

Orientation of an Optically Trapped Non-Spherical Micro-Particle using Iterative Learning Control

Connor Edlund¹, Rachit Shrivastava¹, and Murti V. Salapaka¹

Abstract—Optical trapping of micron to sub-micron sized particles has enabled significant advancements in science and engineering. For spherical particles, the use of dynamic model-based feedback control has resulted in increasingly fine control over particle position and in turn new scientific discoveries. Extending the success of dynamic control to the case of particles of arbitrary shape is difficult as the complex interactions between the particle and laser light do not lend themselves well to dynamic models suitable for real-time use. In this article a learning-based, model-free control approach for the automated manipulation of non-spherical particles is introduced. This approach is instantiated with the automatic out-of-plane rotation of a sub-micron cylinder in dynamic simulation using visual position and orientation feedback.

I. INTRODUCTION

Ashkin in 1970 discovered that the radiation pressure of lasers can be used to apply forces on micron or smaller sized neutral and transparent particles [1]. More importantly, in [2] he demonstrated that when a single laser is focused by a high numerical aperture lens and made incident on a micron sized particle, it can trap the particle; here, *gradient forces* that act in the direction of maximal intensity of the laser provide a stabilizing force that counters the destabilizing forces, resulting in an equilibrium near the focal point of the laser. The use of a *single laser* to hold and manipulate particles opened diverse and widespread applications where traps employed at the micrometer and nanometer scales have yielded breakthroughs in physics and biology. As evidence of the importance of “optical tweezers” and their many applications, Ashkin won the 2018 Nobel Prize in Physics.

While the phenomenon of trapping a transparent particle in the focal point of a strongly focused laser beam is qualitatively well understood, realizing a *dynamical model* that governs the motion of the particle under the laser’s influence for real-time use is challenging. For spherical particles different regimes of the particle size with respect to the laser wavelength require different analysis approaches; when the particle size is much smaller than laser wavelength (the Rayleigh regime) a dipole approximation of the particle is used and conversely when the particle size is much larger than laser wavelength (ray optics regime) geometric optics is used [3]. Empirically it is observed that a particle in between these two size regimes can be trapped, though quantifying the force requires the full electromagnetic scattering problem to be solved [4]. For the simple case of a spherical particle, parametric models of its dynamics in a

laser trap where the parameters such as stiffness and damping are determined from experimental data are obtainable and extensively employed. The parametric dynamic model has enabled the realization of enhanced tracking performance of trapped spherical particles achieved via the high-bandwidth manipulation of lasers coupled with high-bandwidth position sensing at micron and smaller scales [5], [6].

Just as the trapping of spherical particles has revolutionized many areas of science including intracellular transport and microfluidics, the ability to automatically trap and orient micron or sub-micron particles of arbitrary shape will significantly impact areas of biology, physics, and engineered systems at the small scale. However, there are significant challenges in automating the process of trapping and configuring a non-spherical particle in any desired orientation. As alluded to earlier, the physics of a laser interacting with a particle does not lend itself to dynamical models and a non-sphericle particle’s many degrees of freedom make empirical parametric models challenging. Consequently methodical prescription of how to trap non-spherical particles in required configurations has remained largely based on empirical studies and constrained to particular particle shapes. Given the difficulty of realizing tractable models that govern the dynamics of a non-spherical particle, learning-based approaches offer significant potential in automating the movement and orientation of optically trapped, arbitrarily shaped particles.

We now briefly summarize results on controlling the orientation of trapped non-spherical particles with a focus on rotating particle orientation. Red blood cells are trapped and rotated in [7]. The birefringent properties of calcite fragments are used to induce their rotation in [8]. The combination of a Laguerre-Gaussian beam and a plane wave are used to rotate particles in [9]. The orientation of a rod-shaped bacterium is controlled in [10] using a single laser time-shared between multiple points on the bacterium. Orientation of a rod-shaped bacterium is also achieved using a spatial light modulator [11] and by rapidly scanning a trap back and forth across the bacterium [12]. [13] extends the method of [11] to control multiple bacterium simultaneously. In [14] the process of trapping and orienting elliptically shaped diatom plankton using multiple traps is automated via a visual-based approach. A review of manipulation work and its applications is available in [15].

Orientation manipulation enables comprehensive study of the trapped particle. In [16], a method similar to the one in [11] is used to achieve 360° tomographic imaging of a non-spherical cell to determine a three-dimensional map of

*This work was not supported by any organization

¹Department of Electrical and Computer Engineering, University of Minnesota, 200 Union St SE, Minneapolis, MN 55455 murtis@umn.edu

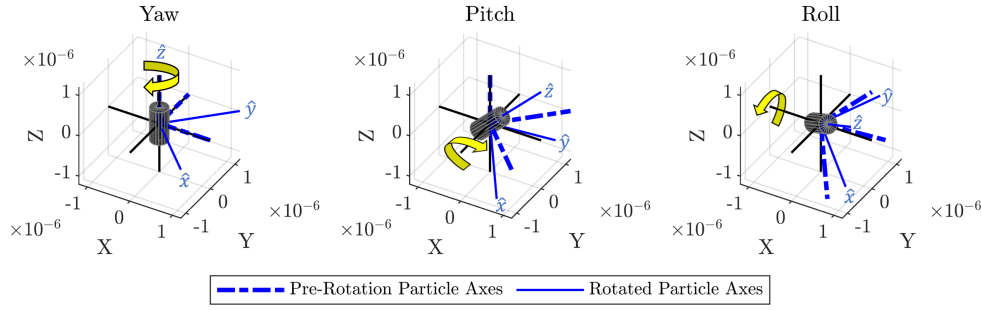


Fig. 1. Rotations of the particle and its coordinate system.

its index of refraction. In [17], the rotation of spherical cells for the purposes of cell surgery and injection is achieved via a dynamic model-based feedback controller. Orientation manipulation is used to study bacterial motility [18]. In non-biological areas, orientation manipulation is useful in micro-assembly [19].

The aforementioned studies concerning the rotation of non-spherical particles use empirical observations together with multiple traps to manipulate particles. Here, often, it is observed that a particle of specific shape can be trapped at multiple locations along its body and these traps serve as end effectors on the particle to achieve the desired orientation. As alluded to earlier, the applicability of laser field based manipulation will be substantially enhanced if the learning of the end effector motion patterns can be automated. In this article, a data-driven approach is provided where the task is instantiated with controlling the out-of-plane rotation of a cylinder using two laser foci whose motion is constrained to the horizontal plane, a common limitation in optical trapping instruments.

Iterative learning (IL) control offers an attractive approach to achieving desired orientation control without requiring a model for controller design [20], [21]. IL control generates a feedforward input that results in a desired manipulation by performing multiple trial iterations of the control task and using the error from previous iterations to better inform the feedforward input for the next trial. In this work, the control task is the out-of-plane rotation of the cylinder and the locations of the two beam foci are the feedforward input.

Toward the learning objective, it is possible to perform the learning iterations via simulation of the particle's dynamics. The problem of deriving a suitable model for the force and torque on the particle is circumvented by using at each simulation time step an open source software [22], [23] to solve for the force and torque on the particle given its position and orientation. The *in-silico* demonstration presented here provides a first study on the effectiveness of data-driven, learning-based methods for determining manipulation control strategies for non-spherical particles.

This work is organized as follows. Section II outlines the Langevin dynamics model for a non-spherical particle and its discretization for simulation. The IL controller and a closed-loop controller are developed in Section III. In Section

IV, the parameters and dimensions of our cylinder trapping problem are defined and simulation results demonstrating the successful out-of-plane rotation of the cylinder are presented. Finally, we offer concluding remarks and directions for future work.

II. SIMULATION FRAMEWORK

A. Particle Dynamics

In this section, a model for a non-spherical particle describing its dynamics under opto-mechanical forces caused by an incident laser is provided. We assume throughout that the particle's translational and rotational motion is uncoupled. The configuration of the particle is determined by the position of its center of mass and its orientation. To track these variables we consider two coordinate systems: an inertial lab coordinate system and a particle coordinate system that is fixed to the particle, both pictured in Figure 1. The axes of the lab coordinates are labeled X , Y , and Z . The laser beam is coincident with the Z axis and is focused by an objective towards the positive Z direction. The particle coordinate system axes are defined using unit vectors $\hat{x}$, $\hat{y}$, and $\hat{z} \in \mathbb{R}^3$, which are expressed in terms of laboratory coordinates. The orientation of the particle is tracked using the yaw, pitch, and roll of the particle coordinate system relative to the lab coordinate system (depicted in Figure 1). Yaw is a rotation about the Z -axis, pitch is a subsequent rotation about the Y -axis, followed by roll which is a rotation about the X -axis. These three angles are represented by the vector $\theta \in \mathbb{R}^3$, while the position of the center-of-mass is represented by vector $x \in \mathbb{R}^3$.

Due to the scale and Brownian motion of the optically manipulated particle, dynamic variables x and θ exhibit Langevin dynamics, as described in [3]. Since the time scale of the optical trap's restoring force is much greater than the relaxation time of the particle's momentum, we will assume over-damped Langevin dynamics which are given by (1) and (2) below:

$$\frac{d}{dt}x(t) = \gamma_x^{-1}F(x(t), \theta(t), \beta(t)) + \sqrt{2D_x}W_x(t), \quad (1)$$

$$\frac{d}{dt}\theta(t) = \gamma_\theta^{-1}T(x(t), \theta(t), \beta(t)) + \sqrt{2D_\theta}W_\theta(t). \quad (2)$$

Here $W_x(t) \in \mathbb{R}^3$, $W_x(t) \sim N(0, I^{3 \times 3})$, $W_\theta(t) \in \mathbb{R}^3$, and $W_\theta(t) \sim N(0, I^{3 \times 3})$. T is the temperature and k_B is Boltzmann's constant. The matrices $\gamma_x \in \mathbb{R}^{3 \times 3}$ and $\gamma_\theta \in \mathbb{R}^{3 \times 3}$ are the translation and rotation drag coefficient matrices, respectively, which relate the velocity of the particle to the friction force from the surrounding fluid. Since the particle considered here is a cylinder and therefore axi-symmetric, the drag coefficient matrices are diagonal with non-zero entries along the diagonal [24] and are therefore invertible. Matrices $D_x \in \mathbb{R}^{3 \times 3}$ and $D_\theta \in \mathbb{R}^{3 \times 3}$ are the translation and rotation diffusion matrices of the particle, respectively. These two diffusion matrices quantify the variance of the Brownian motion affecting the particle. They are related to the particle's drag coefficient matrices by the Einstein relation, which is $D_x = k_B T \gamma_x^{-1}$ for translation and $D_\theta = k_B T \gamma_\theta^{-1}$ for rotation. Values for both the friction coefficient matrices and the diffusion matrices for a cylinder are taken from literature [24]. The function $F(x(t), \theta(t), \beta(t))$ is the force the laser beam exerts on the particle. This force is orientation-dependent due to the particle being non-spherical and is dependent on the location, $\beta(t) \in \mathbb{R}^3$, of the beam. The same is true for $T(x(t), \theta(t), \beta(t))$, which is the torque the beam exerts on the particle.

The particle coordinate system is determined using the diffusion and drag coefficient matrices. Typically the diffusion matrices D_x and D_θ are reported in a particle coordinate system whose origin is the particle's center-of-diffusion. Similarly, the drag coefficient matrices γ_x and γ_θ are defined in a particle coordinate system whose origin is the center-of-reaction (see [24] for definitions of these two centers). Since a cylinder is axisymmetric, both its center-of-diffusion and center-of-reaction coincide with its center-of-mass [24]. Consequently the particle coordinate systems for both the diffusion and drag coefficient matrices are the same. For convenience we take the particle coordinate system $[\hat{x} \ \hat{y} \ \hat{z}]$ (with which we define the orientation of the particle) to coincide with the particle coordinate system of the diffusion and friction coefficient matrices. This results in the axis $\hat{z}$ being coincident with the cylinder's axis of symmetry and the origin coincident with the center of the cylinder, as depicted in Figure 1.

B. Numerical Simulation

For numerical simulation the dynamics in (1) and (2) are discretized and stepped through time following the procedure outlined in [3]. The discrete-time dynamics of x are given by:

$$\begin{aligned} x(i+1) \\ = x(i) + \Delta t \gamma_x^{-1} F(x(i), \theta(i), \beta(i)) + \sqrt{2D_x \Delta t} W_x(i), \end{aligned} \quad (3)$$

where $W_x(i) \in \mathbb{R}^3$ and $W_x(i) \sim N(0, I^{3 \times 3})$. Since D_x is given in particle coordinates the stochastic increment $\sqrt{2D_x \Delta t} W_x(i)$ is determined in particle coordinates then transformed to laboratory coordinates to update x .

Transforming from particle to laboratory coordinates is done using the coordinate transformation matrix $\Sigma_{p-l}(i) = [\hat{x}(i) \ \hat{y}(i) \ \hat{z}(i)] \in \mathbb{R}^{3 \times 3}$. Similarly, since γ_x is reported in particle coordinates, we represent the force in particle coordinates using $\Sigma_{p-l}^{-1}(i)$, then scale it by $\Delta t \gamma_x^{-1}$ and transform the result to laboratory coordinates.

To update the orientation as done in [3], we first determine in particle coordinates the increments in yaw, pitch, and roll due to the stochastic term and the torque applied to the cylinder as follows:

$$\begin{aligned} \Delta \theta_p = \Delta t \gamma_\theta^{-1} \Sigma_{p-l}^{-1}(i) T(x(i), \theta(i), \beta(i)) \\ + \sqrt{2D_\theta \Delta t} W_\theta(i), \end{aligned} \quad (4)$$

where the subscript p indicates particle coordinates, $W_\theta(i) \in \mathbb{R}^3$, $W_\theta(i) \sim N(0, I^{3 \times 3})$, and $\Sigma_{p-l}^{-1}(i)$ is the aforementioned transformation matrix. We take the rotational perturbation $\Delta \theta_p$ and apply it to the unit vectors of the particle coordinate system expressed in particle coordinates. When expressed in particle coordinates these unit vectors before perturbation are $[\hat{x}_p \ \hat{y}_p \ \hat{z}_p] = I^{3 \times 3}$. The rotated particle coordinate axes are:

$$[\hat{x}'_p \ \hat{y}'_p \ \hat{z}'_p] = R_x(\Delta \theta_{p,x}) R_y(\Delta \theta_{p,y}) R_z(\Delta \theta_{p,z}) I^{3 \times 3}, \quad (5)$$

where the superscript $(\cdot)'$ indicates the updated unit vectors and $R_x(\Delta \theta_{p,x})$, $R_y(\Delta \theta_{p,y})$, $R_z(\Delta \theta_{p,z}) \in \mathbb{R}^{3 \times 3}$ are the standard rotation matrices. These rotated particle coordinate axes are then transformed to lab coordinates, which also updates the coordinate transformation matrix as follows:

$$\Sigma_{p-l}(i+1) = \Sigma_{p-l}(i) [\hat{x}'_p \ \hat{y}'_p \ \hat{z}'_p]. \quad (6)$$

The force and torque terms in (3) and (4) are determined at each time step using the Optical Tweezer Toolbox's "forcetorque" function [22], [23]. This function uses the T-matrix method which is a framework for the efficient repeated solving of an electromagnetic (EM) scattering problem. The scattering problem amounts to taking an incident wave and finding the scattered wave created by the particle. The T-matrix method requires the EM scattering problem to be solved once, after which the scattered waveform is expressed as $b = Ta$, where a and b are the multipole expansions of the incident and scattered waves, respectively, and T is the T-matrix. Multiple methods exist for solving the scattering problem and the best choice is informed by the constraints and dimensions of the problem. We have selected the discrete dipole approximation (DDA) method based on the guidance in [25]. A built-in function of the Optical Tweezer Toolbox is used to solve for the T-matrix using the DDA method. The T-matrix depends only on the properties of the scattering particle [4], so once it is known it can be used repeatedly to solve the EM scattering problem of optical trapping for different particle orientations and positions. The "forcetorque" function uses the T-matrix

and incident beam information to determine the scattered beam and estimate the force and torque on the particle. This algorithm is implemented in MATLAB© with a time step of $\Delta t = 8 \times 10^{-7}$ s, which keeps the dynamics in the over-damped regime [3]. A more in-depth review of the toolbox's underlying theory can be found in [4].

III. CONTROL STRATEGY

An important objective here is to learn the motion profile of a laser's focal point that will result in the desired manipulations of a non-spherical particle. The difficulty of deriving a suitable dynamic model makes learning-based approaches desirable. The problem is instantiated by learning the motion profiles of two laser beams' focal points that result in the out-of-plane rotation of a cylinder. When a cylinder is successfully trapped in a single beam, it will assume a vertical orientation with its axis of symmetry coincident with the beam axis. The learned motion profiles for the two foci need to rotate the cylinder from a vertical stable equilibrium orientation to a horizontal orientation where its axis of symmetry is aligned with the X -axis. The rotation needs to be achieved without moving either of the two focal points in the Z -direction as is required in some other approaches [13], [17].

The two beams are created by using an acousto-optic deflector to rapidly multiplex a single beam between two desired beam foci locations along the X -axis. The multiplexing rate is at a much higher bandwidth than the time constant of the particle's dynamics. Consequently, to the cylinder the multiplexed laser appears to be two individual laser beams. To avoid switching the laser between beam focal point positions faster than realistically possible, a minimum time delay between laser shifts is enforced. To accomplish rotation with the movement of the two focal point locations both a closed-loop controller and an IL controller are used, as illustrated in the block diagram of Figure 2.

A. Closed-Loop Controller

The purpose of the closed-loop controller is to ensure that the center of the cylinder stays at the origin of the laboratory coordinate system while it is being rotated to the desired horizontal position aligned along the X -axis. The feedback error signal employed is the difference between the center of the cylinder's X -position and the origin. The controller output is the multiplexing duty ratio δ , which is the fraction of the time the laser beam spends in one position to the time spent in the other. To start the duty ratio is $\delta = 0.5$, where the beam spends an equal amount of time at each of the two focal point positions. Reducing the time spent at one focal point position reduces the effective optical power at that location. Consequently, if the error indicates the cylinder is sliding away from the origin towards one of the two focus locations, the controller reduces the power at that focus location and increases the power at the other by adjusting the duty ratio accordingly. This is accomplished using a simple proportional controller. To avoid unrealistic duty cycles, duty

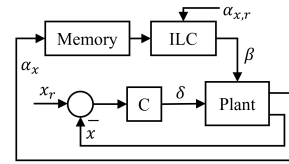


Fig. 2. Control Diagram

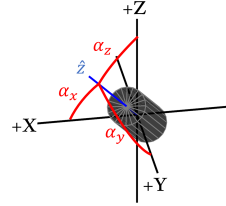


Fig. 3. Defining the cylinder's orientation using $\hat{z}$'s direction angles relative to the X , Y , and Z axes.

cycles less than 0.25 or greater than 0.75 are set to zero or one, respectively.

B. Iterative Learning Controller

Rotating the cylinder is achieved using the motion profiles of the two beam focal points and we learn their motion profiles using an IL controller. For learning, the IL controller relies on multiple trial iterations of the same duration during which the desired manipulation is attempted using a feedforward (i.e. pre-determined) input. The time-series error between the desired performance of the system and its actual performance is stored in memory during each iteration. In between iterations, the IL controller processes the error and uses it to modify the feedforward input for the next iteration in order to reduce the observed error. Typically the error seen in early iterations is quite high and the IL controller changes the feedforward input considerably with each update. As iterations progress the IL controller hones in on a better feedforward input, the error begins to decrease, and its updates to the feedforward input get smaller and smaller. Eventually the desired performance is achieved and iterations are ceased. For the control problem considered here, the feedforward input is the motion profiles of the two focal points the laser beam is multiplexed between and our desired manipulation is the rotation of the cylinder.

As the goal of the iterative learning controller is to rotate the cylinder, we define the error it operates on using the orientation of the cylinder's $\hat{z}$ -axis, as pictured in Figure 3. We track the direction angle, $\alpha_x \in \mathbb{R}$, the $\hat{z}$ -axis makes with the laboratory coordinate system X -axis. In Figure 3 the direction angles $\alpha_y \in \mathbb{R}$ and $\alpha_z \in \mathbb{R}$ the $\hat{z}$ axis makes with the Y and Z laboratory coordinate axes, respectively, are also pictured. The angle α_x at time instant k in iteration j is labeled as $\alpha_x^{(j)}(k)$. The chosen reference trajectory, denoted $\alpha_{r,x}(k) \in \mathbb{R}$ for time instant k , for angle α_x is pictured in Figure 4. In this figure $\alpha_{r,x}$ goes from $\pi/2$ to 0 indicating a rotation to a horizontal orientation. The transition from $\pi/2$ to 0 in the reference trajectory is not instantaneous as

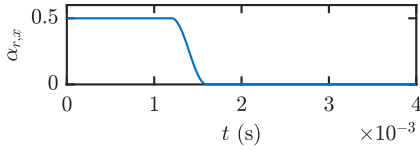


Fig. 4. Reference Angle Trajectory $\alpha_{r,x}(t)$

this results in the IL controller producing smoother motion profiles for the two focal points. With the reference trajectory defined we can now define the error as follows:

$$e^{(j)}(k) = \sin(\alpha_{r,x}(k)) - \sin(\alpha_x^{(j)}(k)), \quad (7)$$

where $k = [1, 2, \dots, n_s]$ is the index of the sampling instances of the cylinder's orientation, with n_s being the total number of samples. We take the sine of the reference angle and actual angle so there is no error penalty if the cylinder turns one direction or another to reach the horizontal configuration.

With the error defined we now detail the motion profiles the IL controller will generate from it. As mentioned earlier, during each of the learning trials the rotation of the cylinder is attempted using the pre-determined, feedforward motion of the two focal points the laser beam is multiplexed between. The motion profiles for the first and second focal points are denoted $\beta_1^{(j)}(i) \in \mathbb{R}^3$ and $\beta_2^{(j)}(i) \in \mathbb{R}^3$, respectively, for time instant i in iteration j . The two foci are only moved along the X -axis of the laboratory coordinate system where only the X -components, $\beta_{1,x}^{(j)}$ and $\beta_{2,x}^{(j)}$, of the motion profiles are non-zero. Furthermore, the X -position of the second focal point is determined by $\beta_1^{(j)}(i) = -\beta_2^{(j)}(i)$ for all time instants i and iterations j . Consequently the only control variable is the X -component of the first focal point's motion profile, $\beta_{1,x}^{(j)}$, which the IL controller generates. The IL process is initialized with a motion profile that keeps the foci at the origin, i.e. $\beta_1^{(1)}(i) = \beta_2^{(1)}(i) = [0 \ 0 \ 0]^T$ for all time instances i . The IL controller generates the X -component of the first focal point's motion profile, $\beta_{1,x}^{(j+1)}$, for the next iteration $j+1$ using the last iteration's time-series error $e^{(j)}$ and the X -component of its motion profile for the first focal point, $\beta_{1,x}^{(j)}$.

Exactly how the IL controller will learn from the time-series error $e^{(j)}$ to better determine the X -component of the first focal point's motion profile depends on the style of IL controller employed. Here a proportional-derivative (PD) IL controller is used as described in [26]. The PD-IL controller is introduced first in its most basic form, then afterwards an additional tuning parameter is added. The basic form is:

$$\begin{aligned} \beta_{1,x}^{(j+1)}(i) \\ = \beta_{1,x}^{(j)}(i) - K_p e^{(j)}(i+1) - K_d (e^{(j)}(i+1) - e^{(j)}(i)), \end{aligned} \quad (8)$$

for $i \in [1, 2, \dots, n_{dt} - 1]$, where n_{dt} is the number of time-steps. Constants $K_p \in \mathbb{R}$ and $K_d \in \mathbb{R}$ are used to tune the controller, which is typically done by trial-and-error. Note

that the j^{th} -trial error at the next time-step $e^{(j)}(i+1)$ impacts the motion profile for the $(j+1)^{th}$ -trial at time i through the proportional term, K_p , and the derivative term, K_d . The resulting action is non-causal.

The ability of the PD-IL controller to preview the error with a longer time horizon is possible using an additional $n_{adv} \in \mathbb{Z}^+$ forward time-steps:

$$\begin{aligned} \beta_{1,x}^{(j+1)}(i) = \beta_{1,x}^{(j)}(i) - K_p e^{(j)}(i+1+n_{adv}) \\ - K_d (e^{(j)}(i+1+n_{adv}) - e^{(j)}(i+n_{adv})), \end{aligned} \quad (9)$$

where $i \in [1, 2, \dots, n_{dt} - n_{adv}]$. For time indices $n_{dt} - n_{adv} < i < n_{dt} - 1$, the PD-IL controller reverts to the form in (8). After trial-and-error tuning to determine K_p , K_d , and n_{adv} , it was found that a proportional-only IL controller ($K_d = 0$) with $n_{adv} = 300$ works best for this application.

C. Controller Sample Rate

As mentioned in the preceding closed-loop and iterative learning controller designs, we have accounted for a finite measurement sample rate of the cylinder's position and orientation. For obtaining the cylinder's position and orientation, image based tracking is a viable option with precedence in literature [14], [12], [27]. To achieve rotation and position stabilization in the time-scales considered here, a high-speed camera would be used as often done in microfluidics [28]. High-speed cameras are available with frame rates exceeding 100 kHz. For these simulations we assume a camera and image processing system can provide a position and orientation sample rate of 10 kHz. The angle orientation of the cylinder can be estimated offline since it is only needed in between iterations, while the position can be estimated online in between image samples using threshold pixel value detection. In the results section we discuss the trade-offs of using lower sample rates.

IV. RESULTS

A series of simulations are performed to confirm that the IL controller learns a motion profile for the two focal points that, together with the closed-loop system, successfully rotates the cylinder from a vertically oriented equilibrium position to a horizontally oriented position. The learning iterations are performed and their results examined. The resulting two focal point motion profiles are analyzed to provide a rationale for its features from a physical viewpoint. To underscore the utility of this learned motion profile, the IL controller is then replaced with a feedforward controller that executes the final motion profile learned by the IL controller and the ensuing results are assessed.

The simulated optical tweezers system considered uses a 1.2 numerical aperture objective lens with a vertically polarized, 1064 nm, continuous-wave, Gaussian-profile laser beam operating at 50 mW. Temperature in the sample chamber is assumed to be 296 K with a medium whose viscosity is $9.359 \times 10^{-4} \text{ N} \cdot \text{s/m}$ and refractive index is $\eta_m = 1.33$. The cylinder is 800 nm in length and 400 nm in

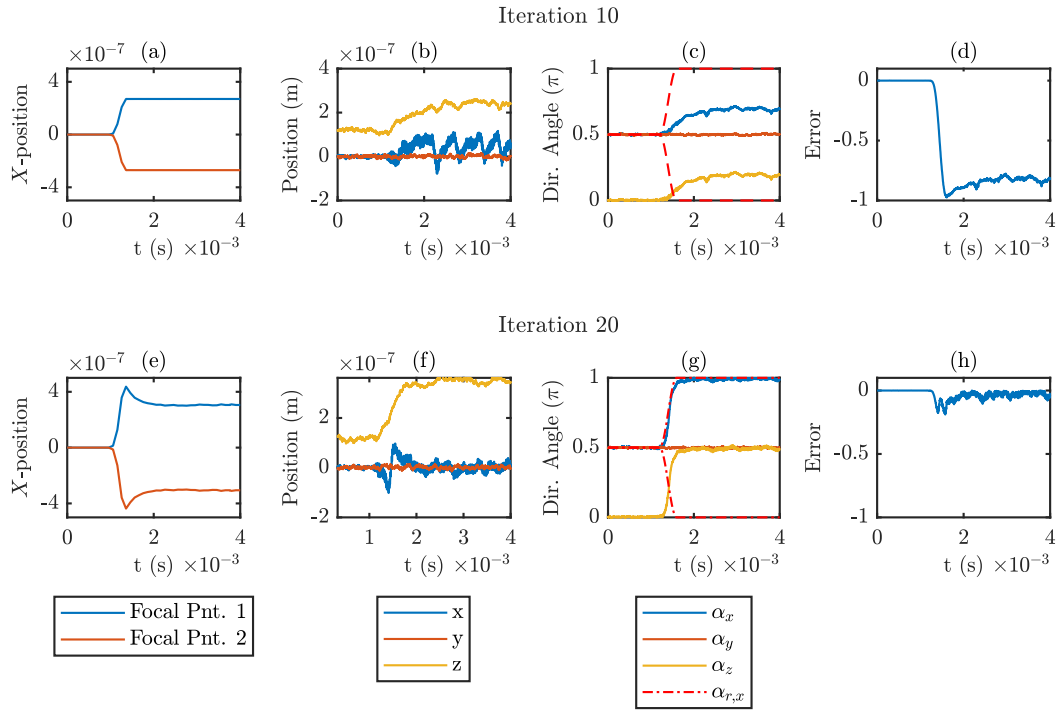


Fig. 5. Results for iterations 10 and 20 of the IL process. For each iteration there are four plots arranged horizontally; the leftmost plots show the motion profiles of the two beams' focal points used to generate the results in the three plots to the right.

diameter with a refractive index of $\eta_c = 1.59$. As mentioned in Section II this cylinder follows the dynamics in (1) and (2), which are presented in discretized form in (3) and (4). The Optical Tweezer Toolbox [22], [23] provides the force and torque on the particle.

First the simulation setup for the IL process is detailed. Each iteration has a simulated time duration of 4 ms, during which the learned motion profiles for the two focal points are applied. In each iteration the cylinder is allowed 1 ms to reach an equilibrium before the IL controller action is implemented. For the first iteration both foci are kept at the origin for the entire duration, i.e. $\beta_1^{(1)}(i) = \beta_2^{(1)}(i) = [0 \ 0 \ 0]^T$ for all time instants i . In between the following iterations the IL controller updates the two motion profiles using the error defined in (7) of the previous run. Iterations are ceased when the desired performance is achieved.

Results for the first 20 iterations of this learning process are used to evaluate its performance, with the results of the 10th and 20th iterations shown in Figure 5. The IL controller takes the first eight or nine iterations to learn a profile that perturbs the cylinder and in iteration 10 these perturbations are noticeable. For both iterations in Figure 5 there are four plots arranged horizontally. Plots (a) and (e) contain the X -position of both focal points as a function of time. These motion profiles are the feedforward inputs that the IL controller generates. The focal point motion profiles for iteration 10 and 20 are the outputs of the IL controller after iterations 9 and 19, respectively. In iteration 10 the cylinder is subjected to the motion profile in plot

(a), resulting in the dynamics plotted in (b)-(d). Similarly, for iteration 20 the motion profile in plot (e) results in the dynamics plotted in (f)-(h). The plots (b) and (f) are the displacements of the center of the cylinder as a function of time. As apparent, the X and Y coordinates oscillate around the origin, as maintained by the closed-loop controller. In the 20th iteration, the Z -position of the center of the cylinder starts at around $Z = 1 \times 10^{-7}$ m, its equilibrium position when trapped vertically, and moves to a new equilibrium around $Z = 3.5 \times 10^{-7}$ m.

The plots (c) and (g) show the direction angles $[\alpha_x \ \alpha_y \ \alpha_z]$ the cylinder's axis of symmetry makes with the laboratory XYZ coordinate axes as a function of time. The reference angle trajectory $\alpha_{r,x}$ is also plotted, along with its image mirrored across the line at $\pi/2$; both are plotted as the error defined in (7) allows angle α_x to go from $\pi/2$ to π or from $\pi/2$ to 0 as both paths indicate the cylinder reaching the horizontal position. The plots (d) and (h) depict the resulting error, defined by (7), as a function of time. The direction angle plot for iteration 10 shows that the cylinder begins to rotate, but stops at an intermediate angle instead of reaching the fully horizontal position. The error plot to the right shows a considerable error for 10th iteration. In iteration 20 we see that α_x closely follows the reference angle trajectory and α_z moves from 0 to $\pi/2$, corroborating a successful rotation to the horizontal position. Meanwhile angle α_y remains at $\pi/2$ indicating the cylinder has rotated to a horizontal position aligned with the X -axis as desired. The error for iteration 20 is small in magnitude with

some fluctuations around zero-error while in the horizontal position. This is due to the horizontally trapped position being slightly less stable than the vertically trapped position and the efforts of the closed-loop controller in keeping the cylinder centered at the origin. The learning process depicted in Figure 5 was repeated ten more times and all of these repeat learning processes learned a beam profile effectively identical to the one of iteration 20 in Figure 5 and achieved rotation with similar low error.

Intuition as to why the two focal point motion profiles of iteration 20 result in the successful rotation of the cylinder can be gleaned from the iteration's results in Figure 5. The two focal points begin to diverge from each other on the X -axis at approximately the 1 ms mark, and as seen in the direction angle plot, the rotation of the cylinder follows soon after. The small duration of focal point actuation before rotation happens is necessary to achieve rotation and the actuation begins before the typical point in time the error used by the IL controller becomes nonzero. A causal IL controller policy would not be effective in determining the necessary pre-actuation from the error. Here $n_{adv} = 300$ is used, as the added error preview allows the IL controller to learn the pre-actuation needed to match the reference angle trajectory. After this small duration of pre-actuation, the diverging focal point positions cause the cylinder to begin tilting at which point one focal point will be closer in X -coordinate to the top and the other will be closer in the X -coordinate to the bottom. This creates opposing influence on the cylinder's ends; one focal point attracts the top of the cylinder, the other the bottom. The focal points continue to diverge until they are just past $\pm 4 \times 10^{-7} m$, slightly exceeding the length of the cylinder, which given how the top and bottom of the cylinder are attracted to the focal points, goads the cylinder into a horizontal orientation. The focal points then quickly return to approximately $\pm 3 \times 10^{-7} m$, which maintains the cylinder in its now horizontal orientation.

Meanwhile the closed-loop controller works to maintain the center of the cylinder at the origin. Figure 6 shows again in larger detail the X -position of the cylinder during iteration 20. The bottom plot in Figure 6 shows the beam multiplexing duty ratio δ as a function of time. A duty ratio greater than 0.5 increases the amount of time the multiplexed laser beam spends at the focal point location on the positive X -axis, while a duty ratio less than 0.5 increases its time at the focal point location on the negative X -axis. Figure 6 demonstrates that the feedback controller acts to maintain the X -position near zero; close to 1.5 s it is evident that the controller alters the duty ratio to first counter the negative excursion of the X -position, leading to a positive overshoot of the X -position. Then duty ratio is eventually brought back to 0.5; the time constant is seen to be close to 2 ms .

Now that the two focal point motion profiles have been learned, their ability to function as a feedforward input in the absence of the IL controller is tested. Specifically, in the control system shown in Figure 2 the "Memory" and "ILC" blocks are removed with the closed-loop controller remaining. The motion profiles of the two focal points used

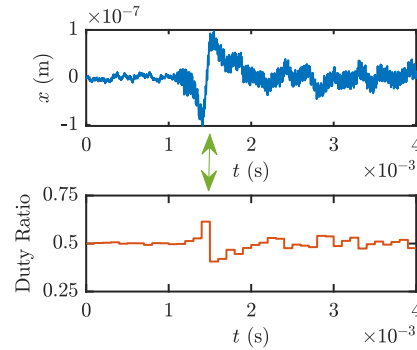


Fig. 6. X -position vs. beam multiplexing duty ratio for iteration 20.

to produce the results of iteration 20 are applied to the closed-loop system as a feedforward input. This exercise is repeated 50 times to ensure the performance is robust. The time-series results from each run are superimposed over one another in the same plots and included here as Figure 7. Each of the position and orientation variables in this figure are plotted separately so it can be seen there are no outliers. In the top three plots, the X , Y , and Z -positions all exhibit identical behavior seen previously for iteration 20 in Figure 5. The same holds for the direction angles. Angle α_x is seen to diverge from $\pi/2$ to either π or 0, indicating that each run is reaching the horizontal orientation even if they may rotate different directions to do so. The α_z angle corroborates this, and angle α_y is equal to $\pi/2$ at all times indicating the horizontal orientation is aligned with the X -axis as desired. The exercise of applying the learned beam focal point motion profiles as a feed-forward input 50 times was repeated using the learned motion profiles of 10 other repetitions of the learning process, with results for all identical to those pictured in Figure 7.

All results for this study were generated using a controller sample rate of 10 kHz as mentioned in Section III. In simulation it was seen that rotation can be achieved with sample rates as low as 5 kHz , but at the price of increased noise. With the lower sample rate there are more fluctuations in the position and orientation of the cylinder when in the horizontally trapped position. A sample rate of 10 kHz offered the best balance of performance and practicality.

V. CONCLUSIONS

The results of this work demonstrate via simulation the successful and repeatable out-of-plane rotation of a sub-micron cylinder via an automated iterative learning control process. Both the iterative learning and closed-loop controllers used do not require a model of the particle's dynamics for design. This circumvents the aforementioned issue of finding a dynamic model of the particle-laser beam interactions suitable for real-time use, which has restricted previous manipulation strategies to empirical methods on restricted particle shapes. The work here serves as a first proof-of-concept for the broader goal of automating the micro-manipulation of any arbitrarily shaped particle. Achieving

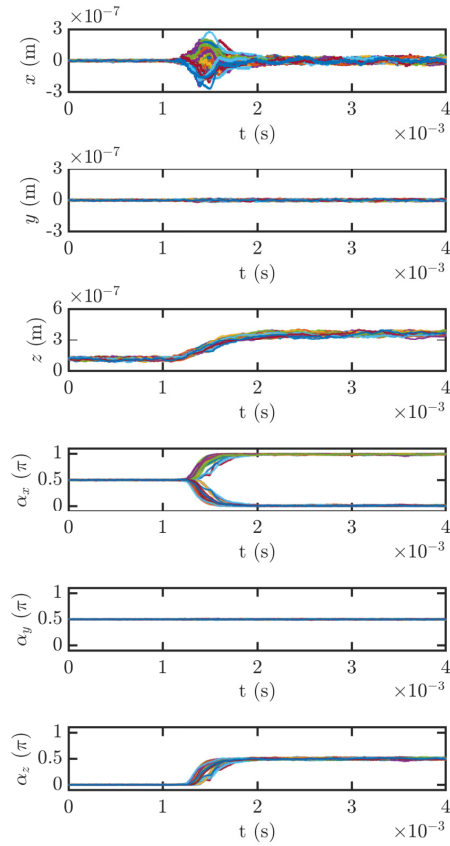


Fig. 7. 50 runs using ILC generated feedforward input.

this goal will enable fast and flexible fine-manipulation at the micron and sub-micron scales, opening doors for advancements in many areas of science. An experimental implementation of the example given in this work is the next step, followed by the extension of this method to greater degrees of control and particles of arbitrary shape.

REFERENCES

- [1] A. Ashkin, "Acceleration and trapping of particles by radiation pressure," *Physical review letters*, vol. 24, no. 4, p. 156, 1970.
- [2] A. Ashkin, J. M. Dziedzic, J. E. Bjorkholm, and S. Chu, "Observation of a single-beam gradient force optical trap for dielectric particles," *Optics letters*, vol. 11, no. 5, pp. 288–290, 1986.
- [3] P. H. Jones, O. M. Maragò, and G. Volpe, *Optical tweezers: Principles and applications*. Cambridge University Press, 2015.
- [4] T. A. Nieminen, N. du Preez-Wilkinson, A. B. Stilgoe, V. L. Loke, A. A. Bui, and H. Rubinsztajn-Dunlop, "Optical tweezers: Theory and modelling," *Journal of Quantitative Spectroscopy and Radiative Transfer*, vol. 146, pp. 59–80, 2014.
- [5] H. Sehgal, T. Aggarwal, and M. V. Salapaka, "High bandwidth force estimation for optical tweezers," *Applied Physics Letters*, vol. 94, no. 15, p. 153114, 2009.
- [6] S. Roychowdhury, S. Bhaban, S. Salapaka, and M. Salapaka, "Design of a constant force clamp and estimation of molecular motor motion using modern control approach," in *2013 American Control Conference*. IEEE, 2013, pp. 1525–1530.
- [7] S. Sato, M. Ishigure, and H. Inaba, "Optical trapping and rotational manipulation of microscopic particles and biological cells using higher-order mode nd: Yag laser beams," *Electronics letters*, vol. 27, no. 20, pp. 1831–1832, 1991.
- [8] M. E. Friese, T. A. Nieminen, N. R. Heckenberg, and H. Rubinsztajn-Dunlop, "Optical alignment and spinning of laser-trapped microscopic particles," *Nature*, vol. 394, no. 6691, pp. 348–350, 1998.
- [9] L. Paterson, M. P. MacDonald, J. Arlt, W. Sibbett, P. Bryant, and K. Dholakia, "Controlled rotation of optically trapped microscopic particles," *Science*, vol. 292, no. 5518, pp. 912–914, 2001.
- [10] K. Visscher, G. Brakenhoff, and J. Krol, "Micromanipulation by "multiple" optical traps created by a single fast scanning trap integrated with the bilateral confocal scanning laser microscope," *Cytometry: The Journal of the International Society for Analytical Cytology*, vol. 14, no. 2, pp. 105–114, 1993.
- [11] V. Bingelyte, J. Leach, J. Courtial, and M. Padgett, "Optically controlled three-dimensional rotation of microscopic objects," *Applied Physics Letters*, vol. 82, no. 5, pp. 829–831, 2003.
- [12] G. Carmon and M. Feingold, "Rotation of single bacterial cells relative to the optical axis using optical tweezers," *Optics letters*, vol. 36, no. 1, pp. 40–42, 2011.
- [13] F. Hörner, M. Woerdemann, S. Müller, B. Maier, and C. Denz, "Full 3d translational and rotational optical control of multiple rod-shaped bacteria," *Journal of biophotonics*, vol. 3, no. 7, pp. 468–475, 2010.
- [14] Y. Tanaka, H. Kawada, K. Hirano, M. Ishikawa, and H. Kitajima, "Automated manipulation of non-spherical micro-objects using optical tweezers combined with image processing techniques," *Optics Express*, vol. 16, no. 19, pp. 15 115–15 122, 2008.
- [15] D. Palima and J. Glückstad, "Gearing up for optical microrobotics: micromanipulation and actuation of synthetic microstructures by optical forces," *Laser & Photonics Reviews*, vol. 7, no. 4, pp. 478–494, 2013.
- [16] Y.-c. Lin, H.-C. Chen, H.-Y. Tu, C.-Y. Liu, and C.-J. Cheng, "Optically driven full-angle sample rotation for tomographic imaging in digital holographic microscopy," *Optics letters*, vol. 42, no. 7, pp. 1321–1324, 2017.
- [17] M. Xie, A. Shakoar, Y. Shen, J. K. Mills, and D. Sun, "Out-of-plane rotation control of biological cells with a robot-tweezers manipulation system for orientation-based cell surgery," *IEEE Transactions on Biomedical Engineering*, vol. 66, no. 1, pp. 199–207, 2018.
- [18] T. L. Min, P. J. Mears, L. M. Chubiz, C. V. Rao, I. Golding, and Y. R. Chemla, "High-resolution, long-term characterization of bacterial motility using optical tweezers," *Nature methods*, vol. 6, no. 11, pp. 831–835, 2009.
- [19] P. J. Rodrigo, L. Kelemen, C. A. Alonzo, I. R. Perch-Nielsen, J. S. Dam, P. Ormos, and J. Glückstad, "2d optical manipulation and assembly of shape-complementary planar microstructures," *Optics express*, vol. 15, no. 14, pp. 9009–9014, 2007.
- [20] M. Garden, "Learning control of actuators in control systems," Jan. 12 1971, uS Patent 3,555,252.
- [21] M. Uchiyama, "Formation of high-speed motion pattern of a mechanical arm by trial," *Transactions of the Society of Instrument and Control Engineers*, vol. 14, no. 6, pp. 706–712, 1978.
- [22] T. A. Nieminen, V. L. Loke, A. B. Stilgoe, G. Knöner, A. M. Brańczyk, N. R. Heckenberg, and H. Rubinsztajn-Dunlop, "Optical tweezers computational toolbox," *Journal of Optics A: Pure and Applied Optics*, vol. 9, no. 8, p. S196, 2007.
- [23] I. C. Lenton, A. A. Bui, T. A. Nieminen, A. B. Stilgoe, and H. Rubinsztajn-Dunlop, "Optical tweezers toolbox: full dynamics simulations for particles of all sizes," in *Optical Trapping and Optical Micromanipulation XV*, vol. 10723. International Society for Optics and Photonics, 2018, p. 107232B.
- [24] J. G. de la Torre and V. A. Bloomfield, "Hydrodynamic properties of complex, rigid, biological macromolecules: theory and applications," *Quarterly reviews of biophysics*, vol. 14, no. 1, pp. 81–139, 1981.
- [25] X. Qi, T. A. Nieminen, A. B. Stilgoe, V. L. Loke, and H. Rubinsztajn-Dunlop, "Comparison of t-matrix calculation methods for scattering by cylinders in optical tweezers," *Optics letters*, vol. 39, no. 16, pp. 4827–4830, 2014.
- [26] S. Arimoto, "Mathematical theory of learning with applications to robot control," in *Adaptive and Learning Systems*. Springer, 1986, pp. 379–388.
- [27] D. B. Phillips, D. M. Carberry, S. Simpson, H. Schäfer, M. Steinhart, R. Bowman, G. M. Gibson, M. J. Padgett, S. Hanna, and M. Miles, "Optimizing the optical trapping stiffness of holographically trapped microrods using high-speed video tracking," *Journal of Optics*, vol. 13, no. 4, p. 044023, 2011.
- [28] J. S. Dudani, D. R. Gossett, T. Henry, and D. Di Carlo, "Pinched-flow hydrodynamic stretching of single-cells," *Lab on a Chip*, vol. 13, no. 18, pp. 3728–3734, 2013.