-\rho_k)\hat{\sigma}^2(\hat{a}_{k-I}^k)\mathfrak{M}(S_{k-1}^k) + \rho_k\sigma^2(a_{k-I}^k)\hat{\mathfrak{M}}(\hat{S}_{k-1}^k))^2}{2\sigma^2(a_{k-I}^k)\hat{\sigma}^2(\hat{a}_{k-I}^k)((1-\rho_k)\hat{\sigma}^2(\hat{a}_{k-I}^k) + \rho_k\sigma^2(a_{k-I}^k))}.
\end{aligned}$$

BIBLIOGRAPHY

- [1] G. Binnig, C. F. Quate and C. Gerber. Atomic force microscope. *Phys. Rev. Lett.*, 56:930, 1986.
- [2] R. Garcia, R. Magerle and R. Perez. Nanoscale compositional mapping with gentle forces. *Nature Materials*, vol. 6:405-411, June 2007.
- [3] P. Girard. Electrostatic force microscopy: principles and some applications to semiconductors. *IOP Nanotechnology*, vol. 12:485-490, 2001.
- [4] L. Bond, S. Allen, M. C. Davies, C. J. Roberts, A. P. Shivji, S. J. B. Tendler, P. M. Williams and J. Zhang. Differential scanning calorimetry and scanning thermal microscopy analysis of pharmaceutical materials.. *International Journal of Pharmaceutics*, vol. 243:71-82, 2002.
- [5] A. Lewis, A. Radko, N. B. Ami, D. Palanker and K. Lieberman. Near-field scanning optical microscopy in cell biology. *Trends in Cell Biology*, vol. 9, Issue 2:70-73, Feb 1999.
- [6] J. Xie, M. Hu, S. Ling and H. Du. Fabrication and characterization of piezoelectric cantilever. *Sensors and Actuators A*, vol. 126:182-186, 2006.
- [7] Nancy A. Burnham and Richard J. Colton. Measuring the nanomechanical properties and surface forces of materials using an atomic force microscope. *J. Vac. Sci. Technol.*, A 7 (4), Jul/Aug 1989.
- [8] A. Alessandrini and P. Facci. AFM: a versatile tool in biophysics. *Meas. Sci. Technol*, Vol. 16, pp. R65-R92, April 2005.

- [9] Seiji Akita, Hidehiro Nishijima, Takayoshi Kishida and Yoshikazu Nakayama. Nanoin-
dentation of polycarbonate using carbon nanotube tip. *International Microprocesses and
Nanotechnology Conference*, pages 228 – 229, 2000.
- [10] Kavcic Aleksandar and Jose M. F. Moura. The Viterbi Algorithm and Markov Noise
Memory. *IEEE Transactions on Information Theory*, 46 , Issue: 1:291–301, 2000.
- [11] M. Ashhab, M. V. Salapaka, M. Dahleh and I. Mezic. Dynamical analysis and control of
micro-cantilevers. *Automatica*, 1999.
- [12] G. Casella and R. Berger. *Statistical Inference*. Duxbury Thomson Learning, second
edition, 2002.
- [13] B. W. Chui, H. J. Mamin, B. D. Terris, D. Rugar and T. W. Kenny. Sidewall-implanted
dual-axis piezoresistive cantilever for afm data storage readback and tracking. pages 12–17,
Jan 1998.
- [14] P. Baechtold A. Sebastian H. Pozidis D. R. Sahoo, W. Haeberle and E. Eleftheriou. On
intermittent-contact mode sensing using electrostatically-actuated micro-cantilevers with
integrated thermal sensors. *Proceeding of the American Control Conference*, pages 2034
– 2039, June 2008.
- [15] D. L. DeVoe and A. P. Pisano. Modeling and optimal design of piezoelectric cantilever
microactuators. *Journal of Microelectromechanical Systems*, 6(3):266–270, Sep 1997.
- [16] U. Dürig. Fundamentals of micromechanical thermoelectric sensors. *Journal of Applied
Physics*,, 98:044, 2005.
- [17] Fedor N Dzegilenko, Deepak Srivastava and Subhash Saini. Nanoscale etching and inden-
tation of a silicon(001) surface with carbon nanotube tips. *Nanotechnology*, 10:253257,
1999.
- [18] E. Eleftheriou et al. Millepede - A MEMS-based Scanning-Probe Data-Storage System.
IEEE Trans. on Magnetism, 39, no. 2:938–945, 2003.

- [19] E. Eleftheriou et al. A nanotechnology-based approach to data storage. In *VLDB '2003: Proceedings of the 29th international conference on Very large data bases*, pages 3–7. VLDB Endowment, 2003.
- [20] Jr. Forney, G. Maximum-likelihood sequence estimation of digital sequences in the presence of intersymbol interference. *IEEE Transactions on Information Theory*, 18, Issue: 3:363 – 378, 1972.
- [21] R. García and R. Pérez. Dynamic atomic force microscopy methods. *Surf. Sci. Rep.*, 47:197, 2002.
- [22] F. J. Giessibl. Advances in atomic force microscopy. *Rev. Mod. Phys.*, 75(3):949, July 2003.
- [23] D. W. Wiesmann U. Drechsler M. Despont T. Albrecht H. Pozidis, W. Hberle and E. Eleftheriou. Demonstration of thermomechanical recording at 641 gbit/in². *IEEE Transactions on Magnetism*, 4:25312536, 2004.
- [24] S. Hu and A. Raman. Chaos in atomic force microscopy. *Physics Review Letters*, pages 036107:1–3, January 2006.
- [25] T. Kailath, A. H. Sayed and B. Hassibi. *Linear Estimation*. Prentice Hall, 2000.
- [26] H. Vincent Poor. An Introduction to Signal Detection and Estimation, 2nd Ed. *Springer-Verlag*, 1994.
- [27] J. Moon. The role of sp in data-storage systems. *IEEE Signal Processing Magazine*, 15(4):54–72, Jul 1998.
- [28] R.D. Cideciyan, F. Dolivo, R. Hermann, W. Hirt and W. Schott. A PRML system for digital magnetic recording. *IEEE Journal on Selected Areas in Communications*, 10(1):38–56, Jan 1992.
- [29] R. Wood and D. Petersen. Viterbi detection of class IV partial response on a magnetic recording channel. *IEEE Transactions on Communications*, 34(5):454–461, May 1986.

- [30] J. Moon and L.R. Carley. Performance comparison of detection methods in magnetic recording. *IEEE Transactions on Magnetics*, 26(6):3155–3172, Nov 1990.
- [31] Jaekyun Moon and Jongseung Park. Pattern-dependent noise prediction in signal-dependent noise. *IEEE Journal on Selected Areas in Communications*, 19(4):730–743, Apr 2001.
- [32] H. Thapar and A. Patel. A class of partial response systems for increasing storage density in magnetic recording. *IEEE Transactions on Magnetics*, 23(5):3666–3668, Sep 1987.
- [33] D. R. Sahoo, A. Sebastian and M. V. Salapaka. Harnessing the transient signals in atomic force microscopy. *International Journal of Robust and Nonlinear Control, Special Issue on Nanotechnology and Micro-biology*, 15:805–820, 2005.
- [34] M. V. Salapaka, H. S. Bergh, J. Lai, A. Majumdar, and E. McFarland. Multi-mode noise analysis of cantilevers for scanning probe microscopy. *Journal of Applied Physics*, 81(6):2480–2487, March 1997.
- [35] M. V. Salapaka, D. Chen and J. P. Cleveland. Linearity of amplitude and phase in tapping-mode atomic force microscopy. *Phys. Rev. B.*, 2000.
- [36] S. Salapaka, M. Dahleh and I. Mezic. On the Dynamics of a Harmonic Oscillator Undergoing Impacts with a Vibrating Platform. *Nonlinear Dynamics*, 24:333–358, 2001.
- [37] A. Sebastian, A. Gannepalli and M.V. Salapaka. A review of the systems approach to the analysis of dynamic-mode atomic force microscopy. *IEEE Transactions on Control Systems Technology*, 15(5):952–959, Sept. 2007.
- [38] A. Sebastian, M. V. Salapaka, D. Chen and J. P. Cleveland. Harmonic and power balance tools for tapping-mode atomic force microscope. *Journal of Applied Physics*, 89 (11):6473–6480, Jun. 2001.

- [39] R. D. Shachter. Bayes-Ball: The Rational Pastime (for Determining Irrelevance and Requisite Information in Belief Networks and Influence Diagrams). . In *Uncertainty in Artificial Intelligence: Proceedings of the Fourteenth Conference*, pages 480–487, 1998.
- [40] B. D. Terris, S. A. Rishton, H. J.Mamin, R. P. Ried and D. Rugar. Atomic forcemicroscope-based data storage: track servo and wear study. *Applied Physics A Materials*, 6:S809S813, 1998.
- [41] P. Vettiger, G. Cross, M. Despont, U. Drechsler, U. Durig, B. Gotsmann, W. Haberele, M. A. Lantz, H. Rothuizen, R. Stutz and G. Binnig. The millipede-nanotechnology entering data storage. *IEEE Transactions on Nanotechnology*, 1((1)), 2002.
- [42] D. Wiesmann, U. Duerig, B. Gotsmann, A. Knoll, H. Pozidis, F. Porro and R. Vecchione. Characterization of electrostatically coupled microcantilevers. *Innovative Mass Storage Technologies*, page 19, 2007.
- [43] R. Wisendanger. *Scanning Probe Microscopy and Spectroscopy*. Cambridge University Press, 1994.
- [44] Q. Zhong, D. Inniss, K. Kjoller and V B Elings. Fractured polymer/silica fiber surface studied by tapping mode atomic force microscopy. *Surface Science Letters*, 290:L688–L692, 1993.
- [45] T. Ando, N. Kodera, E. Takai, D. Maruyama, K. Saito and A. Toda. A high-speed atomic force microscope for studying bilological macromolecules. *Proceedings of National Academy of Science*, 98(22):034106, 2001.
- [46] L. Bahl, J. Cocke, F. Jelinek and J. Raviv. Optimal decoding of linear codes for minimizing symbol error rate (corresp.). *IEEE Transactions on Information Theory*, 20(2):284 – 287, Mar 1974.
- [47] S. Kasas, N. H. Thomsona, B. L. Smith, H. G. Hansma, et al. Escherichia coli rna polymerase activity observed using atomic force microscopy. *Biochemistry*, 36(3):461 – 468, 1997.

- [48] D. O. Koralek, W. F. Heinz, M. D. Antonik, A. Baik and J. H. Hoh. Probing deep interaction potentials with white-noise-driven atomic force microscope cantilevers. *IEEE T. Instrum. Meas.*, 76:2952, 2000.
- [49] N. Kumar, P. Agarwal, A. Ramamoorthy and M. Salapaka. Channel modeling and detector design for dynamic mode high density probe storage. In *42nd Conference on Information Sciences and Systems*. IEEE, 2008.
- [50] D.R. Sahoo, P. Agarwal and M.V. Salapaka. Transient force atomic force microscopy: A new nano-interrogation method. In *American Control Conference, 2007. ACC '07*, pages 2135–2140, 2007.
- [51] E. Biglieri. High-level modulation and coding for nonlinear satellite channels. *IEEE Transactions on Communications*, 32(5):616 – 626, May 1984.
- [52] Jr. Forney, G.D. The viterbi algorithm. *Proceedings of the IEEE*, 61(3):268 – 278, March 1973.
- [53] R. G. Gallager. *Information Theory and Reliable Communication*. New York, NY: Wiley, 1968.
- [54] A. Kavcic and J.M.F. Moura. Correlation-sensitive adaptive sequence detection. *IEEE Transactions on Magnetics*, 34(3):763 –771, May 1998.
- [55] A. Kavcic and A. Patapoutian. A signal-dependent autoregressive channel model. *Magnetics, IEEE Transactions on*, 35(5):2316 –2318, Sep 1999.
- [56] N. Kumar, P. Agarwal, A. Ramamoorthy and M. Salapaka. Maximum-likelihood sequence detector for dynamic mode high density probe storage. *IEEE Transactions on Communications*, 58(6):1686 –1694, June 2010.
- [57] N. Kumar, P. Agarwal, A. Ramamoorthy and M. Salapaka. Maximum likelihood sequence detector for dynamic mode high density probe storage *IEEE GlobeCom 2009*.

- [58] N. Kumar, A. Ramamoorthy and M. Salapaka. Performance evaluation for ML sequence detection in ISI channels with Gauss Markov Noise *IEEE GlobeCom 2010*.
- [59] N. Kumar, G. Saraswat, P. Agarwal, A. Ramamoorthy and M. Salapaka. High-speed nanoimaging using dynamic mode AFM: A MAP detection approach. *44th Asilomar Conference on Signals, Systems and Computers 2010*.
- [60] I. Lee and J.M. Cioffi. Performance analysis of the modified maximum likelihood sequence detector in the presence of data-dependent noise. *26th Asilomar Conference on Signals, Systems and Computers*, pages 961 –964 vol.2, 26-28 1992.
- [61] Y. J. Liu, I. Oka and E. Biglieri. Error probability for digital transmission over nonlinear channels with application to tcm. *IEEE Transactions on Information Theory*, 36(5):1101 –1110, Sep 1990.
- [62] A. Viterbi. Error bounds for convolutional codes and an asymptotically optimum decoding algorithm. *IEEE Transactions on Information Theory*, 13(2):260 – 269, Apr 1967.
- [63] Andrew J. Viterbi and Jim K. Omura. *Principles Of Digital Communication And Coding*. McGraw-Hill, Inc. New York, NY, USA, 1979.
- [64] W. Zeng and J. Moon. Modified viterbi algorithm for a jitter-dominant 1-d2 channel. *IEEE Transactions on Magnetics*, 28(5):2895 –2897, Sep 1992.
- [65] Jian-Gang Zhu and Haiyun Wang. Noise characteristics of interacting transitions in longitudinal thin film media. *IEEE Transactions on Magnetics*, 31(2):1065 –1070, Mar 1995.