

μ -Synthesis-Based Generalized Robust Framework for Grid-Following and Grid-Forming Inverters

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Abstract—Grid-following and grid-forming inverters are integral components of microgrids and for integration of renewable energy sources with the grid. For grid following (GFL) inverters, which need to emulate controllable current sources, a significant challenge is to address the large uncertainty of the grid impedance. For grid forming (GFM) inverters, which need to emulate a controllable voltage source, large uncertainty due to varying loads has to be addressed. This article presents a generalized control framework by leveraging the voltage-current duality in the plant dynamic model of GFL and GFM inverters. The modeling of uncertainties is also generalized under the control framework by quantifying the uncertainties in grid impedance parameters and the uncertainties in equivalent loading parameters for GFL and GFM inverters, respectively. Based on the generalized control framework, a μ -synthesis-based robust control design methodology is proposed for both GFL and GFM inverters. The control objectives, while designing the proposed optimal controllers, are reference tracking, disturbance rejection, and harmonic compensation capability with *i*) sufficient *LCL* resonance damping under large variations of grid impedance uncertainty for GFL inverters and *ii*) with enhanced dynamic response under large variations of equivalent loading uncertainty for GFM inverters. A combined system-in-the-loop, controller hardware-in-the-loop, and power hardware-in-the-loop based experimental validation on 10-kVA microgrid system with two physical inverter systems is conducted in order to evaluate the efficacy and viability of the proposed controllers.

Index Terms—Grid-following inverter, grid-forming inverter, $\mathcal{H}_\infty$ -based loop shaping, parametric uncertainty, robust control.

I. INTRODUCTION

WITH the proliferation of inverter-interfaced distributed energy resources, there is a renewed emphasis on local microgrids that provide operational flexibility and aid sustainability for the energy infrastructures. Both grid-forming (GFM) and grid-following (GFL) inverters have become essential components that have pivotal roles to play in such microgrids operating both in grid-tied and islanded mode. During islanded mode, GFM inverters maintain a stable voltage and frequency

of the microgrid in the absence of grid. On the other hand, GFL inverters are usually operated to supply power where voltage and frequency are maintained either by the grid or other GFM inverters [1]. In the hierarchical structure of microgrid control system, *inner* voltage-control loops, regulating voltage to specified values, are responsible for GFM inverters to emulate controllable voltage sources. Similarly, GFL inverters emulate controllable current sources by regulating currents via *inner* current-control loop [2].

Typically GFL inverters are connected to grid via *LCL* filters for high-frequency attenuation caused by switching. Multiple important factors are considered in the design stage of GFL inverters including

- 1) resonances caused by low-damping of the *LCL* filter in GFL inverters, while connected to weak grid, which could lead to system instability. Here, proper damping of such resonance is crucial while designing [3];
- 2) Uncertainty in variation of grid impedance parameters significantly influence the robustness of the output current controller. For example, increase in grid inductance requires reduction in the gain and bandwidth of the current controller to keep the system stable that leads to degradation of tracking performance and disturbance rejection capability. Here, there is a need for control design that delivers optimal performance while guaranteeing stability for the range of grid impedance encountered in practice [4];
- 3) Grid impedance variation causes variation in resonance frequency of the inverter system that impacts active damping methods. Here, robustness of the active damping to remain effective under uncertainty is required [5];
- 4) Most importantly, the controller should result in good tracking performance, disturbance rejection, and harmonic compensation capability while remaining implementable in low-cost microcontrollers. Various types of control schemes and their advancements for GFL inverters are proposed in the literature [6].

Major current-control schemes for GFL inverters, reported in existing literature, are compared in Fig. 1(a). In summary, existing methods include classical (i.e., proportional/integral/resonant controller-based), hysteresis, sliding-mode, model predictive, repetitive, linear-quadratic regulator, $\mathcal{H}_\infty$ -based control schemes as reported in [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], [28], [29],

Manuscript received 5 July 2022; revised 18 October 2022; accepted 29 November 2022. Date of publication 2 December 2022; date of current version 26 December 2022. This work was supported by Advanced Research Projects Agency-Energy (ARPA-E) through the project titled “Rapidly Viable Sustained Grid” under Grant DE-AR0001016. Recommended for publication by Associate Editor Y. Xue. (Corresponding author: Soham Chakraborty.)

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Color versions of one or more figures in this article are available at <https://doi.org/10.1109/TPEL.2022.3226224>.

Digital Object Identifier 10.1109/TPEL.2022.3226224

Ref.	LCL Damping	Robustness Against Grid Impedance Parameter Variation	Ref.	Improved Dynamic Response	Robustness Against Equivalent Load Parameter Variation
[3] [7]–[16]	• New filter topology • Additional feedforward of grid voltage, grid current, inner capacitor current • State feedback control • Lack robust damping	• Lack robustness under varying unknown grid impedance • Designed with known, fixed, nominal grid impedance	[34] [36]–[38] [39]–[41]	• Multiloop control structure • Feedback of inductor current, inner capacitor current • Disturbance observer design • Dominant pole elimination • Need multiple measurements	• Lack robustness during variation of equivalent loading • Designed controller for rated loading condition • No loading uncertainty
[4], [5] [17]–[22] [23]–[28]	• Active/Passive damping • Additional outer current inner current loop • Inherent active damping using controller shaping • Challenging during varying grid impedance	• Lack robustness under varying unknown grid impedance • Grid impedance uncertainty not designed for controller synthesis	[42]–[44] [45]–[48] [49]–[51]	• Multiloop control structure with sliding mode, LQR, $\mathcal{H}_2/\mathcal{H}_\infty$, $\mathcal{H}_\infty/\mathcal{H}_\infty$ control • Complicated, loopy structure with multiple measurements • Challenging during varying equivalent loading conditions	• Lack robustness for full range variation of equivalent loading • Lack robust performance • Designed controller for simple load uncertainty, without non-linear load
Proposed	✓ Single loop control leads reduction in measurements ✓ Inherent active damping using controller shaping ✓ Robust damping under varying grid impedance	✓ Grid impedance uncertainty designed for controller synthesis ✓ Exhibit robustness under varying unknown grid impedance	Proposed	✓ $\mathcal{H}_\infty$-based robust control ✓ Single loop control structure ✓ No need of feedforward loop ✓ Reduction in measurements ✓ Enhancement of dynamic response by controller shaping	✓ Equivalent loading uncertainty designed for controller synthesis ✓ Exhibit robustness under the variation of equivalent loading

(a)

(b)

Fig. 1. Summary of major current-control and voltage-control strategies for grid-following and grid-forming inverters proposed since last decade, respectively.

[30], [31], [32]. Most of the existing schemes emphasize on the enhancement of the performance of the current controller under a fixed and large value of grid impedance parameter. Addition of a nonvarying grid impedance parameters during controller synthesis can enhance the controller performance to an extent but unable to guarantee the robust performance criterion under varying grid impedance parameter condition. Moreover, broadly these schemes use either passive damping with or without new filter topology or active damping using either additional measurements for feedforward control action. These additional measurements increase the cost by employing multiple sensors.

Similarly, designing the voltage controller for GFM inverter is essentially a multiobjective task. The design considerations include

- 1) reference tracking, disturbance rejection, and harmonic compensation in presence of various linear and nonlinear loads;
- 2) the voltage controllers are required to provide compensation to dynamic variations of the output load current and improve the dynamic response [33];
- 3) unknown nature of the output loading of GFM inverter can significantly alter the system behavior. Here during heavily loaded condition of GFM inverter, transient performance is severely compromised [34].

Therefore, the voltage controller should be robust enough against unknown loading uncertainties to perform all the aforementioned tasks. Numerous voltage control strategies are proposed in the literature during past decades for GFM inverter system [35]. Major voltage-control schemes, reported in existing literature, are shown in Fig. 1(b). In summary, there are nested-loop classical (i.e., proportional/integral/resonant controller-based), sliding-mode, model predictive, repetitive, linear-quadratic regulator, $\mathcal{H}_\infty$ -based, $\mathcal{H}_2$ -based control schemes as reported in [34], [35], [36], [37], [38], [39], [40], [41], [42], [43], [44], [45], [46], [47], [48], [49], [50], [51]. Most of the advancements have focused on either multiple nested-loop structures with advanced control techniques or adding extra measurements as feedforward

for enhancing the dynamic performance. This results in either increased complication of the control structure that lead to difficulties in implementation or increase in the capital cost of employing multiple sensors. Also, most of the schemes face are unable to guarantee robust performance of the voltage controller under varying loading condition of the inverter. As a result, these techniques face deteriorating dynamic response in controller performance under varying and uncertain equivalent loading. Moreover, there are some advanced control strategies applicable for GFM inverter system such as GFM inverter enabled virtual power plant control with inertia support capability, virtual synchronous generator control, sliding-mode control-based grid-supporting inverters, virtual oscillator control, etc., that can found in [52], [53], [54], [55], and [56]. However, these strategies are fundamentally responsible for enhancing the “Primary Control” layer of microgrid control hierarchy by generating voltage reference signal for “Zero-level Control” layer. The premise of the proposed work lies in the “Zero-level Control” layer and is agnostic to the methods employed in the “Primary Control.”

A generalized and unified control framework is required that can be utilized to synthesize a current controller for GFL inverters as well as a voltage controller for GFM inverters while considering the respective model uncertainties in order to ensure robust stability and performance. In this essence, this article presents a generalized μ -synthesis-based robust control framework. The contributions of this work are as follows.

- 1) A generalized and unified control framework is proposed, leveraged by the voltage-current duality in the plant dynamic model of GFL and GFM inverters, in order to synthesize respective controllers.
- 2) The respective model uncertainties (i.e., imposed by varying grid impedance for GFL inverter and by varying equivalent loading for GFM inverter) are also included in the generalized framework after modeling.
- 3) Based on the generalized control framework, a μ -synthesis-based robust control design methodology is proposed for GFL and GFM inverters that results in

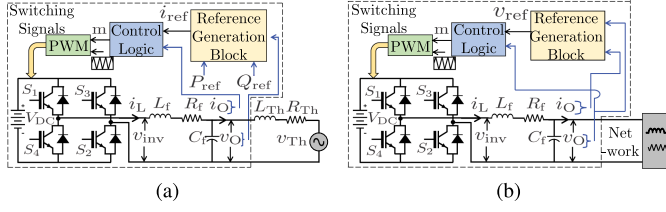


Fig. 2. 1- ϕ (a) grid-following inverter, and (b) grid-forming inverter.

controllers that are single-loop, hence simple and cost-effective and guarantee robust stability and robust performance under model uncertainties.

- 4) The synthesized optimal controller has enhanced reference tracking, disturbance rejection, and harmonic compensation capability with i) sufficient LCL resonance damping under large variations of grid impedance uncertainty for GFL inverters and ii) with enhanced dynamic response under large variations of equivalent loading uncertainty for GFM inverters.

A combined system-in-the-loop (SIL), controller-hardware-in-the-loop (CHIL), and power-hardware-in-the-loop (PHIL)-based experiment is conducted to evaluate the efficacy and viability of the proposed controllers. Moreover, CHIL-based experiment is conducted to compare the performance of the proposed μ -synthesized controller with some existing modern controllers for both GFL and GFM inverters.

This article is organized as follows. In Section II, the problem formulation and motivation of the study are presented. In Section III, the proposed generalized μ -synthesis-based robust control framework is described. Section IV provides the controller synthesis and corresponding stability analysis. In Section V, the experimental setup and corresponding results are described. Finally, Section VI concludes the article.

II. PROBLEM FORMULATION AND MOTIVATION

In this section, control problem formulation will be discussed for inverter system operating in GFL and GFM mode. For both GFL and GFM inverters, a single-phase H-bridge inverter is considered comprising a dc bus, V_{DC} , four switching devices, S_1, S_2, S_3, S_4 , and an LCL filter with L_f, L_g, R_f, R_g , and C_f as inverter and grid side filter inductors, parasitic resistances, and filter capacitor, respectively. A control layer is employed with sinusoidal PWM switching technique to generate the switching signals for power circuit.

A. Problem Formulation for Grid-Following Inverter

A GFL inverter is connected to a distribution grid, represented by the Thevenin equivalent voltage source v_{Th} in series with the Thevenin equivalent impedance $Z_{Th} := R_{Th} + j\omega_o L_{Th}$. ω_o (in rad/s) is the frequency of the distribution network. For simplicity, the Thevenin equivalent impedance accounts grid side filter parameters also (i.e., L_g, R_g). Various components of GFL inverter system are shown in Fig. 2(a). In this operation, the inverter operates with an output current control strategy to regulate real and reactive power output, where the voltage and

frequency of the distribution network are determined by another source such as the grid or other GFM inverters. The goal of the current controller is to generate a controlled voltage signal v_{inv} by switching signals such that the output current i_O , tracks the reference signal i_{ref} , generated by reference generation block as discussed in Appendix A. The dynamics of the inverter are described as

$$L_f \frac{d\langle i_L \rangle}{dt} + R_f \langle i_L \rangle = \langle v_{inv} \rangle - \langle v_O \rangle \quad (1)$$

$$L_{Th} \frac{d\langle i_O \rangle}{dt} + R_{Th} \langle i_O \rangle = \langle v_O \rangle - \langle v_{Th} \rangle \quad (2)$$

$$C_f \frac{d\langle v_O \rangle}{dt} = \langle i_L \rangle - \langle i_O \rangle \quad (3)$$

where $\langle \cdot \rangle$ signifies the average values of the corresponding variable over one switching cycle (T_s) [33]. Laplace transformation and algebraic manipulation with (1), (2), and (3) results in

$$i_O(s) = \mathcal{G}_{inv}^{GFL}(s)v_{inv}(s) - \mathcal{G}_{Th}^{GFL}(s)v_{Th}(s) \quad (4)$$

where $\mathcal{G}_{inv}^{GFL}(s)$ and $\mathcal{G}_{Th}^{GFL}(s)$ are transfer functions parameterized by L_f, R_f, C_f, L_{Th} , and R_{Th} , as given in (5) and (6) shown at the bottom of the next page. As observed in (4), distribution network has two-fold impacts on the open-loop plant dynamics. $v_{Th}(s)$ acts as an exogenous disturbance signal to the plant, which can be addressed by classical disturbance rejection problem. Both $\mathcal{G}_{inv}^{GFL}(s)$ and $\mathcal{G}_{Th}^{GFL}(s)$ consist of L_{Th} and R_{Th} as parameters. Variation of these parameters due to changing distribution network topology introduces uncertainties in the plant model [24], [25]. Fig. 3(a) and (b) shows the Bode plot of $\mathcal{G}_{inv}^{GFL}(s)$ that clearly shows the large variation in frequency response of the open-loop plant model due to grid inductance variation. Fig. 3(a) shows that the equivalent LCL resonant peak is sensitive to grid inductance L_{Th} . For stiff grid with small L_{Th} (leading to a large resonant frequency), the resultant resonant frequency is larger than the bandwidth of the controller. However, such a controller when used for a sufficiently weak grid with large L_{Th} , the resultant resonance peak may enter the pass-band of the current controller that in turn results in instability [24], [25]. As a result, this uncertainty in grid impedance parameters introduces challenges with respect to performance under uncertainty of grid impedance. With this motivation, this article designs a single-loop μ -synthesis-based stabilizing controller for GFL inverter that has robust active damping, tracking performance, disturbance rejection, and harmonic compensation under grid impedance uncertainties.

B. Problem Formulation for Grid-Forming Inverter

Various components of a GFM inverter are shown in Fig. 2(b). A GFM inverter is connected to an electrical network of a distribution system. In this mode, the inverter operates with an output voltage control strategy to generate a stable voltage and frequency for the electrical network. The goal of the voltage control logic is to generate a controlled voltage signal v_{inv} by switching signals such that the output voltage v_O tracks the reference signal v_{ref} generated by reference generation layer as discussed in Appendix B. The dynamics of the inverter are

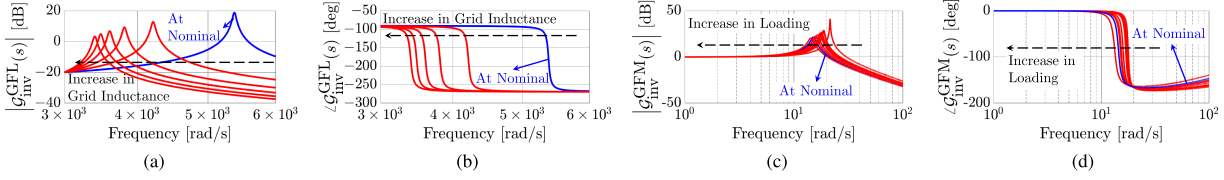


Fig. 3. Bode plots of (a) magnitude and (b) phase of $\mathcal{G}_{inv}^{GFL}(s)$ of (4) and (c) magnitude and (d) phase of $\mathcal{G}_{inv}^{GFM}(s)$ of (13), respectively.

described as

$$L_f \frac{d\langle i_L \rangle}{dt} + R_f \langle i_L \rangle = \langle v_{inv} \rangle - \langle v_O \rangle, \quad (7)$$

$$C_f \frac{d\langle v_O \rangle}{dt} = \langle i_L \rangle - \langle i_O \rangle, \quad (8)$$

where $\langle \cdot \rangle$ signifies the average values of the corresponding variable over one switching cycle (T_s) [33]. Laplace transformation and algebraic manipulation with (7) and (8) result

$$v_O(s) = \mathcal{G}_1(s)v_{inv}(s) - \mathcal{G}_2(s)i_O(s) \quad (9)$$

where $\mathcal{G}_1(s)$ and $\mathcal{G}_2(s)$ are transfer functions parameterized by L_f , R_f , C_f , as given in (10) and (11) and described by

$$\mathcal{G}_1(s) = \frac{1}{[L_f C_f]s^2 + [R_f C_f]s + 1} \text{ and} \quad (10)$$

$$\mathcal{G}_2(s) = \frac{L_f s + R_f}{[L_f C_f]s^2 + [R_f C_f]s + 1}. \quad (11)$$

For the modeling of i_O , it is essential to understand the impact of the external network on the open-loop plant. In this work, the disturbance is modeled by a parallel combination of two components. First component is a linear admittance, $Y_{Load}(s) := 1/(L_{Load}s + R_{Load})$, with unknown R_{Load} and L_{Load} elements in series combination; the grid side filter parameters are included in R_{Load} and L_{Load} . Another component is a parallel combination of current sources consisting of both fundamental and harmonic components [57] and defined as $i_h(s) := \sum_k i_k(s)$, where k is odd and $i_k(s)$ is the k th harmonic current. As a result, $i_O(s)$ is characterized as

$$i_O(s) = Y_{Load}(s)v_O(s) + i_h(s). \quad (12)$$

Combining (9) and (12), system's closed-loop is described by

$$v_O(s) = \mathcal{G}_{inv}^{GFM}(s)v_{inv}(s) - \mathcal{G}_{Load}^{GFM}(s)i_h(s) \quad (13)$$

where $\mathcal{G}_{inv}^{GFM}(s)$ and $\mathcal{G}_{Load}^{GFM}(s)$ are transfer functions parameterized by L_f , R_f , C_f , L_{Load} , and R_{Load} , as given in (14) and (15) shown at the bottom of the next page.

As observed in (13), load has impacts on the open-loop plant dynamics of the GFM inverter. First, both $\mathcal{G}_{inv}^{GFM}(s)$ and $\mathcal{G}_{Load}^{GFM}(s)$ consist of L_{Load} and R_{Load} as parameters, which introduce uncertainties in the plant model. Variation of these parameters

due to loading of the inverter due to changing generation-load imbalance in the distribution network introduces uncertainties in the plant model [42], [43]. Second, $i_h(s)$ imposed by the fundamental and harmonic current load, acts as an exogenous disturbance signal to the plant. Where the latter one is a classical disturbance rejection problem, the former one poses robustness issues. Fig. 3(a) and (d) depicts the Bode plot of $\mathcal{G}_{inv}^{GFM}(s)$ that shows the variation in frequency response of the open-loop plant model due to variation in equivalent loading. It is also evident that the uncertain loading variation across the inverter causes significant change in the plant dynamics, especially the effective resonant frequency of the inverter system as shown in Fig. 3(c). This deteriorates the transient response of the GFM inverter severely as well as overall system stability [34], [45] with the above motivation, this article designs a single-loop μ -synthesis-based stabilizing controller that has robust tracking performance, disturbance rejection, and harmonic compensation with improved transient operation under the full range variation in equivalent loading.

III. μ -SYNTHESIS-BASED GENERALIZED FRAMEWORK FOR CONTROLLER SYNTHESIS

Motivated by Sections II-A and II-B, this section will describe the generalized control framework and required modeling for μ -synthesis-based controller synthesis.

A. μ -Synthesis-Based Generalized Framework

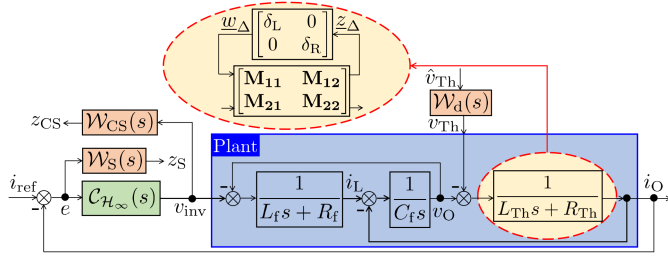
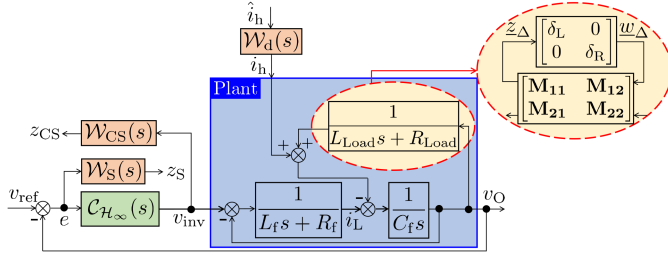
The following observations are common for both GFL and GFM inverter system:

I) The open-loop plant model $\mathcal{G}_{inv}^{GFL}(s)$ of (5) and $\mathcal{G}_{inv}^{GFM}(s)$ of (14) are third-order model with different numerators for GFL and GFM inverter, respectively. The disturbance model, $\mathcal{G}_{Th}^{GFL}(s)$ of (6) and $\mathcal{G}_{Load}^{GFM}(s)$ of (15), are third-order models for GFL and GFM, respectively. Moreover, there is a voltage-current duality in the plant dynamic model of GFL and GFM inverter. Here for both the models, v_{inv} is the *plant input* signal i_O , v_O are the *plant output* and v_{Th} , i_h are the *disturbance* signal for GFL and GFM inverter, respectively, as given in (4) and (13).

II) The open-loop plant and disturbance models have similar parametric uncertainties imposed by L_{Th} , R_{Th} and L_{Load} and

$$\mathcal{G}_{inv}^{GFL}(s) = 1/[(C_f L_f L_{Th})s^3 + C_f(L_{Th}R_f + L_f R_{Th})s^2 + (C_f R_f R_{Th} + L_f + L_{Th})s + (R_f + R_{Th})] \quad (5)$$

$$\mathcal{G}_{Th}^{GFL}(s) = [(C_f L_f)s^2 + (C_f R_f)s + 1]\mathcal{G}_{inv}^{GFL}(s) \quad (6)$$

Fig. 4. Proposed μ -synthesis-based robust controller for GFL Inverter.Fig. 5. Proposed μ -synthesis-based robust controller for GFM Inverter.

$R_{L_{Load}}$ for both GFL and GFM inverter, respectively. The similarity is evident in Figs. 4 and 5, where the open-loop plant models are shown using block diagram inside the blue boxes and corresponding uncertainty block using dashed ovals for both GFL and GFM inverter, respectively.

III) Robust tracking performance, disturbance rejection, and harmonic compensation capabilities are desired for controllers of GFL and GFM inverters under the uncertainties.

Items I) and II) motivate designing a generalized control framework for GFL and GFM inverters based on these similarity and duality property. Third point warrants the μ -synthesis-based robust controller synthesis for addressing multiple objectives. A systematic approach is presented below for designing controller, $C_{H_\infty}(s)$ of Figs. 4 and 5 for GFL and GFM inverter, respectively, using the generalized μ -synthesis-based robust control framework.

B. Modeling of Uncertainty

In both the GFL and GFM inverter open-loop plant model, the model uncertainty is present as parametric uncertainties in first-order transfer function (dashed ovals in Figs. 4 and 5, respectively). A systematic approach is followed here for characterizing and modeling the uncertainty for both the cases as discussed in the following.

1) *Characterizing the Uncertainty for GFL Inverter:* Variations in grid impedance (i.e., L_{Th} and R_{Th}) results in real-parametric uncertainties in the GFL plant model. The short-circuit ratio (SCR) is often used to characterize the grid stiffness and can be employed in determining the Thevenin equivalent impedance of the grid at the point-of-connection. SCR is defined as $(V_{PC}^N)^2 / [S_B \sqrt{(\omega_N L_{Th})^2 + R_{Th}^2}]$, where V_{PC}^N and ω_N are the nominal voltage and frequency at point-of-connection, and S_B is the rated apparent power of the GFL inverter. Usually the grid at point-of-connection is considered as weak when the SCR is less than 3 [57], [58]. In this work, with a prespecified SCR (< 3) and X/R ratio (< 10), the nominal grid impedance parameters, L_{Th}^{Nom} and R_{Th}^{Nom} , are determined. By considering $\pm 100\%$ variations over nominal values, it is assumed that $L_{Th} \in [\underline{L}_{Th}, \bar{L}_{Th}]$ and $R_{Th} \in [\underline{R}_{Th}, \bar{R}_{Th}]$. It is to be noted that stiff to extremely weak grid conditions are accommodated with this uncertainty characterization. As a result

$$L_{Th} := L_{Th}^{Nom} + w_{Th}^L \delta_L, \quad R_{Th} := R_{Th}^{Nom} + w_{Th}^R \delta_R \quad (16)$$

where $\delta_L, \delta_R \in [-1, 1]$, $L_{Th}^{Nom} = \frac{1}{2}[\bar{L}_{Th} + \underline{L}_{Th}]$, $R_{Th}^{Nom} = \frac{1}{2}[\bar{R}_{Th} + \underline{R}_{Th}]$, $w_{Th}^L = \frac{1}{2}[\bar{L}_{Th} - \underline{L}_{Th}]$, $w_{Th}^R = \frac{1}{2}[\bar{R}_{Th} - \underline{R}_{Th}]$.

2) *Characterizing the Uncertainty for GFM Inverter:* In this case, variation in equivalent loading (i.e., R_{Load} and L_{Load}) results in real-parametric uncertainties in the GFM plant model. In this work, the linear part of the loading is modeled by a series combination of equivalent unknown R_{Load} and L_{Load} element. These elements are at nominal while the GFM inverter loading is at rated condition. Considering rated VA loading as S_{rated} , with active power P_{rated} , and reactive power Q_{rated} , the following hold for nominal values:

$$R_{Load}^{Nom} = V_N^2 \frac{P_{rated}}{S_{rated}^2}, \quad L_{Load}^{Nom} = V_N^2 \frac{Q_{rated}}{S_{rated}^2} \frac{1}{\omega_N} \quad (17)$$

where V_N and ω_N are the nominal voltage and frequency of the network, respectively. By considering no-loading and over-loading (200% loading) of GFM, it is assumed that $L_{Load} \in [\underline{L}_{Load}, \bar{L}_{Load}]$, $R_{Load} \in [\underline{R}_{Load}, \bar{R}_{Load}]$. As a result

$$L_{Load} := L_{Load}^{Nom} + w_{Load}^L \delta_L \quad (18)$$

$$R_{Load} := R_{Load}^{Nom} + w_{Load}^R \delta_R \quad (19)$$

where $\delta_L, \delta_R \in [-1, 1]$, $L_{Load}^{Nom} = \frac{1}{2}[\bar{L}_{Load} + \underline{L}_{Load}]$, $R_{Load}^{Nom} = \frac{1}{2}[\bar{R}_{Load} + \underline{R}_{Load}]$, $w_{Load}^L = \frac{1}{2}[\bar{L}_{Load} - \underline{L}_{Load}]$, and $w_{Load}^R = \frac{1}{2}[\bar{R}_{Load} - \underline{R}_{Load}]$.

3) *Generalized Representation of Uncertainty:* In synthesizing the controller with defined uncertainties in $R \in [\underline{R}, \bar{R}]$ and $L \in [\underline{L}, \bar{L}]$ when appearing in the form of $1/(Ls + R)$, linear fractional transformation (LFT) [59] can be utilized to convert

$$\mathcal{G}_{inv}^{GFM}(s) = [L_{Load}s + R_{Load}] / [(C_f L_f L_{Load})s^3 + C_f (L_{Load} R_f + L_f R_{Load})s^2 + (C_f R_f R_{Load} + L_f + L_{Load})s + (R_f + R_{Load})] \quad (14)$$

$$\mathcal{G}_{Load}^{GFM}(s) = \mathcal{G}_{inv}^{GFM}(s) [(L_{Load} L_f)s^2 + (L_{Load} R_f + L_f R_{Load})s + (R_{Load} R_f)] / [L_{Load}s + R_{Load}]. \quad (15)$$

the model into an upper LFT, $F_U(\mathbf{M}, \Delta)$, given as

$$F_U(\mathbf{M}, \Delta) = \frac{1}{sL + R} = \mathbf{M}_{22} + \mathbf{M}_{21}\Delta[\mathbf{I} - \mathbf{M}_{11}\Delta]^{-1}\mathbf{M}_{12}$$

$$\text{with } \mathbf{M} = \begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22} \end{bmatrix}, \Delta = \begin{bmatrix} \delta_L & 0 \\ 0 & \delta_R \end{bmatrix}, \delta_L \in [-1, 1], \delta_R \in [-1, 1]$$

$$\mathbf{M}_{11} = \mathbf{M}_{22} \begin{bmatrix} sw^L & w^R \\ sw^L & w^R \end{bmatrix}, \mathbf{M}_{12} = \mathbf{M}_{22} \begin{bmatrix} 1 & 1 \end{bmatrix}^\top, \mathbf{M}_{21} = \mathbf{M}_{22} \begin{bmatrix} sw^L & w^R \end{bmatrix}$$

where $\mathbf{M}_{22} = -1/[sL_{\text{Nom}} + R_{\text{Nom}}]$, $w^L = \frac{1}{2}[\bar{L} - \underline{L}]$, $w^R = \frac{1}{2}[\bar{L} + \underline{L}]$. It is important to note here that it is a generalized representation of uncertainty for both GFL and GFM inverter, where $L = L_{\text{Th}}$, $R = R_{\text{Th}}$ for GFL inverter and $L = L_{\text{Load}}$, $R = R_{\text{Load}}$ for GFM inverter.

C. Shaping of Transfer Functions

The closed-loop objectives in designing the feedback control based on proposed robust controller $\mathcal{C}_{\mathcal{H}_\infty}(s)$, as shown in Figs. 4 and 5 for GFL and GFM inverter are as follows:

- i) *Reference Tracking*: i_O tracks i_{ref} for GFL and v_O tracks v_{ref} for GFM inverter with minimum tracking error.
- ii) *Disturbance Rejection*: Effects of v_{Th} on i_O for GFL and effect of i_{h} on v_O for GFM inverter are attenuated.
- iii) *Control Effort Reduction*: v_{inv} satisfies the respective bandwidth limitations for GFL and GFM inverter.

Based on the objectives, user-defined weighting transfer functions, $\mathcal{W}_S(s)$, $\mathcal{W}_{CS}(s)$, $\mathcal{W}_d(s)$, are designed. The guidelines for designing the weighting functions are provided below.

1) *Selection of $\mathcal{W}_S(s)$* : To shape the sensitivity transfer function, the weighting function $\mathcal{W}_S(s)$ is introduced so that i) the tracking error e ($e := i_{\text{ref}} - i_O$ and $e := v_{\text{ref}} - v_O$ for GFL and GFM inverter, respectively) at fundamental frequency is small; ii) resonance phenomenon of the system is actively damped. $\mathcal{W}_S(s)$ is modeled to have peaks around ω_N and system's resonant frequency ω_r (different in GFL and GFM plant), with second-order roll-off $k_{S,1}(s)$ and formed as

$$\mathcal{W}_S(s) = k_{S,1}(s) \frac{s^2 + 2k_{S,2}\zeta\omega_N s + \omega_N^2}{s^2 + 2\zeta\omega_N s + \omega_N^2} \frac{s^2 + 2k_{S,3}\zeta\omega_r s + \omega_r^2}{s^2 + 2\zeta\omega_r s + \omega_r^2}$$

where $k_{S,2}$ and $k_{S,3}$ are selected to exhibit peaks and ζ addresses the off-nominal frequency around the nominal values.

2) *Selection of $\mathcal{W}_{CS}(s)$* : $\mathcal{W}_{CS}(s)$ is designed to suppress high-frequency control effort to shape the performance of v_{inv} for both GFL and GFM controller. Hence, it is designed as a high-pass filter with cutoff frequency at switching frequency with the form

$$\mathcal{W}_{CS}(s) = k_{CS} \frac{s + k_{CS,1}\omega_N}{s + k_{CS,2}\omega_N}, \text{ where } k_{CS,1} \ll k_{CS,2}.$$

3) *Selection of $\mathcal{W}_d(s)$* : $\mathcal{W}_d(s)$ emphasizes the disturbances at fundamental and harmonic frequencies imposed by v_{Th} and i_{h} and emphasized by exogenous signal $\hat{v}_{\text{Th}}$ and $\hat{i}_{\text{h}}$ for GFL and GFM inverter, respectively, as shown in Figs. 4 and 5. It is designed by a low-pass filter $k_d(s)$ with peaks at selected

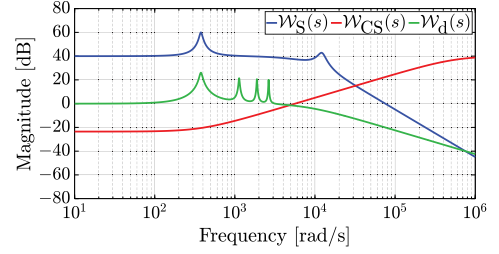


Fig. 6. Bode plots of magnitudes of selected weighting transfer functions.

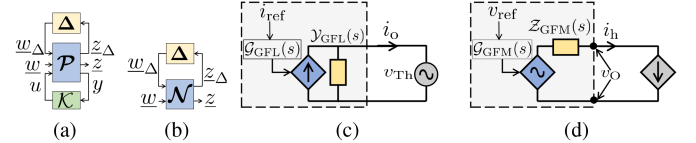


Fig. 7. Generalized (a) $\Delta\text{-P-K}$ control configuration, (b) $\Delta\text{-N}$ control configuration, of Figs. 4 or 5 and closed-loop equivalent circuit model of (c) grid-following inverter system, (d) grid-forming inverter system.

frequencies and is given by

$$\mathcal{W}_d(s) = k_d(s) \prod_{h=1,3,5,7} \frac{s^2 + 2k_{d,h}\zeta h\omega_N s + h^2\omega_N^2}{s^2 + 2\zeta h\omega_N s + h^2\omega_N^2}$$

where the values of $k_{d,h}$ are selected based on the regulated limits of third, fifth, seventh harmonics in network voltage and current injection with respect to fundamental [60]. A representative of selected weighting functions are shown in Fig. 6.

D. Preparing the Generalized Plant

In preparation for robust controller design $\mathcal{C}_{\mathcal{H}_\infty}(s)$, the multiloop closed-loop block diagram in Figs. 4 and 5 for GFL and GFM inverter, respectively, are consolidated into the general control configuration in Fig. 7(a) [59]. Here, $\mathcal{P}(s)$ is the generalized multi-input-multi-output (MIMO) plant, $\mathcal{K}(s)$ is the proposed $\mathcal{C}_{\mathcal{H}_\infty}(s)$ controllers to be designed for GFL and GFM inverter and Δ is the structured uncertainty. $\underline{w}$ is a vector of the exogenous inputs (e.g., reference, disturbance), $\underline{z}$ are the exogenous outputs (e.g., signals to be regulated). y and u are the controller input and output signals, respectively. $\underline{z}_\Delta$ and $\underline{w}_\Delta$ are the vector of input and output signals of structured uncertainty block. Note that in this continuous-time modeling framework, all variables are functions of the Laplace variable s ; not explicitly shown for notational convenience. As a result, the generalized MIMO plant maps $\begin{bmatrix} \underline{w}_\Delta & \underline{w} & u \end{bmatrix}^\top$ to $\begin{bmatrix} \underline{z}_\Delta & \underline{z} & y \end{bmatrix}^\top$ as follows:

$$\begin{bmatrix} \underline{z}_\Delta \\ \underline{z} \\ y \end{bmatrix} = \begin{bmatrix} \mathcal{P}_{\Delta\Delta} & \mathcal{P}_{\Delta w} & \mathcal{P}_{\Delta u} \\ \mathcal{P}_{z\Delta} & \mathcal{P}_{zw} & \mathcal{P}_{zu} \\ \mathcal{P}_{y\Delta} & \mathcal{P}_{yw} & \mathcal{P}_{yu} \end{bmatrix} \begin{bmatrix} \underline{w}_\Delta \\ \underline{w} \\ u \end{bmatrix} \quad (20)$$

where the input and output signals are tabulated in Table I for both GFL and GFM inverter system. The detailed MIMO transfer function models of (20) for both GFL and GFM inverter

Type	$\mathcal{P}(s)$ [$\mathcal{P}_{\Delta\Delta} \in \mathbb{C}^{2 \times 2}$, $\mathcal{P}_{\Delta w} \in \mathbb{C}^{2 \times 2}$, $\mathcal{P}_{\Delta u} \in \mathbb{C}^{2 \times 1}$, $\mathcal{P}_{z\Delta} \in \mathbb{C}^{2 \times 2}$, $\mathcal{P}_{zw} \in \mathbb{C}^{2 \times 2}$, $\mathcal{P}_{zu} \in \mathbb{C}^{2 \times 1}$, $\mathcal{P}_{y\Delta} \in \mathbb{C}^{1 \times 2}$, $\mathcal{P}_{yw} \in \mathbb{C}^{1 \times 2}$, $\mathcal{P}_{yu} \in \mathbb{C}$]		
GFL	$\mathcal{P}_{\Delta\Delta} = \mathbf{M}_{11} - \frac{\mathcal{B}_{\text{GFL}}}{1 + \mathcal{M}_{22}\mathcal{B}_{\text{GFL}}} \mathbf{M}_{12}\mathbf{M}_{21}$	$\mathcal{P}_{\Delta w} = \begin{bmatrix} \mathbf{0}_{2 \times 1} & \frac{\mathcal{B}_{\text{GFL}}\mathcal{W}_d\mathcal{M}_{22}}{1 + \mathcal{M}_{22}\mathcal{B}_{\text{GFL}}} \mathbf{M}_{12} \end{bmatrix}$	$\mathcal{P}_{\Delta u} = \left(\mathcal{A}_{\text{GFL}} - \frac{\mathcal{B}_{\text{GFL}}\mathcal{A}_{\text{GFL}}\mathcal{M}_{22}}{1 + \mathcal{M}_{22}\mathcal{B}_{\text{GFL}}} \mathbf{M}_{12} \right) \mathbf{M}_{12}$
	$\mathcal{P}_{z\Delta} = \begin{bmatrix} -\frac{\mathcal{W}_s}{1 + \mathcal{M}_{22}\mathcal{B}_{\text{GFL}}} \mathbf{M}_{21} \\ \mathbf{0}_{1 \times 2} \end{bmatrix}$	$\mathcal{P}_{zw} = \begin{bmatrix} \mathcal{W}_s & \frac{\mathcal{W}_s\mathcal{W}_d\mathcal{M}_{22}}{1 + \mathcal{M}_{22}\mathcal{B}_{\text{GFL}}} \\ 0 & 0 \end{bmatrix}$	$\mathcal{P}_{zu} = \begin{bmatrix} -\frac{\mathcal{W}_s\mathcal{M}_{22}\mathcal{A}_{\text{GFL}}}{1 + \mathcal{M}_{22}\mathcal{B}_{\text{GFL}}} \\ \mathcal{W}_{\text{CS}} \end{bmatrix}$
	$\mathcal{P}_{y\Delta} = -\frac{1}{1 + \mathcal{M}_{22}\mathcal{B}_{\text{GFL}}} \mathbf{M}_{21}$	$\mathcal{P}_{yw} = \begin{bmatrix} 1 & \frac{\mathcal{M}_{22}\mathcal{W}_d}{1 + \mathcal{M}_{22}\mathcal{B}_{\text{GFL}}} \end{bmatrix}$	$\mathcal{P}_{yu} = -\frac{\mathcal{M}_{22}\mathcal{A}_{\text{GFL}}}{1 + \mathcal{M}_{22}\mathcal{B}_{\text{GFL}}}$
GFM	$\mathcal{P}_{\Delta\Delta} = \mathbf{M}_{11} - \mathcal{B}_{\text{GFM}}\mathbf{M}_{12}\mathbf{M}_{21}$	$\mathcal{P}_{\Delta w} = \begin{bmatrix} \mathbf{0}_{2 \times 1} & -\mathcal{W}_d\mathcal{B}_{\text{GFM}}\mathbf{M}_{12} \end{bmatrix}$	$\mathcal{P}_{\Delta u} = \mathcal{A}_{\text{GFM}}\mathbf{M}_{12}$
	$\mathcal{P}_{z\Delta} = \begin{bmatrix} -\mathcal{W}_s\mathcal{B}_{\text{GFM}}\mathbf{M}_{21} \\ \mathbf{0}_{1 \times 2} \end{bmatrix}$	$\mathcal{P}_{zw} = \begin{bmatrix} \mathcal{W}_s & -\mathcal{W}_s\mathcal{W}_d\mathcal{B}_{\text{GFM}} \\ 0 & 0 \end{bmatrix}$	$\mathcal{P}_{zu} = \begin{bmatrix} -\mathcal{W}_s\mathcal{A}_{\text{GFM}} \\ \mathcal{W}_{\text{CS}} \end{bmatrix}$
	$\mathcal{P}_{y\Delta} = \mathcal{B}_{\text{GFM}}\mathbf{M}_{21}$	$\mathcal{P}_{yw} = \begin{bmatrix} 1 & \mathcal{W}_d\mathcal{B}_{\text{GFM}} \end{bmatrix}$	$\mathcal{P}_{yu} = -\mathcal{A}_{\text{GFM}}$

Fig. 8. Generalized MIMO transfer function models $\mathcal{P}(s)$ of (20) for both GFL inverter control of Fig. 4 and GFM inverter control of Fig. 5.TABLE I
INPUT-OUTPUT MAPPING OF THE GENERALIZED PLANT

Signals	GFL Inverter	GFM Inverter
$\underline{w}$	$[i_{\text{ref}} \ \hat{v}_{\text{Th}}]^\top$ of Fig. 4	$[v_{\text{ref}} \ \hat{i}_{\text{h}}]^\top$ of Fig. 4
$\underline{z}$	$[z_{\text{S}} \ z_{\text{CS}}]^\top$ of Fig. 4	$[z_{\text{S}} \ z_{\text{CS}}]^\top$ of Fig. 5
u	v_{inv} of Fig. 4	v_{inv} of Fig. 5
y	e of Fig. 4	e of Fig. 5
w_{Δ}	w_{Δ} of Fig. 4	w_{Δ} of Fig. 5
z_{Δ}	z_{Δ} of Fig. 4	z_{Δ} of Fig. 5

systems are provided in Fig. 8, where

$$\begin{aligned} \mathcal{A}_{\text{GFL}} &= \frac{1}{[L_f C_f]s^2 + [R_f C_f]s + 1} \\ \mathcal{B}_{\text{GFL}} &= \frac{L_f s + R_f}{[L_f C_f]s^2 + [R_f C_f]s + 1} \\ \mathcal{A}_{\text{GFM}} &= \frac{1}{[L_f C_f]s^2 + [R_f C_f + L_f]s + [1 + R_f]} \\ \mathcal{B}_{\text{GFM}} &= \frac{L_f s + R_f}{[L_f C_f]s^2 + [R_f C_f + L_f]s + [1 + R_f]}. \end{aligned}$$

$\mathcal{P}$ and $\mathcal{K}$ can be combined together by a lower LFT to form $\mathcal{N}$ in Fig. 7(b) as follows:

$$\begin{aligned} \mathcal{N} &= \begin{bmatrix} \mathcal{P}_{\Delta\Delta} & \mathcal{P}_{\Delta w} \\ \mathcal{P}_{z\Delta} & \mathcal{P}_{zw} \end{bmatrix} + \frac{\mathcal{K}}{[1 - \mathcal{P}_{yu}\mathcal{K}]^{-1}} \begin{bmatrix} \mathcal{P}_{\Delta u} \\ \mathcal{P}_{zu} \end{bmatrix} \begin{bmatrix} \mathcal{P}_{y\Delta} \\ \mathcal{P}_{yw} \end{bmatrix}^\top \\ &= \begin{bmatrix} \mathcal{N}_{\Delta\Delta} & \mathcal{N}_{\Delta w} \\ \mathcal{N}_{z\Delta} & \mathcal{N}_{zw} \end{bmatrix}, \text{ where } \begin{bmatrix} z_{\Delta} \\ z \end{bmatrix} = \mathcal{N} \begin{bmatrix} w_{\Delta} \\ w \end{bmatrix}. \end{aligned} \quad (21)$$

Therefore, the uncertainty closed-loop transfer function from $\underline{w}$ to $\underline{z}$, $\underline{z} = \mathcal{M}\underline{w}$ is related to $\mathcal{N}$ and Δ by an upper LFT, where $\mathcal{M} = \mathcal{N}_{zw} + \mathcal{N}_{z\Delta}\Delta(\mathbf{I} - \mathcal{N}_{\Delta\Delta}\Delta)^{-1}\mathcal{N}_{\Delta w}$.

IV. CONTROLLER SYNTHESIS AND STABILITY ANALYSIS

With reference to the general control configuration of Fig. 7(a), the standard μ -synthesis-based optimal control problem is to find all stabilizing controllers $\mathcal{K}(s)$ by solving

$$\min_{\mathcal{K}(s) \text{ stabilizing}} \|\mathcal{N}\|_{\infty} \quad (22)$$

where $\|\cdot\|_{\infty}$ refers to the μ -synthesis norm. This problem can be readily solved using the MATLAB Robust Control Toolbox. An

algorithm for solving (22) along with the theoretical underpinnings of this optimization problem can be found in [59]. Upon finding a stabilizing controller, the requirement of stability and performance of the closed-loop system are needed to be checked and can be summarized as follows:

$$\text{Nominal Stable(NS)} : \mathcal{N} \text{ is internally stable} \quad (23)$$

$$\text{Nominal Performance(NP)} : \|\mathcal{N}_{zw}\|_{\infty} < 1 \ \& \ \text{NS} \quad (24)$$

$$\text{Robust Stable(RS)} : \mu_{\Delta}(\mathcal{N}_{\Delta\Delta}) < 1, \forall \omega, \ \& \ \text{NS} \quad (25)$$

$$\text{Robust Performance(RP)} : \mu_{\Delta}(\mathcal{N}) < 1, \forall \omega, \ \& \ \text{NS} \quad (26)$$

where $\mu_{\Delta}(\mathcal{N}_{\Delta\Delta})$ and $\mu_{\Delta}(\mathcal{N})$ are the structured singular values of $\mathcal{N}_{\Delta\Delta}$ and $\mathcal{N}$ for the allowed structure of Δ and $\bar{\Delta} := \text{diag}(\Delta, \Delta_P)$, respectively, with Δ_P being an unstructured uncertainty [59]. It is necessary to check whether stabilizing controller $\mathcal{K}(s)$ of (22) satisfies all the conditions of (23)–(26) to analyze robust performance of the controller. In this work, an iterative approach is followed for μ -synthesis problem (i.e., finding the stabilizing controller that minimizes a given μ -condition). Design guidelines for selecting parameters of $\mathcal{W}_S(s)$, $\mathcal{W}_{\text{CS}}(s)$, $\mathcal{W}_d(s)$ are provided below.

- 1) During the design of $\mathcal{W}_S(s)$, $k_{S,1}(s)$, $k_{S,2}$, $k_{S,3}$, and ζ are required to be selected. A desirable selection of $k_{S,1}(s)$ is a second-order roll-off low-pass filter with cutoff frequency of around 10–15 times the fundamental frequency. It primarily determines the bandwidth of the resulting controller. The choice of high value of $k_{S,2}$, $k_{S,3}$ ensure to have significantly high value of $\mathcal{W}_S$ at fundamental power frequency (this leads to significantly less tracking error) and at system's resonance frequency (this lead to sufficient active damping of resonance). Moreover, in microgrid scenarios, where frequency can vary significantly, it may be necessary to increase ζ to broaden the frequency range where $\mathcal{W}_S$ is large.
- 2) During the design of $\mathcal{W}_{\text{CS}}(s)$, k_{CS} , $k_{\text{CS},1}$, and $k_{\text{CS},2}$ are required to be selected. It is important to note that $\mathcal{W}_{\text{CS}}(s)$ corresponds to shaping the performance of u of Fig. 7(a), i.e., v_{inv} of Figs. 4 or 5. Therefore, $\mathcal{W}_{\text{CS}}(s)$ is usually designed to suppress high-frequency control effort produced by the resulting controller. The suitable choice of $k_{\text{CS},1}$, and $k_{\text{CS},2}$ ($k_{\text{CS},1} \ll k_{\text{CS},2}$) should be such that the cutoff frequency is around the switching frequency of the

TABLE II
STABILITY ASSESSMENT OF RESULTING CONTROLLER

Controller	NS	NP	RS	RP
GFL Inverter	✓	0.49	0.91	0.94
GFM Inverter	✓	0.47	0.94	0.98

inverter. Moreover, selection of k_{CS} will be determined based on the desired gain of the high-pass filter.

- During the design of $\mathcal{W}_d(s)$, $k_d(s)$, $k_{d,1}$, $k_{d,3}$, $k_{d,5}$, and $k_{d,7}$ are required to be selected. $\mathcal{W}_d(s)$ characterizes the response of expected disturbances. For systems with linear and nonlinear loads (such as diode rectifier loads and switching power supplies, etc.), disturbance currents have components primarily at fundamental frequency and harmonics at higher orders. The value of $k_{d,1}$, $k_{d,3}$, $k_{d,5}$, and $k_{d,7}$ can be determined based on the relative harmonic distortion limits of various nonlinear loads as provided in [60]. By designing the $k_d(s)$ as a low-pass filter with cut-off frequency less than the fundamental power frequency, low-order disturbance rejection is emphasized.

The selected weights are shown in Appendix C. The μ -synthesis-based optimal controller design procedure based on the generalized robust framework contains the following steps:

Step 1: Based on the design guidelines, select the performance weights (i.e., $\mathcal{W}_S(s)$, $\mathcal{W}_{CS}(s)$, $\mathcal{W}_d(s)$).

Step 2: Based on Section III-B, characterize the uncertainties of the model (i.e., to determine the $\mathbf{M}$ of Section III-B3).

Step 3: Construct the generalized plant $\mathcal{P}$. An analytical form of the generalized plant for both GFL and GFM inverter system is given in Fig. 8.

Step 4: Solve the μ -synthesis problem, as stated in (22), with the following iterations:

i) *Synthesis stage:* Synthesize an $\mathcal{H}_\infty$ controller for the generalized plant $\mathcal{P}$, with initial choice of parameters as guided by Step 1-4.

ii) *Checking stage:* Check the conditions of (23)–(26) are all satisfied or not. If conditions are satisfied, μ -synthesized optimal controller is achieved as the problem in (22) is solved. If the conditions are not satisfied, return to Step-1 and adjust either performance weights (i.e., $\mathcal{W}_S(s)$, $\mathcal{W}_{CS}(s)$, $\mathcal{W}_d(s)$) or uncertainty weights (i.e., w^L , w^R) and continue until conditions of (23)–(26) are all satisfied.

The optimal controller $\mathcal{K}(s)$ will have an order similar to the order of $\mathcal{P}$. Thus, before implementation in actual inverter control board, model order reduction is used to obtain a lower order controller using MATLAB's *balred* command. Moreover, bilinear transformation is used in the discretization stage of the resulting controller.

1) *Analysis of Resulting Controller for GFL Inverter:* Following the procedure of synthesizing the optimal controller for GFL inverter, a 13th order $\mathcal{C}_{\mathcal{H}_\infty}(s)$ of Fig. 4 is found to perform well. The closed-loop stability and desired performances are met as summarized in Table II. The closed-loop model for GFL inverter with negative feedback loop transfer function with resulting controller $\mathcal{C}_{\mathcal{H}_\infty}(s)$ in Fig. 4 can be derived by substituting $v_{inv}(s) = \mathcal{C}_{\mathcal{H}_\infty}(s)[i_{ref} - i_O]$ in (4). It can be written

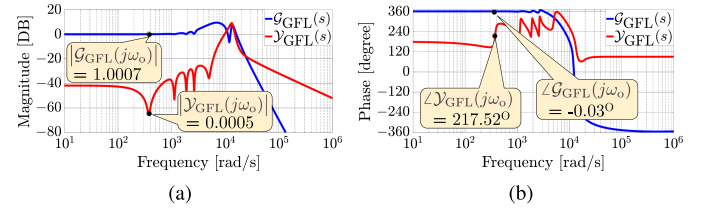


Fig. 9. Bode plots (a) magnitudes, (b) phase, of $\mathcal{G}_{GFL}$, $\mathcal{Y}_{GFL}$ of Fig. 7(c).

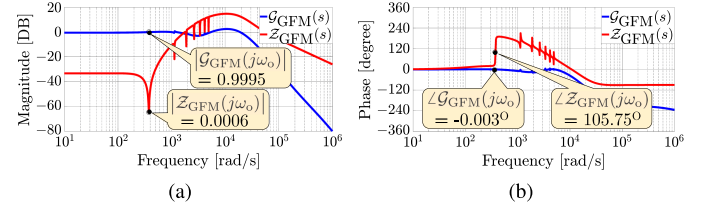


Fig. 10. Bode plots: (a) magnitudes, (b) phase of $\mathcal{G}_{GFM}$, $\mathcal{Z}_{GFM}$ of Fig. 7(d).

as $i_O = \mathcal{G}_{GFL}(s)i_{ref} - \mathcal{Y}_{GFL}(s)v_{Th}$ and represented as Norton's equivalent model connected to a voltage source as shown in Fig. 7(c). For an example, at nominal plant condition with resulting optimal controller, the Bode plot of $\mathcal{G}_{GFL}(s)$ and $\mathcal{Y}_{GFL}(s)$ are shown in Fig. 9(a) and (b), respectively. It is observed that $|\mathcal{G}_{GFL}(j\omega_N)|$, $\angle\mathcal{G}_{GFL}(j\omega_N)$ and $|\mathcal{Y}_{GFL}(j\omega_N)|$ are ≈ 1 , $\approx 0^\circ$ and ≈ 0 , respectively, that leads to $i_O \approx i_{ref}$ at fundamental frequency.

2) *Analysis of Resulting Controller for GFM Inverter:* Following the procedure of synthesizing the optimal controller for GFM inverter, a 14th order $\mathcal{C}_{\mathcal{H}_\infty}(s)$ of Fig. 5 is found to be sufficient and performs well. The closed-loop stability and desired performances are met as summarized in Table II. The closed-loop model for GFM inverter with negative feedback loop transfer function with resulting controller, $\mathcal{C}_{\mathcal{H}_\infty}(s)$, in Fig. 5 can be derived by substituting $v_{inv}(s) = \mathcal{C}_{\mathcal{H}_\infty}(s)[v_{ref} - v_O]$ in (13). It can be written as $v_O = \mathcal{G}_{GFM}(s)v_{ref} - \mathcal{Z}_{GFM}(s)i_h$ and represented as Thevenin's equivalent model connected across a current source as shown in Fig. 7(d). For an example, at nominal plant condition with resulting optimal controller, the Bode plot of $\mathcal{G}_{GFM}(s)$ and $\mathcal{Z}_{GFM}(s)$ are shown in Fig. 10(a) and (b), respectively. It is observed that $|\mathcal{G}_{GFM}(j\omega_N)|$, $\angle\mathcal{G}_{GFM}(j\omega_N)$ and $|\mathcal{Z}_{GFM}(j\omega_N)|$ are ≈ 1 , $\approx 0^\circ$, and ≈ 0 , respectively, which leads to $v_O \approx v_{ref}$ at fundamental frequency.

V. EXPERIMENTAL RESULTS AND VERIFICATION

A. Experimental Configuration

A combined system-in-the-loop (SIL), controller hardware-in-the-loop (CHIL), and power hardware-in-the-loop (PHIL) based experimental validation is conducted in order to evaluate the efficacy and viability of the proposed μ -synthesis-based controller for single-phase GFL and GFM inverters. The ratings and parameters of the inverter systems are tabulated in Table III. Moreover, the efficiency curve of the inverter system under study operating at 240 V RMS is shown in Fig. 11. The laboratory-based experimental setup is shown in Fig. 12. The experimental configuration is shown in Fig. 13.

TABLE III
1-PHASE INVERTER SYSTEM UNDER STUDY

Inverter	Value
Ratings (1- ϕ)	240 V, 60 Hz, 1.67 kVA, 0.9 pf
Inverter Parameters	$V_{dc} = 500$ V, $f_{sw} = 20$ kHz
Filter Parameters	$L_f = 2$ mH, $R_f = 0.2$ Ω , $C_f = 20$ μ F

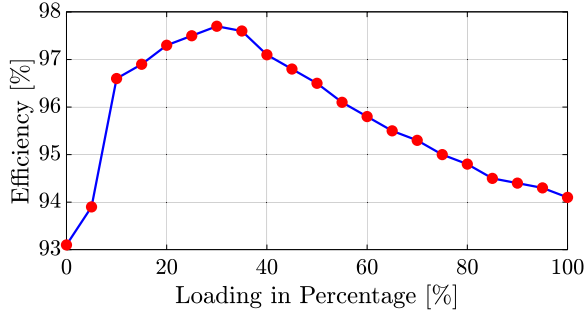


Fig. 11. Efficiency curve of the 1- ϕ , 60 Hz, 1.67 kVA inverter under study operating at 240 V RMS.

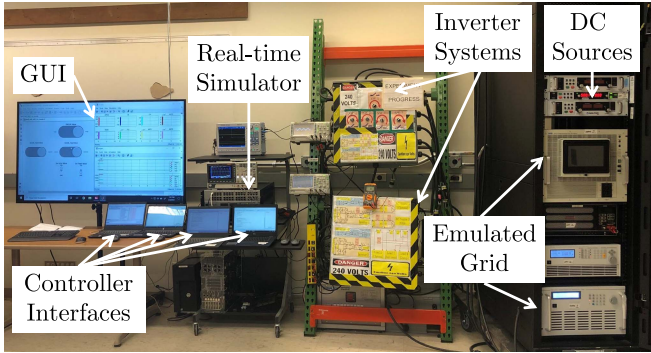


Fig. 12. Figure of the laboratory-based experimental hardware setup.

1) *Real-Time Simulation and SIL Configuration*: A residential subnetwork of North American low-voltage distribution feeder from CIGRE Task Force C6.04.02 [61], affiliated with CIGRE Study Committee C6 is emulated using eMEGASIM platform inside the OP5700 RT-simulator (RTS) manufactured by OPAL-RT. The original ratings of load at each bus and line parameters are modified in order to make it compatible with the voltage rating and power capacity of the laboratory. Moreover, the test system is modified by including sufficient nonlinear loads at various buses while respecting the recommended limits of harmonic distortions mentioned in [60]. Usually real-time simulation applied to the domain of power engineering is classified into two categories: 1) fully real-time simulation [e.g., system-in-the-loop (SIL)], and 2) hardware-in-the-loop (HIL) real-time simulation. A fully real-time simulation (or SIL) requires the entire system (including control, protection, and other accessories) to be modeled inside the simulator and does not involve external interfacing or inputs/outputs [62]. As part of SIL-setup, one GFM and one GFL inverter are emulated entirely (i.e., both power circuit and the control with proposed μ -synthesis-based controller) inside the RTS, connected at Bus₁ and Bus₁₂, respectively, as shown in Fig. 13.

2) *Controller Hardware-in-the-Loop Configuration*: In the CHIL configuration, the interface block diagram is shown in Fig. 14(a). This stage of validation is used in this work for rapid controller prototyping with the proposed μ -synthesis based robust controller for both GFL and GFM inverter systems. As part of CHIL-setup, one GFM and one GFL inverter system are emulated with only power circuit inside the RTS, connected at Bus₁₀ and Bus₁₄, respectively, as shown in Fig. 14(a). The proposed μ -synthesis-based control logic of both GFL and GFM inverter systems are realized on two Texas-Instruments TMS320F28379D, 16/12-bit floating-point 200-MHz Delfino microcontroller boards interfaced with RTS [c-HUT of Fig. 14(a)].

3) *Power Hardware-in-The-Loop Configuration*: In the PHIL configuration, the interface block diagram is shown in Fig. 14(b). This stage of validation is used in this work for real hardware validation with the proposed μ -synthesis-based robust controller for both GFL and GFM inverter systems. As part of PHIL-setup, one GFM (HUT-1 in Fig. 13) and one GFL inverter (HUT-2 in Fig. 13), connected at Bus₁₂ and Bus₁₀, respectively, are physically realized [p-HUT of Fig. 14(b)]. In HUT-1, the physical inverter system, fed by MAGNA-POWER programmable dc power supply (configured with user-defined $I - V$ characteristic of battery cell), is interfaced with low-cost Texas-Instruments TMS320F28379D, 16/12-bit floating-point 200 MHz Delfino microcontroller boards employed with proposed μ -synthesis control logic for GFM inverter with “Overload Mitigation Controller” based overcurrent protection scheme [63]. On the other hand, in HUT-2 the physical inverter system, fed by another MAGNA-POWER programmable dc power supply (configured with user-defined $I - V$ characteristic of solar photovoltaic array), is interfaced with another low-cost Texas-Instruments TMS320F28379D, 16/12-bit floating-point 200 MHz Delfino microcontroller boards employed with proposed μ -synthesis control logic for GFL inverter with “Instantaneous Current Limiter” based overcurrent protection scheme [64]. The ideal transformer model (ITM) based PHIL interface logic [65] is adapted for both the hardware-under-tests (HUTs’). The power terminals of the inverters are connected with a power amplifiers of Fig. 14(b). NHR 9410 regenerative grid simulator and Chroma 61605 programmable ac power source are used as the power amplifiers. The sensors to measure inverter output current as shown in Fig. 14(b) are realized by AEMC MN-260 current probe. Moreover, a diode-bridge rectifier based load, realized using a “Chroma 63800 Series AC-DC Electronic Load Bank,” is also considered as nonlinear load under test.

B. Results and Discussions

Four test cases (two test cases each for GFL and GFM inverters) are demonstrated by emulating a sequence of events. Two test cases for GFL inverters are as follows:

- CASE-1: The emulated distribution network of Fig. 13 is running in off-grid mode and P-Q reference of GFL inverters jumps up by 50% due to increased demand.

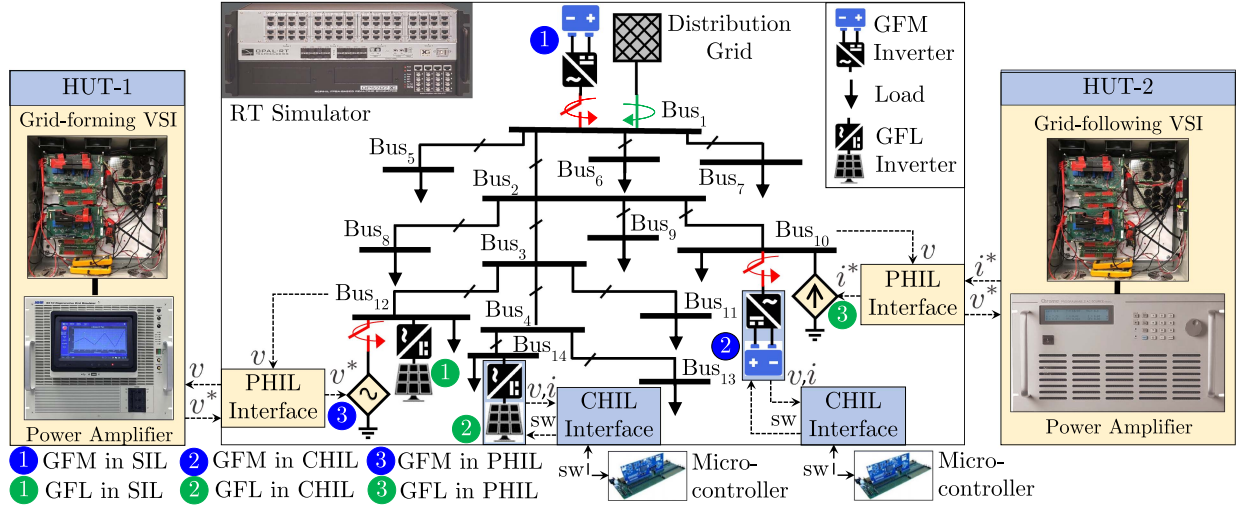


Fig. 13. Figure of the configuration of SIL-CHIL-PHIL-based experimental setup.

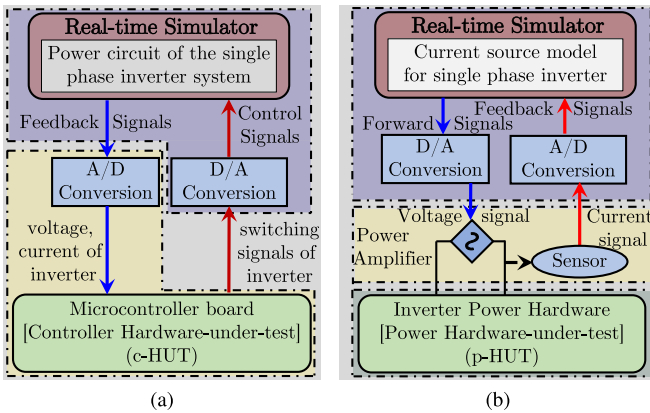


Fig. 14. Hardware-in-the-loop configuration and interfacing blocks for (a) controller hardware-in-the-loop and (b) power hardware-in-the-loop.

- CASE-2: The network is running in off-grid mode and experiences a topology change which results in 30% increase in equivalent Thevenin impedance at PCC of GFM inverters.
- CASE-3: The emulated distribution network of Fig. 13 has an on-grid to off-grid mode transition. GFM inverters will have a maximum jump in loading from no-load condition (during on-grid mode) to full-load condition (during off-grid mode).
- CASE-4: The same network has an off-grid to on-grid mode transition. GFM inverters will have another maximum jump in loading from full-load condition (during off-grid mode) to no-load condition (during on-grid mode).

Clearly, CASE-1, CASE-2, CASE-3, and CASE-4 are designed in order to capture the robust performance of proposed GFL and GFM control during maximal model uncertainty, respectively.

Fig. 15(a) and (b) shows the current reference tracking capability of the proposed μ -synthesis-based optimal controller for GFL inverter (at Bus₁₄ of Fig. 13) in CHIL demonstration of CASE-1 and CASE-2, respectively. RMS instantaneous current tracking error (rICTE in %), defined as $100 \times \text{RMS}(i_{\text{ref}} - i_o) / [\sqrt{2} \text{RMS}(i_{\text{ref}})]$, is used for assessing the tracking performance of the current controller. It is observed in Fig. 15(a)

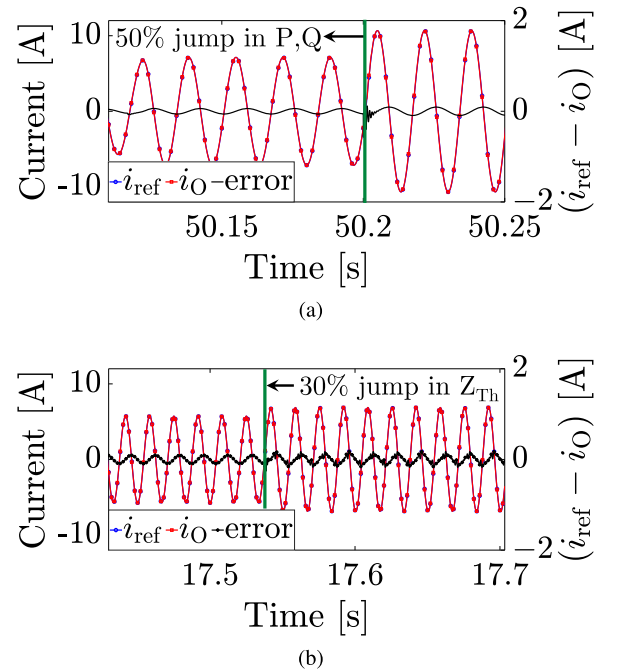


Fig. 15. Current reference tracking capability of GFL inverter with proposed μ -synthesis-based optimal controller in CHIL setup at Bus₁₄ of Fig. 13 in (a) CASE-1, (b) CASE-2.

that both the current reference and output current increases during 50% increase in P-Q setpoints due to adopted reference generation of Appendix A. The proposed optimal controller has significantly small error in current reference tracking before (rICTE $\approx 1.7\%$) and after (rICTE $\approx 1.9\%$) the transition in CASE-1. Similarly, Fig. 15(b) shows that the proposed optimal controller has significantly small error in current reference tracking before (rICTE $\approx 1.9\%$) and after (rICTE $\approx 2.1\%$) the jump of equivalent Thevenin impedance in CASE-2. Fig. 16 shows the current response of GFL inverter (HUT-2 at Bus₁₀ of Fig. 13) as a part of PHIL demonstration of the same event. Here the result is focused on determining the harmonic compensation

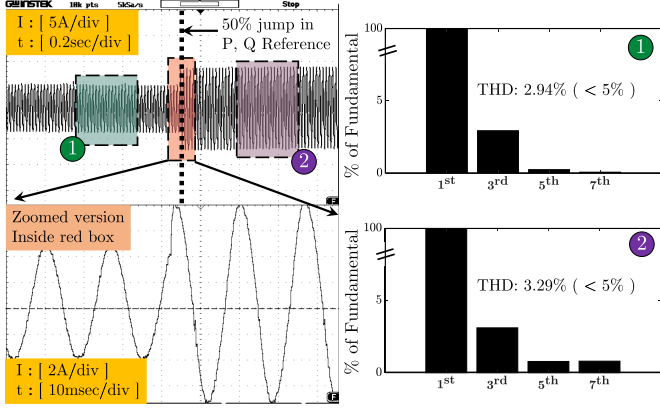


Fig. 16. Current reference tracking capability of GFL inverter in PHIL setup at Bus₁₀ of Fig. 13 in CASE-1 with the proposed optimal controller.

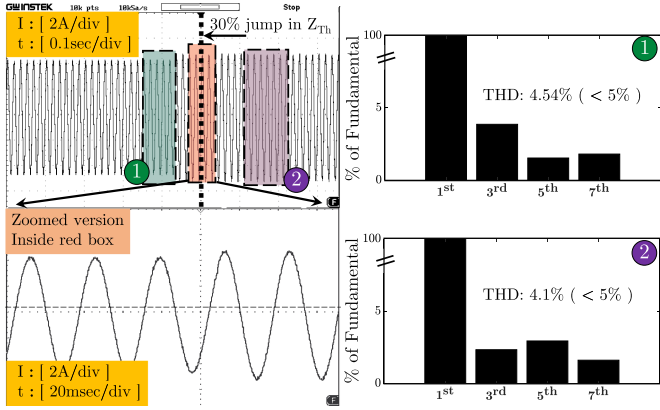


Fig. 17. Current reference tracking capability of GFL inverter in PHIL setup at Bus₁₀ of Fig. 13 in CASE-2 with the proposed optimal controller.

capability of the proposed optimal controller for GFL inverter during varying reference set-point. It is observed that the total demand distortion (TDD) of current waveform is $< 5\%$ before and after the transition as recommended in [60]. Thus, the proposed μ -synthesis-based controller for GFL inverter shows good reference tracking and harmonic compensation capability in CASE-1. Similarly, Fig. 17 shows the current response of GFL inverter (HUT-2 at Bus₁₀ of Fig. 13) as a part of PHIL demonstration of the same event. Here the result is focused on determining the harmonic compensation capability of the proposed μ -synthesis-based optimal controller for GFL inverter during model uncertainty. It is observed here that the TDD of current waveform is $< 5\%$ before and after the transition as recommended in [60]. Thus, the data corroborates the efficacy of the proposed μ -synthesis-based controller for GFL inverter for good reference tracking and harmonic compensation capability in CASE-2. The CHIL and PHIL results substantiate the fact that the proposed μ -synthesis-based optimal controller for GFL is showing robust performance by making sure to have good reference tracking, disturbance rejection, and harmonic compensation capability under significant in the plant model uncertainty while supplying reference active and reactive power as shown in Fig. 18(a) and (b).

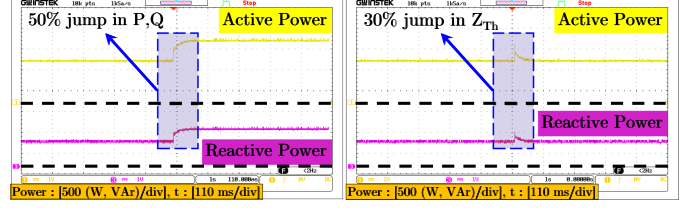


Fig. 18. Output active and reactive power of the GFL inverter with proposed μ -synthesis-based optimal controller in PHIL setup at Bus₁₀ of Fig. 13 in (a) CASE-1 (b) CASE-2.

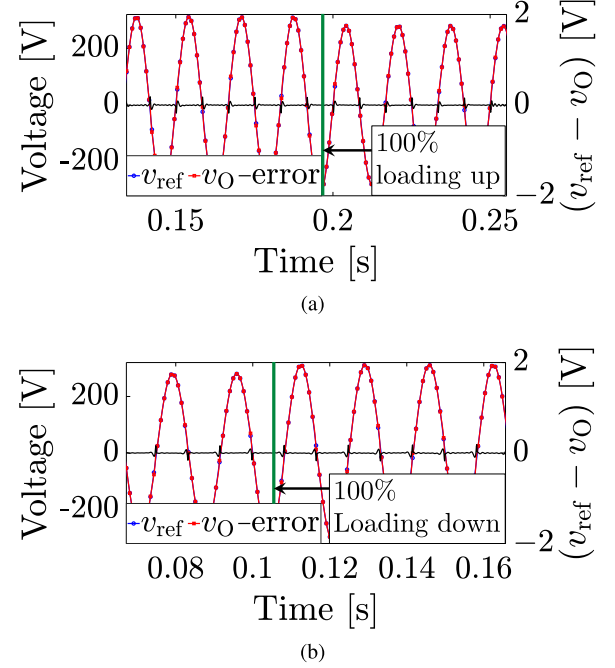


Fig. 19. Voltage reference tracking capability of GFM inverter with the proposed μ -synthesis-based optimal controller in CHIL setup at Bus₁₀ of Fig. 13 in (a) CASE-3 and (b) CASE-4.

Fig. 19(a) and (b) shows the voltage reference tracking capability of the proposed μ -synthesis-based optimal controller for GFM inverter (at Bus₁₀ of Fig. 13) in CHIL demonstration of CASE-3 and CASE-4, respectively. RMS instantaneous voltage tracking error (rIVTE in %), defined as $100 \times \text{RMS}(v_{\text{ref}} - v_o) / [\sqrt{2} \text{RMS}(v_{\text{ref}})]$, is used for assessing the tracking performance of the voltage controller. It is observed in Fig. 19(a) that both the voltage reference and output voltage drop during jump in loading (no-load to full-load) due to adopted droop-controlled reference generation of Appendix B. The proposed μ -synthesis-based optimal controller has significantly small error in voltage reference tracking as shown in Fig. 19(a) before (rIVTE $\approx 0.2\%$) and after (rIVTE $\approx 0.1\%$) the increase of equivalent loading. Similarly, it is observed in Fig. 19(b) that both the voltage reference and output voltage, increase during drop in loading (full-load to no-load). The proposed optimal controller has significantly small error in voltage reference tracking before (rIVTE $\approx 0.2\%$) and after (rIVTE $\approx 0.1\%$) the decrease in equivalent loading. Fig. 20 shows the voltage response of

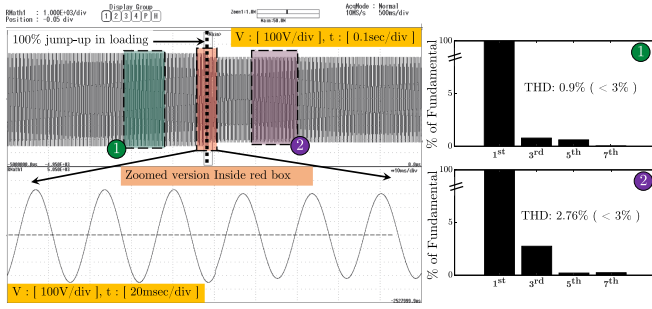


Fig. 20. Voltage reference tracking capability of GFM inverter in PHIL setup at Bus₁₂ of Fig. 13 in CASE-3 with the proposed optimal controller.

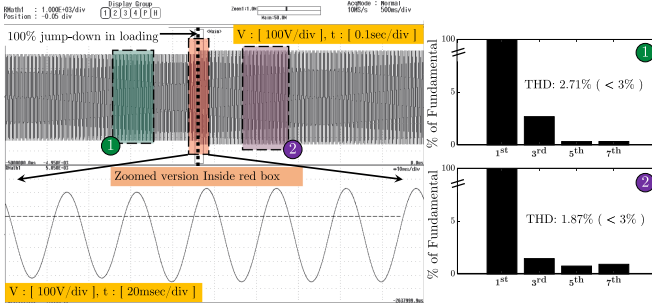


Fig. 21. Voltage reference tracking capability of GFM inverter in PHIL setup at Bus₁₂ of Fig. 13 in CASE-4 with the proposed optimal controller.

GFM inverter (HUT-1 at Bus₁₂ of Fig. 13) as a part of PHIL demonstration of the same event. Here the result is focused on determining the harmonic compensation capability of the proposed optimal controller for GFM inverter during model uncertainty change due to loading. It is observed that the total harmonic distortion (THD) of voltage waveform is $< 3\%$ before and after the transition. This is significantly less than the voltage distortion limit ($< 8\%$) as recommended in [60]. Thus, the data corroborated the advantage of the proposed μ -synthesis-based controller for GFM inverter shows robust performance during model uncertainty caused in CASE-3. Similarly, Fig. 21 shows the voltage response of GFM inverter (HUT-1 at Bus₁₂ of Fig. 13) as a part of PHIL demonstration of the same event. Here the result is focused on determining the harmonic compensation capability of the proposed optimal controller for GFM inverter during model uncertainty change due to loading. It is observed that the total harmonic distortion (THD) of voltage waveform is $< 3\%$ before and after the transition. This is significantly less than the voltage distortion limit ($< 8\%$) as recommended in [60]. Thus GFM inverter shows robust performance during model uncertainty caused in CASE-4. The CHIL and PHIL results substantiate the fact that the proposed optimal controller for GFM is showing robust performance by having good reference tracking, disturbance rejection, and harmonic compensation capability under significant in the plant model uncertainty while sharing the active and reactive power demand as shown in Fig. 22(a) and (b).

An experimental study on the inverter systems are conducted to validate the plug-and-play (PnP) performance of the inverter

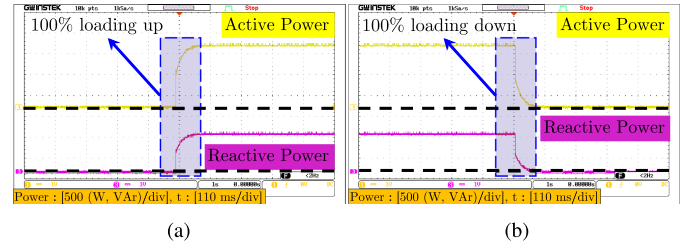


Fig. 22. Output active and reactive power of the GFM inverter with proposed μ -synthesis-based optimal controller in PHIL setup at Bus₁₂ of Fig. 13 in (a) CASE-3 (b) CASE-4.

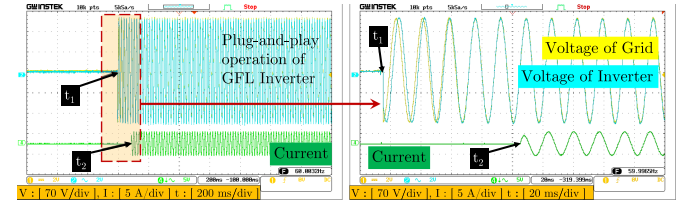


Fig. 23. Experimental results showcasing the plug-and-play performance of the GFL inverter with the proposed μ -synthesis-based current controller.

while operating in GFM mode with the proposed μ -synthesis-based optimal voltage controller and GFL mode with the proposed μ -synthesis-based optimal current controller. Accredited standards like [58], [66], [67] provide a basic guidelines for testing PnP capabilities of GFM and GFL inverter in a microgrid scenario. In PnP testing, the following two test scenarios are validated in the same test system for the inverters of HUT-1 and HUT-2:

- the performance during plugging-in of a GFL inverter system at the point-of-connection with the microgrid network after a proper synchronization and during successive start by injecting power into the microgrid network as requested,
- the performance during starting and then plugging-in of the GFM inverter at the point-of-connection with the loads in the microgrid network after transitioning from on-grid to off-grid mode.

Fig. 23 shows the voltage and current waveform of the GFL inverter (HUT-2) with the proposed μ -synthesis-based optimal current controller during the PnP operation. At time $t = t_1$, as highlighted in Fig. 23, the GFL inverter starts synchronization process with respect to the voltage waveform (yellow colored waveform) at point-of-connection and within 3-4 cycles the synchronization is achieved by aligning the inverter voltage (cyan colored waveform) with the grid voltage. At $t = t_2$, the GFL inverter is synchronized and plugged-in with the grid and starts injecting power into the network. Similarly, Fig. 24 shows the voltage and current waveform of the GFM inverter (HUT-1) with the proposed μ -synthesis-based optimal voltage controller during the PnP operation. At time $t = t_1$, as highlighted in Fig. 24, the GFM inverter starts from black-start and generates smooth and stable voltage waveform across its terminal (cyan colored waveform). It is observed that after having a ramp up time of 2-3 cycles, the GFM inverter generates a stable voltage across its terminal. At $t = t_2$ the GFM inverter is plugged-in

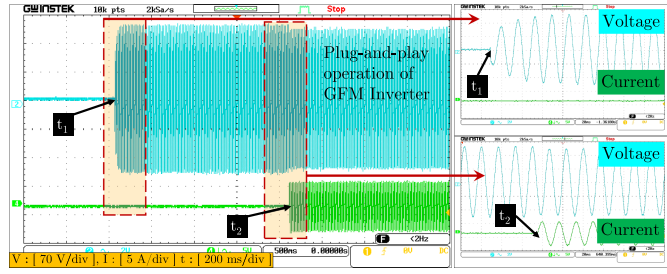


Fig. 24. Experimental results showcasing the plug-and-play performance of the GFM inverter with the proposed μ -synthesis-based voltage controller.

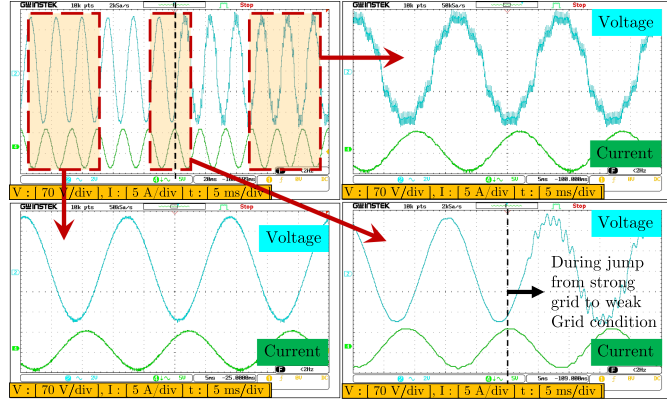


Fig. 25. Experimental results showcasing the performance of the GFL inverter with the proposed μ -synthesis-based optimal current controller while connected with weak grid network.

with the loads in the microgrid and starts injecting power into the network. These PnP operations are achieved in the absence of any sort of communication among the inverters and the microgrid as recommended in the standards.

Moreover, it is important to validate the performance of the GFL inverter with the proposed μ -synthesis-based optimal current controller while the inverter is interfaced with a weak grid. An experimental study is conducted on the GFL inverter (HUT-2) when it is interfaced with a weak grid, having low short-circuit ratio ($SCR < 2$) emulated by OPAL-RT and programmable voltage source. Fig. 25 shows the voltage and current waveform of the GFL inverter. During this experiment, the point-of-connection of the emulated grid has a transition from strong grid to weak grid condition as observed in the voltage waveform of the grid (the cyan colored waveform). It is observed here that the current output of the GFL inverter with the proposed optimal controller have less impact on the controller performance during, before, and after the transition from strong grid to weak grid condition. This corroborates the fact that the proposed μ -synthesis-based current controller is robust enough in its performance while experiencing a wide range of variation in grid impedance.

C. Performance Comparison

To showcase the advantages of the proposed μ -synthesis-based controller on robust performance, a CHIL-based performance comparison study is conducted with two controllers of the

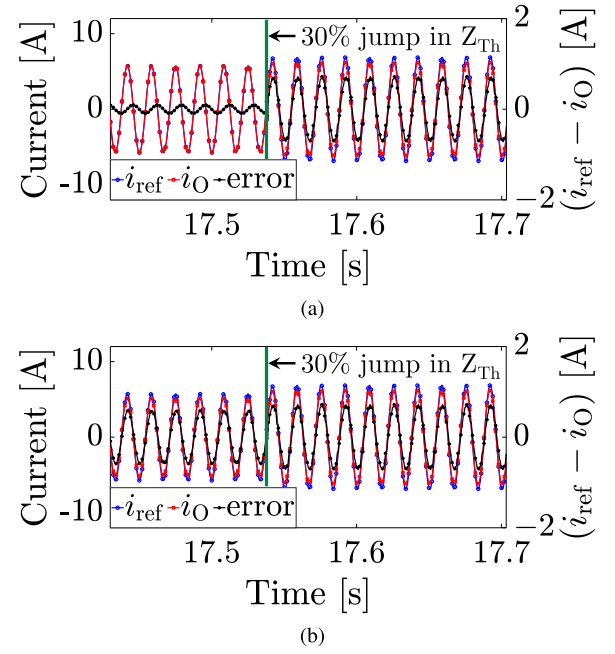


Fig. 26. Performance comparison of the current reference tracking capability of GFL inverter in CHIL setup at Bus₁₄ of Fig. 13 with, (a) $\mathcal{H}_\infty$ -based controller without considering uncertainty of Thevenin impedance, (b) the classical PR controller.

existing literature. The existing current controllers used in this study for comparison purposes are as follows: 1) conventional classical proportional+resonant (PR)-based controller that can be found in many commercial grid-following inverter systems, and 2) an nominal $\mathcal{H}_\infty$ -based current controller considering a fixed grid impedance modeling (i.e., modeling of uncertainty imposed by the variation of the grid impedance is not considered). This method is adopted from the existing work of [29]. Similarly, the existing voltage controllers used in this study for comparison purposes are as follows: 1) conventional classical PR-based controller that can be found in many commercial grid-forming inverter systems, and 2) an nominal $\mathcal{H}_\infty$ -based voltage controller considering a fixed equivalent loading modeling (i.e., the modeling of the uncertainty imposed by variation of equivalent loading is not considered). This method is adopted from the existing work of [43]. Fig. 26(a) and (b) shows the current reference tracking capability of the $\mathcal{H}_\infty$ -based and PR-based current controller for GFL inverter (at Bus₁₄ of Fig. 13) in CHIL demonstration of CASE-2, respectively. It is observed that the nominal $\mathcal{H}_\infty$ -based robust controller has significant error in current reference tracking (rICTE increases from $\approx 2.5\%$ to $\approx 10\%$) after 30% jump in equivalent Thevenin impedance from nominal value. It is observed that the PR-based controller has comparatively larger error both before (rICTE $\approx 6\%$) and after (rICTE $\approx 10\%$) the transition of CASE-2. In comparison, the proposed μ -synthesis-based optimal controller has significantly small error in current reference tracking before (rICTE $\approx 1.9\%$) and after (rICTE $\approx 2.1\%$) the jump of equivalent Thevenin impedance in CASE-2 as shown in Fig. 15(b). Similarly, Fig. 27(a) and (b) shows the voltage reference tracking capability of the $\mathcal{H}_\infty$ -based and PR-based

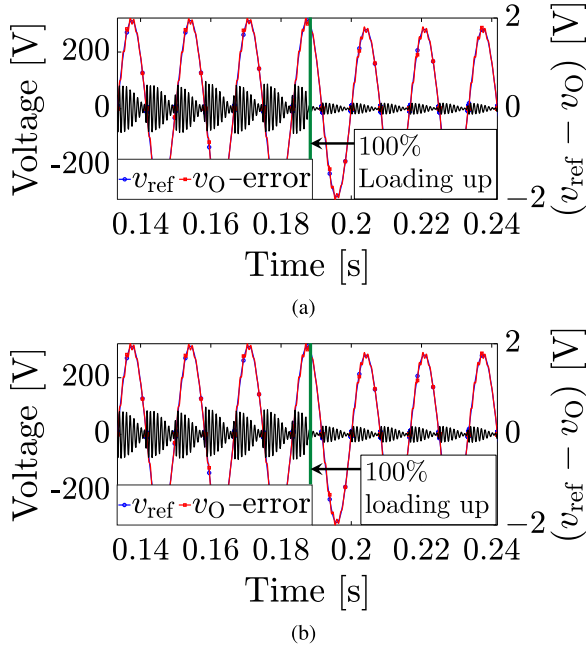


Fig. 27. Performance comparison of the voltage reference tracking capability of GFM inverter in CHIL setup at Bus₁₀ of Fig. 13 with, (a) $\mathcal{H}_\infty$ -based controller without considering uncertainty of equivalent loading, (b) the classical PR controller.

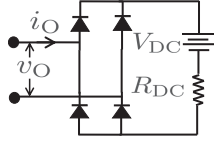


Fig. 28. Schematic of the diode-bridge-based nonlinear load.

voltage controller for GFM inverter (at Bus₁₀ of Fig. 13) in CHIL demonstration of CASE-3, respectively. It is observed that the nominal $\mathcal{H}_\infty$ -based robust controller has significant error in voltage reference tracking (rIVTE $\approx 1.2\%$) at no-load condition. The PR-based controller has comparatively larger error both before (rIVTE $\approx 1.4\%$) and after (rIVTE $\approx 0.8\%$) the transition of CASE-3. In comparison, the proposed μ -synthesis-based optimal controller has significantly small error in voltage reference tracking before (rIVTE $\approx 0.2\%$) and after (rIVTE $\approx 0.1\%$) the increase of equivalent loading as shown in Fig. 19(a).

Moreover, an experimental study is conducted to compare the dynamic and steady-state performance of the proposed μ -synthesis-based optimal voltage controller for GFM inverter under diode-based nonlinear load with respect to the existing nominal $\mathcal{H}_\infty$ -based robust controller and classical PR-based controller. The diode bridge rectifier-based nonlinear load of the form as shown in Fig. 28 is considered in this study. Figs. 29, 30, and 31 show the voltage and current waveform of the GFM inverter with the proposed μ -synthesis-based, nominal $\mathcal{H}_\infty$ -based, and classical PR-based voltage controller. It is observed that the GFM inverter is able to maintain a smooth and steady voltage waveform across the nonlinear load leveraged by the enhanced reference tracking and disturbance rejection capability of the proposed voltage controller in comparison with the existing

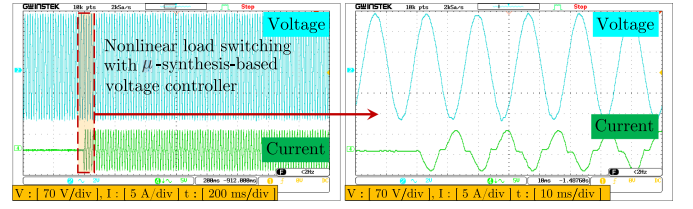


Fig. 29. Experimental results showcasing the dynamic and steady-state performance of the proposed μ -synthesis-based optimal voltage controller employed in the grid-forming inverter terminated at diode-bridge-based nonlinear load.

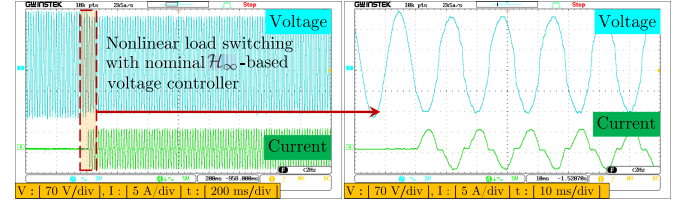


Fig. 30. Experimental results showcasing the dynamic and steady-state performance of the existing nominal $\mathcal{H}_\infty$ -based voltage controller employed in the grid-forming inverter terminated at diode-bridge-based nonlinear load.

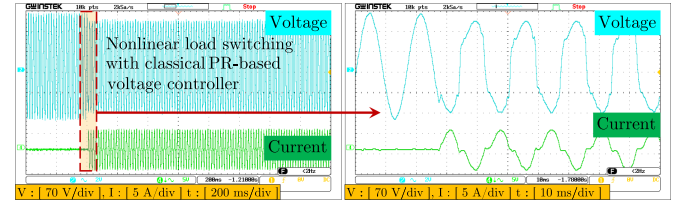


Fig. 31. Experimental results showcasing the dynamic and steady-state performance of the existing PR-based voltage controller employed in the grid-forming inverter terminated at diode-bridge-based nonlinear load.

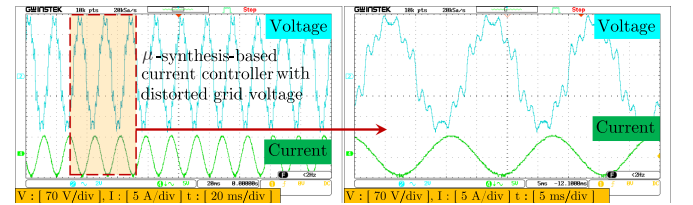


Fig. 32. Experimental results showcasing the performance of the proposed μ -synthesis-based optimal current controller employed in the grid-following inverter terminated at grid with distorted voltage.

voltage controllers. The change in the plant model (plant uncertainty) due to the presence of nonlinear load across the GFM inverter has comparatively less impact on the performance of the proposed voltage controller compared to the existing controllers as observed in the results. The total harmonic distortion (THD) of the voltage waveform are $\approx 2.65\%$, $\approx 9.1\%$, and $\approx 11.7\%$ for proposed μ -synthesis-based, nominal $\mathcal{H}_\infty$ -based, and PR-based voltage controllers, respectively, after the nonlinear load transition.

Similarly, an experimental study is conducted to compare the steady-state performance of the proposed μ -synthesis-based optimal current controller for GFL inverter under distorted grid voltage condition with respect to the existing nominal $\mathcal{H}_\infty$ -based robust controller and classical PR-based controller. Figs. 32, 33,

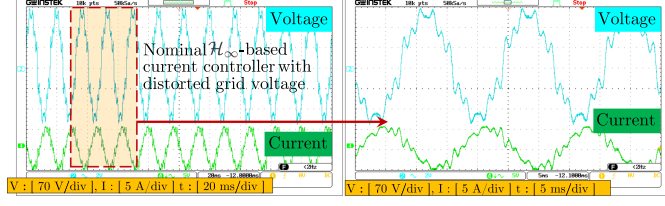


Fig. 33. Experimental results showcasing the performance of the existing nominal $\mathcal{H}_\infty$ -based current controller employed in the grid-following inverter terminated at grid with distorted voltage.

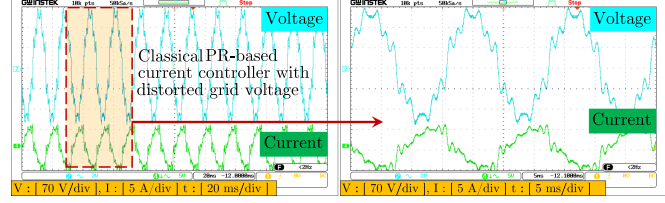


Fig. 34. Experimental results showcasing the performance of the existing PR-based current controller employed in the grid-following inverter terminated at grid with distorted voltage.

and 34 show the voltage and current waveform of the GFL inverter with the proposed μ -synthesis-based, nominal $\mathcal{H}_\infty$ -based, and classical PR-based current controller. It is observed that the GFL inverter is able to generate and inject a fairly smooth and steady sinusoidal current into the grid while interfaced with a distorted grid voltage. The proposed controller is able to nullify the impact of the distortion of the grid voltage on the current output, leveraged by the enhanced reference tracking and disturbance rejection capability of the proposed current controller under large variation of grid impedance and grid voltage distortion in comparison with the existing current controllers. The total demand distortion (TDD) of current waveform is $\approx 3.4\%$, $\approx 7.5\%$, and $\approx 9.4\%$ for proposed μ -synthesis-based, nominal $\mathcal{H}_\infty$ -based, and PR-based current controller, respectively, with the distorted grid voltage.

VI. CONCLUSION

In this article, a generalized μ -synthesis-based robust control framework is proposed utilizing the fact that there is a voltage-current duality in the plant dynamic model of GFL and GFM inverter. The uncertainties in grid impedance parameters and uncertainties in equivalent loading parameters for GFL and GFM inverters are modeled, respectively. The generalized control framework results the controllers that are single-loop, hence simple and cost-effective enough to be implemented, and optimal, in the sense of robustness in performance under uncertainties. The resulting current-controller for GFL inverter provides inherent active damping under grid parameter variation whereas the resulting voltage-controller for GFM inverter enhances the dynamic performance during load transients. A SIL-CHIL-PHIL-based experimental validation evaluates the efficacy and viability of the proposed controllers. Comparison study with some existing works showcase the advantages of the proposed controller.

APPENDIX

A. Reference Generation for Grid-Following Inverter

The outer “Reference Generation Block” of Fig. 2(a) eventually generates i_{ref} signal using its prespecified reference active power P_{ref} , and reactive power Q_{ref} , (defined locally/centrally) and output signals from phase-locked loop (PLL). The expression of i_{ref} is given by

$$i_{\text{ref}} = \sqrt{2} \frac{\sqrt{P_{\text{ref}}^2 + Q_{\text{ref}}^2}}{\tilde{V}} \sin \left(\tilde{\theta} - \arctan \frac{Q_{\text{ref}}}{P_{\text{ref}}} \right) \quad (27)$$

where a 1- ϕ second order generalized integrator-based synchronous reference frame PLL (SOGI-SRF-PLL) operates with its grid-synchronization technique and generates the RMS value $\tilde{V}$, and synchronized phase information $\tilde{\theta}$ of v_o [68].

B. Reference Generation for Grid-forming Inverter

A P - f/Q - V droop control strategy is adopted for outer “Reference Generation Block” of Fig. 2(b). The droop characteristic equations are as follows [69]:

$$\omega = \omega_N - n_P P, \quad V = V_N - m_Q Q \quad (28)$$

$$v_{\text{ref}} = \sqrt{2} V \sin \left(\int \omega dt \right) \quad (29)$$

where ω_N and V_N are nominal frequency (in rad/s) and voltage (RMS), respectively. P and Q are averaged active and reactive power output of GFM inverter. n_P and m_Q are the droop coefficients and the values are typically chosen such that ω and V are within the allowed specification, defined by IEEE 1547 Standard [58], for all $P \in [0, P_{\text{rated}}]$ and $Q \in [-Q_{\text{rated}}, Q_{\text{rated}}]$, respectively. Here, P_{rated} and Q_{rated} are the rated active and reactive power of the GFM inverter.

C. Shaping Transfer Functions

$$\begin{aligned} \mathcal{W}_S &= \frac{5.97 \times 10^9}{(s + 5278)^2} \times \frac{s^2 + 314s + (2\pi 60)^2}{s^2 + 31.4s + (2\pi 60)^2} \\ &\quad \times \frac{s^2 + 1.2 \times 10^4 s + 1.5 \times 10^8}{s^2 + 245s + 1.5 \times 10^8} \\ \mathcal{W}_{CS} &= 100 \times \frac{s + 2\pi 60}{s + 1500 \times 2\pi 60} \\ \mathcal{W}_d &= \frac{7539.8}{s + 7540} \times \frac{s^2 + 628.1s + (2\pi 60)^2}{s^2 + 31.4s + (2\pi 60)^2} \\ &\quad \times \frac{s^2 + 314s + 9 \times (2\pi 60)^2}{s^2 + 31.4s + 9 \times (2\pi 60)^2} \\ &\quad \times \frac{s^2 + 314s + 25 \times (2\pi 60)^2}{s^2 + 31.4s + 25 \times (2\pi 60)^2} \\ &\quad \times \frac{s^2 + 314s + 49 \times (2\pi 60)^2}{s^2 + 31.4s + 49 \times (2\pi 60)^2}. \end{aligned}$$

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